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CURVE RECONSTRUCTION FROM A SET OF MEASURED POINTS

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Abstract: In this article, a method of cubic spline curve fitting to a set of points passing at a prescribed distance from input points obtained by measurement on a coordinate measuring machine is described. When reconstructing the shape of measured object from the points obtained by real measurements, it is always necessary to consider measurement uncertainty (tenths to tens of micrometres). This uncertainty is not zero, therefore interpolation methods, where the resulting curve passes through the given points, do not lead to acceptable results in practice. Also, conventional B-spline approximation methods cannot be used because, for real distances between measured points (tenths to units of millimetres), the distance of the input data from the resulting approximation curve is much greater than the measurement uncertainty considered. The proposed reconstruction method allows to control the maximum distance of the resulting curve from the input data and thus to respect the uncertainty with which the input data was obtained.

Keywords: metrology, coordinate measuring machine, reverse engineering, uncertainty, interpolation, approximation, least squares method

MSC: 65N15, 65M15, 65F08

1. Introduction

The coordinate measuring machine (CMM) is a device used for measuring specific 3D elements and their surfaces to inspect their declared properties. The result of such procedure is a set of discrete points measured on a surface of an object with a probe. Currently, the most commonly used probes are mechanical, optical, laser and white light. Depending on the type of CMM, the probe position can be manually controlled using a handbox with a joystick or it can be controlled by a computer. Computer controlling is mostly used for calibration procedures and physical standards calibrations [3] and [6]. In this case, the position of the probe is constantly compared with the Computer Aided Design (CAD) model and adjusted accordingly.

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This CAD model must be as accurate as possible to maintain a specified accuracy of CMM that ranges from tenths to tens of μm. For example, data used in the following computations was obtained by measuring on CMM Zeiss Xenos with the length measurement error $E0 = 0.3 + L/1000$ μm, where L is the measured length in m, [5] and [8]. Currently, CAD models of basic geometrical elements are Non-Uniform B-Spline (NURBS) [4] representations of these surfaces – i.e. the CAD model of a sphere is a NURBS representation of the sphere fitted to the measured points by the least squares method (LSM). In the case of so-called freeform surfaces (i.e. surfaces whose shapes do not belong to the group of fundamental geometrical 3D objects such as plane, sphere, cone, etc. and with very difficult analytical expression), this approach does not work. Currently, this task of dimensional metrology is often solved with a technology called the reverse engineering (RE) that combines methods of computer graphics, numerical mathematics and statistics. The result is a CAD model of the measured surface where the shape sufficiently accurate with respect to the application considered.

In this paper, 2D situation is considered – i.e. the process of shape fitting is reduced to planar curves. In the following computations, let us consider planar points $[x, z]$ the points of intersection of a measured surface with the vertical plane. Cartesian x- and z-coordinates of these points are the real measurands and the y-coordinates are rounded as a constant. In Section 2, NURBS curves commonly used in CAD systems are presented and one specific example shows their comparison. Section 3 describes our new method of creating a new kind of CAD model of the curve passing through the measured points in a prescribed distance. In Section 4, two practical examples of this proposed method are demonstrated and their comparison with approximation cubic curve widely used in CAD systems is presented. Section 5 summarizes achieved results and suggests possible applications of this proposed method for shape reconstruction of the surface. All data used in the following computations was obtained by measuring of Czech Metrology Institute (CMI) freeform standard Hyperbolic paraboloid on CMM Zeiss Xenos [7].

