SEMINAR UNIFORM SPACES 1975 - 76

Injectivity of polyhedra

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This small note is no discovery - we put it in just for the completeness of the whole text in this publication. We show the possibly known statement that each finite dimensional polyhedron is an injective uniform space. We have not investigated polyhedra of infinite dimension. For the definition of uniform polyhedra see [I].

Statement: Each finite dimensional polyhedron P is injective.

Proof: By Isbell's result, [I], p. 82, P is an uniform absolute neighbourhood retract. Let us embed P "canonically" into the 1-ball B of $\ell_\infty(A)$. Let $i: P \longrightarrow B$ be the embedding. Again by Isbell, [I], p. 42, the ball B is injective and so we have a mapping j of a closed ε-neighbourhood H of P onto P such that $ji = id_P$. As any retract of an injective space is injective as well it suffices to find a retract $r: B \longrightarrow H^{\varepsilon}$. Define a mapping $\varphi : B \longrightarrow R$ such that $\varphi(f) = \max\{k \mid k \in R, k.f \in H^{\varepsilon}\}$. This mapping is uniform because, by an easy computation, if $\sup|f - g| < \alpha$ then $|\varphi(f) - \varphi(g)| < \alpha\, \varepsilon^{-1}$. So we can put $r(f) = (\min(\varphi(f),1)).f$ and the proof is complete.

References

[F] Z. Frolík" Four functors into paved spaces, SUS 1973-74, Publ. Math. Inst., Prague 1975

[I] J. Isbell: Uniform spaces, AMS 1964