2. Modelling of plane curve

For reconstructing the shape of a curve, two approaches can be considered. If we know the kind of a curve (e.g. a line or a parabola) and this curve is not transcendental – i.e. it has a polynomial representation, we can use numerical LSM for computing the required coefficients and the result is an approximation of these points by the polynomial of a given degree. In the case of an unknown curve shape, this approach is impossible and using well known computer graphics methods – approximation and interpolation by a segmented spline cubic curve is more appropriate. Approximation by the clamped uniform B-spline cubic curve (clamped cubic) [1] is a classic and simple approximation method usually used in CAD systems that are based on NURBS representation and then require data in a form of control points. Using another kind of approximation, open Coons B-spline cubic curve, is complicated, because this model is not implemented into CAD systems due to its user-unfriendly features. The position of each knot point of this segmented curve (including the starting point and the end point) depends on 3 control points – i.e. the system does not start drawing the curve until the 3 points have been entered. The problem can be solved as follows: Each segment of this open Coons cubic B-spline can be defined as a Bézier curve, and because these segments are C^2 continuous, this curve can be considered a clamped cubic (with new computed control points) and can be displayed in CAD systems. The simplest solution is an interpolation of given points by a segmented cubic curve. A widely used interpolation by the so called natural spline cubic is formed by C^2 continuous segments of Ferguson cubics. If the software works with another type of an interpolation spline, such as the Rhinoceros used here, it is necessary to convert this spline into the clamped cubic again as for the previous Coons cubic B-spline.

Due to uncertainty of measurement, the situation becomes more difficult, because we have to suppose that the given position of each point is not exact. The real point lies in a circular surrounding of a measured point, where the value of the radius of this circular surrounding is taken from $E0$ of CMM (0.3 μm in our case). The task is to find such a CAD model of the curve that preserves this uncertainty in terms of keeping the distance of the curve from the given points. All three curves mentioned above are depicted in Fig. 1.

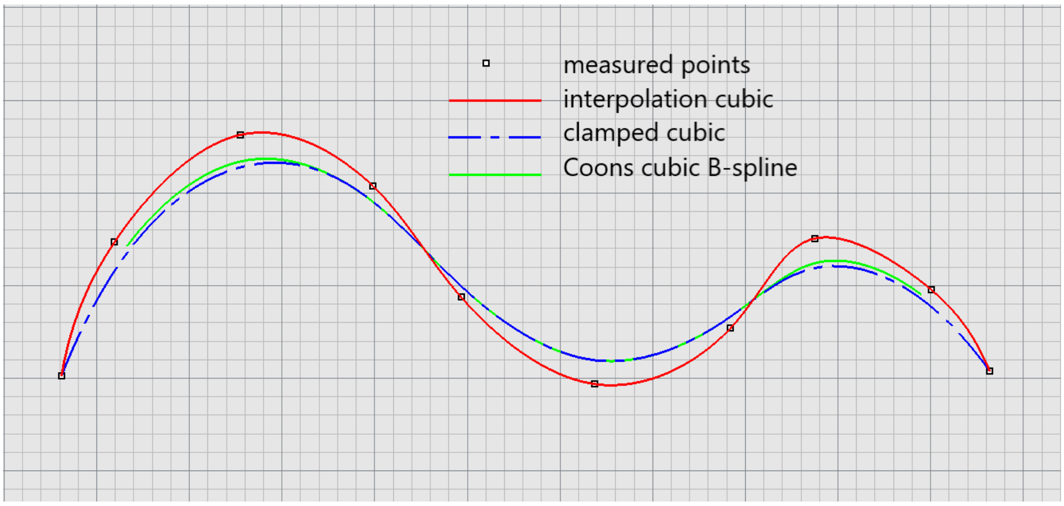

Figure 1: Interpolation cubic, clamped cubic and Coons cubic B-spline.

The interpolation curve goes exactly through the given points; both approximating curves copy the shape of the control polygon, but their distances from the control points are too large. A specific example of 14 measured points is shown in Fig. 2, the coordinates of these points are given in Table 1. The corresponding clamped cubic and Coons cubic B-spline curves were modelled in Rhinoceros. A figure of the approximation curves is not included because the differences between these two curves are not visible and both curves look like the interpolation curve (using the same scale as in Fig. 2). The numerical values of basic statistics for the comparison of these two curves are shown in Table 2. The distance between the curve and a point is measured on a normal line to the curve passing through the given point. These statistics were obtained from statistical analysis tools implemented in Rhinoceros.

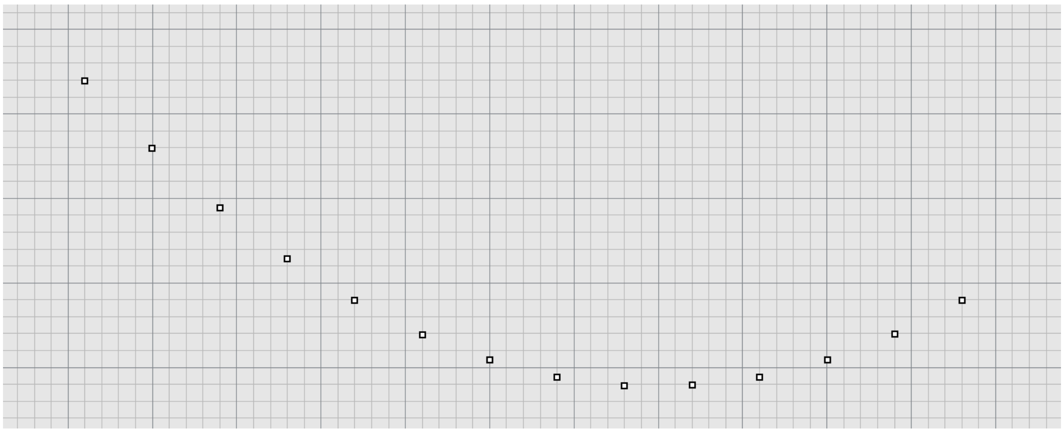

Figure 2: Distribution of 14 measured points.

x	28.0146	24.0136	20.0130	16.0106	12.0058	8.0019	4.0018
z	28.9639	26.9577	25.4530	24.4476	23.9388	23.9376	24.4413
x	0.0046	-3.9951	-7.9970	-11.9979	-16.0016	-20.0030	-24.0063
z	25.4445	26.9470	28.9493	31.4450	34.4486	37.9510	41.9538

Table 1: Coordinates of 14 measured points (in mm).

	Clamped cubic	Coons B-spline
Average distance	0.0821	0.0758
Median distance	0.0808	0.0779
Standard deviation	0.0091	0.0077
Maximum distance	0.0162	0.0844
Minimum distance	0.0695	0.0606

Table 2: Basic statistics of distances obtained from Rhinoceros (in mm).

3. Construction of a new approximation segmented cubic curve

In the new method, both approaches mentioned above are incorporated. Firstly, measured points are interpolated by the segmented cubic spline. At each knot point (i.e. each measured point), the normal line to this curve is computed and a new pair of points on each normal line at the distance of 0.3 μm is constructed. Then, the selection of one point from this pair is made as follows: The original points are approximated with third-degree polynomial

$$P(t) = at^3 + bt^2 + ct + d, \;\; t \in [x_1, x_n],$$

where n is the number of points and coefficients a, b, c, d are computed by LSM (the use of the cubic curve is a sufficient solution of approximation in computer graphics and it is also the simplest one). At each point $[x_i, z_i]$, the value of $P(x_i)$ is compared with z_i. In the case when $P(x_i) > z_i$, the new point on the normal line with the smaller z-coordinate is chosen; for $P(x_i) < z_i$ the second point is chosen. Finally, these new chosen points are interpolated by a spline segmented cubic curve, see Fig. 3; the radius of all circular surroundings is increased to make them visible.

This is not the only solution, but this "close enough" (referred to as CE from now on) curve certainly meets the required condition of the prescribed distance between the points and the curve. Due to the way the new points are selected, this curve can describe the detailed shape of a measured curve more precisely than an interpolation curve.

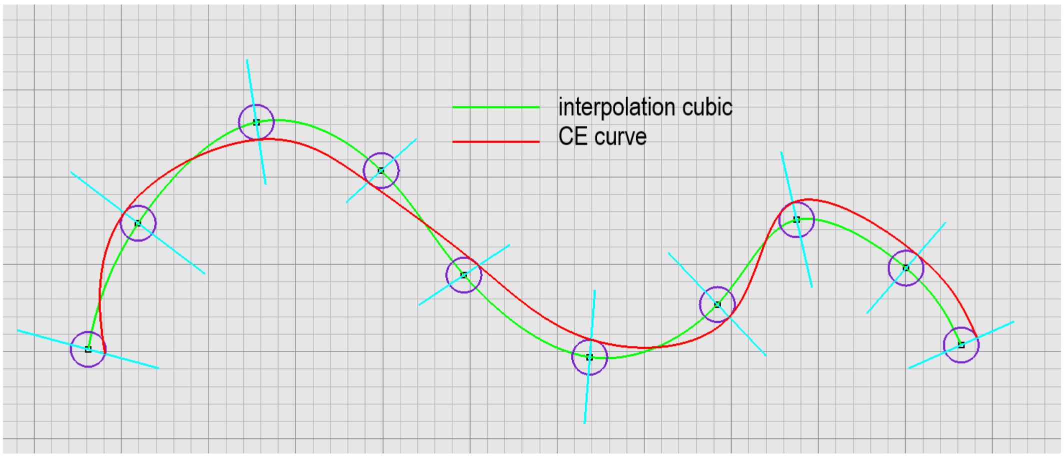

Figure 3: Construction of CE curve.

4. Practical examples and comparisons of results

All calculations were computed by Maple software and all statistics were taken from the software Rhinoceros.

Considering the 14 measured points from Fig. 2, coefficients of the approximation cubic curve given by LSM are presented in Table 3 and an illustrative image of CE curve is shown in Fig. 4 (the circle surroundings are enlarged).

a	b	c	d
0.00000088	0.01563206	-0.31291765	25.44435929

Table 3: Coefficients of the approximating polynomial $P(t)$.

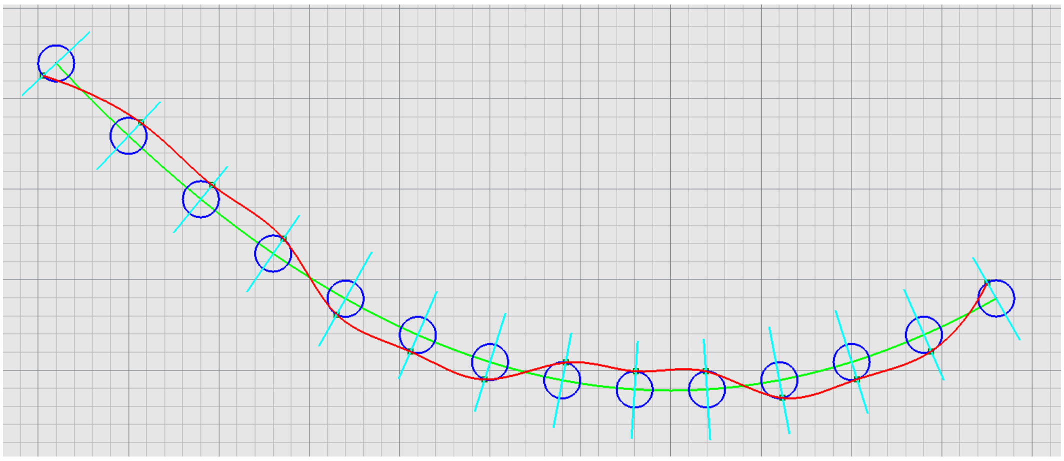

Figure 4: CE curve given by 14 points.

The results of the diagnostics of this CE curve and both classical approximation curves are presented in Table 4. This CE curve fully complies with the condition of uncertainty and is applicable as a detailed CAD model of the curve shape.

	Clamped cubic	Coons B-spline	CE curve
Average distance	0.0821	0.0758	0.0003
Median distance	0.0808	0.0779	0.0003
Standard deviation	0.0091	0.0077	0.0000
Maximum distance	0.0162	0.0844	0.0003
Minimum distance	0.0695	0.0606	0.0003

Table 4: Basic statistics of distances obtained from Rhinoceros (in mm).

Shape reconstruction will be more accurate with more points available. However, regarding the complexity of the mathematical model of a given surface, sometimes it is necessary to reduce the number of the measured points used for the reconstruction. Comparison of the properties of CE curves obtained by reduction of given points is performed on the following model example. 57 measured points and CE 57 curve are

Figure 5: CE curve given by 57 points.

	Clamped 57	Clamped 29	Clamped 15
Average distance	0.0048	0.0193	0.0789
Median distance	0.0048	0.0196	0.0814
Standard deviation	0.0004	0.0016	0.0089
Maximum distance	0.0055	0.0217	0.0873
Minimum distance	0.0038	0.0135	0.0375
	CE 57	CE 29	CE 15
Average distance	0.0003	0.0005	0.0024
Median distance	0.0003	0.0003	0.0007
Standard deviation	0.0000	0.0007	0.0044
Maximum distance	0.0003	0.0051	0.0175
Minimum distance	0.0003	0.0000	0.0000

Table 5: Basic statistics of distances obtained from Rhinoceros (in mm).

shown in Fig. 5. In the x-axis direction, the points are about 1 mm apart. CE 29 and CE 15 curves given by a reduced number of the points were computed, in the case of CE 29, every second point was removed and in the case of CE 15, every fifth point was retained. In all three cases, a clamped cubic curve was modelled and the distances between the inner 55 points and curves were obtained from Rhinoceros. Basic statistics of these distances are summarized in Table 5 and a detailed distance distribution of the clamped cubic curves and CE curves are shown in Fig. 6 and Fig. 7.

The results obtained from CE 29 and CE 15 curves are not significant as from CE 57 but an improvement could be made, for example, by adjusting the distance of the new control points. Comparing these CE curves with clamped curves, CE curves give better results than clamped curves, even when the number of the measured points is reduced.

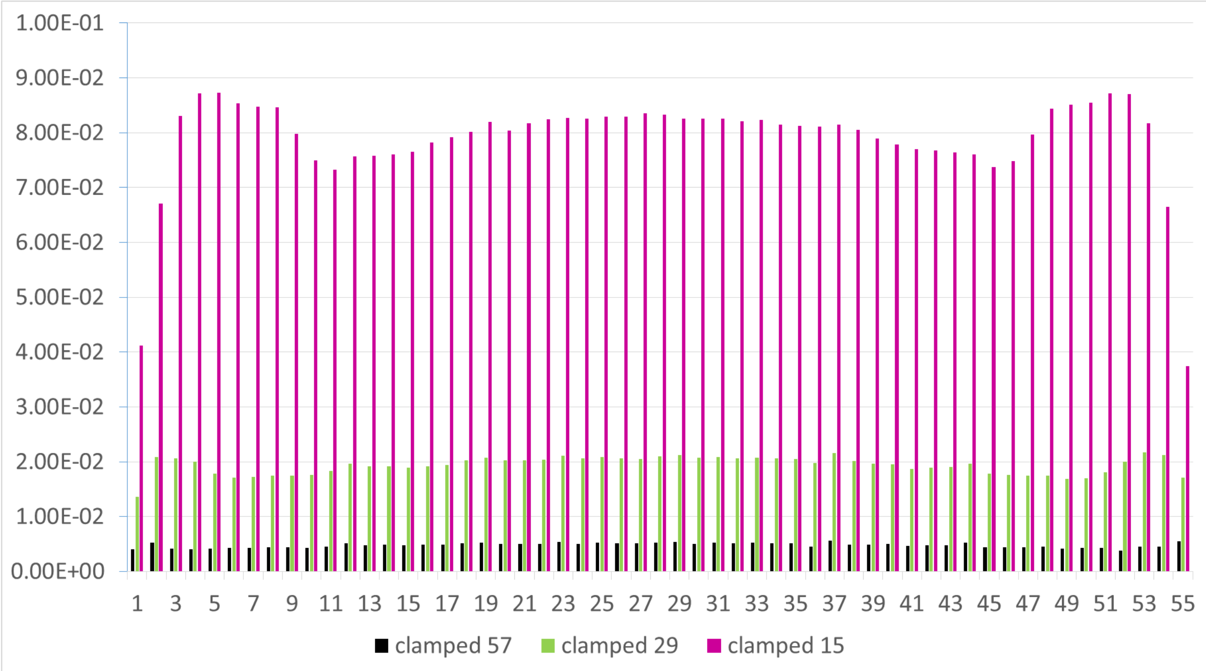

Figure 6: Distribution of distances – clamped cubics.

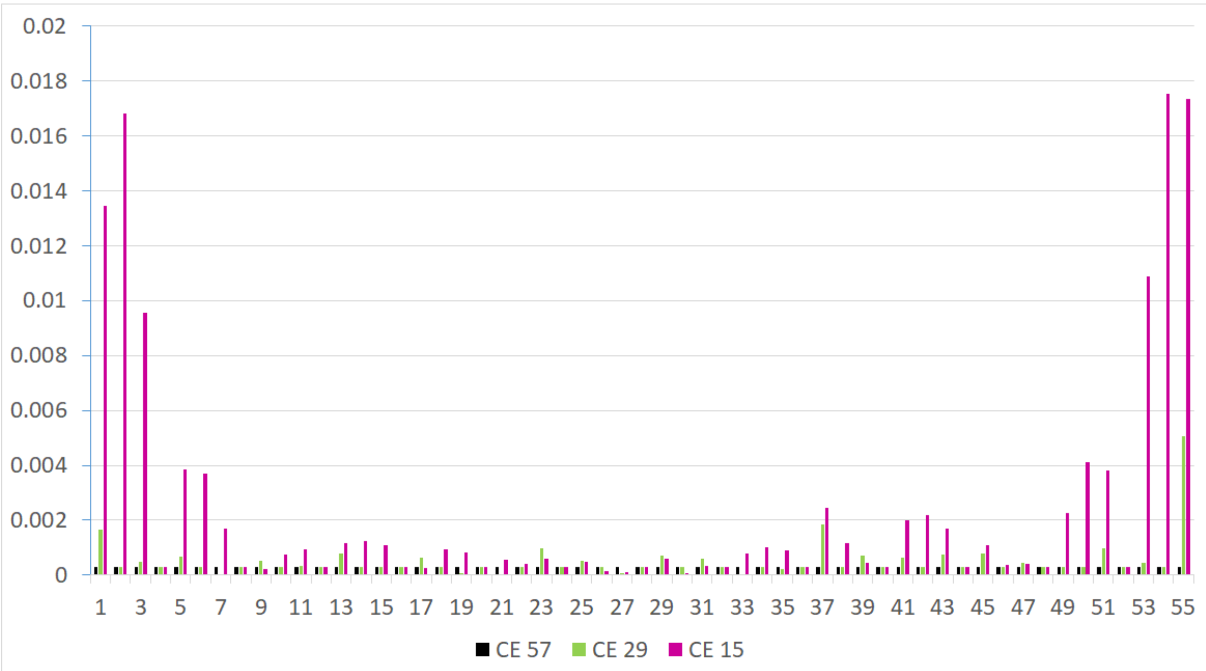

Figure 7: Distribution of distances – CE curves.

5. Conclusion

This contribution is focused on a detailed reconstruction of a planar curve from a set of measured points considering the uncertainty of measurement. The method to accomplish the required curve shape reconstruction including both the approximation and the interpolation approach is described here. The case of a planar curve does not have much use in metrological practice, but it is the first basic step to solve the same problem concerning surfaces [2]. In future work, a procedure to obtain a detailed CAD model of a surface from a set of measured points will be proposed. This method will certainly find great use in the processes of calibration and measurements on freeform surfaces.

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