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WAVELET METHOD FOR OPTION PRICING UNDER THE TWO-ASSET MERTON JUMP-DIFFUSION MODEL

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Abstract: This paper examines the pricing of two-asset European options under the Merton model represented by a nonstationary integro-differential equation with two state variables. For its numerical solution, the wavelet-Galerkin method combined with the Crank-Nicolson scheme is used. A drawback of most classical methods is the full structure of discretization matrices. In comparison, the wavelet method enables the approximation of discretization matrices with sparse matrices. Sparsity is essential for the efficient application of iterative methods in solving the resulting systems and the efficient computation of the matrices arising from the discretization of integral terms. To illustrate the efficiency of the method, we provide the results of numerical experiments concerning a European option on the maximum of two assets.

Keywords: Merton model, wavelet-Galerkin method, integro-differential equation, spline wavelets, Crank-Nicolson scheme, sparse matrix, option pricing

MSC: 65T60, 65M60, 47G20, 60G51

1. Introduction

Thus far, many models for option pricing have been proposed, including the famous Black-Scholes model and various stochastic volatility models. These models assume that the price of the underlying asset is a continuous function in time, which is not always consistent with the behaviour of real market prices. Therefore, several models that assume that jumps can occur in the price have been designed. This paper focuses on a jump-diffusion model, namely the Merton model, for two underlying assets. This model is represented by a nonstationary partial integro-differential equation (PIDE) with two state variables.

Several numerical schemes for the deterministic two-asset Merton model have already been studied, e.g. the time discretization schemes in [2], the finite difference method in [6], and the wavelet-Galerkin method featuring logarithmic prices in [7].

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Let us also mention that the wavelet-Galerkin method for one-asset jump-diffusion models, including the Merton model and the Kou model, was studied in [3] and [4], where the main advantages of the method were sparse and well-conditioned matrices, higher-order convergence, and a small number of parameters needed to represent the solution with the desired accuracy. These promising results are the motivation for the study of the wavelet-Galerkin method for the two-asset Merton model. However, there are several challenges to overcome. First, the method leads to four-dimensional integrals. Second, the standard methods such as the finite difference method, the finite element method, and the Galerkin method with splines, used for PIDEs typically lead to full matrices. On the contrary, the wavelet-Galerkin method used to discretizate the PIDEs leads to matrices that can be approximated by sparse matrices efficiently. Moreover, the differential operator contained in the equation is degenerate, and the initial function is not smooth.

2. The two-asset Merton model

The Merton model designed in [8] assumes that the price S^i_τ of the ith asset at time τ follows the jump-diffusion process

$$S^i_\tau = S^i_0 \exp\left(\left(r - \frac{\sigma_i^2}{2} - \lambda\kappa_i\right)\tau + \sigma_i W^i_\tau + \sum_{k=1}^{N_\tau} Y^i_k\right), \quad i = 1, 2. \tag{1}$$

The parameter r represents a risk free interest rate, and σ_i is the volatility of the asset i corresponding to the diffusion part of the process. The processes W^1_τ and W^2_τ are two Wiener processes with correlation coefficient ρ. Furthermore, N_τ is the Poisson process, where the intensity λ represents the number of jumps. The random variables Y^i_k are independent identically distributed random variables for fixed i, Y^1_k and Y^2_k occur simultaneously, and their correlation coefficient is $\hat{\rho}$. The parameter κ_i is the expected relative jump size, i.e. $\kappa_i = \mathrm{E}(\exp(Y^i_k) - 1)$.

The equation (1) describes a general jump-diffusion process. The concrete model is obtained by specifying the distribution of the random variables Y^i_k. In the Merton model, it is assumed that $Y_k = (Y^1_k, Y^2_k)$ is bivariate normally distributed, which implies that the vector e^{Y_k} has a log-normal distribution with a density

$$f(y_1, y_2) = \frac{K}{y_1 y_2} \exp\left(-\frac{\left(\frac{\ln y_1 - \gamma_1}{\delta_1}\right)^2 + \left(\frac{\ln y_2 - \gamma_2}{\delta_2}\right)^2 - 2\hat{\rho}\left(\frac{\ln y_1 - \gamma_1}{\delta_1}\right)\left(\frac{\ln y_2 - \gamma_2}{\delta_2}\right)}{2\left(1 - \hat{\rho}^2\right)}\right), \tag{2}$$

where $K = 1/2\pi\delta_1\delta_2\sqrt{1 - \hat{\rho}^2}$.

Under these assumptions and using no arbitrage principle and Itô calculus, the deterministic model for the price of an option can be derived, see [8]. Let T be the maturity date, $t = T - \tau$ be time to maturity, S_i be the price of the asset i, and U be the function such that $U\left(S_1, S_2, t\right)$ represents the value of the option for asset prices S_1 and S_2 and time to maturity t. Then U can be computed as the solution of the partial integro-differential equation

$$\frac{\partial U}{\partial t} - \mathcal{L}_D(U) - \mathcal{L}_I(U) = 0, \quad S_1 > 0,\ S_2 > 0,\ 0 < t \leq T, \tag{3}$$

where $\mathcal{L}_D$ is a degenerate elliptic operator defined as

$$\begin{aligned} \mathcal{L}_D(U) = & \frac{\sigma_1^2 S_1^2}{2} \frac{\partial^2 U}{\partial S_1^2} + \rho \sigma_1 \sigma_2 S_1 S_2 \frac{\partial^2 U}{\partial S_1 \partial S_2} + \frac{\sigma_2^2 S_2^2}{2} \frac{\partial^2 U}{\partial S_2^2} \\ & + (r - \lambda \kappa_1) S_1 \frac{\partial U}{\partial S_1} + (r - \lambda \kappa_2) S_2 \frac{\partial U}{\partial S_2} - (r + \lambda) U, \end{aligned} \tag{4}$$

and $\mathcal{L}_I$ is an integral operator given by

$$\mathcal{L}_I(U) = \lambda \int_0^\infty \int_0^\infty U(S_1 y_1, S_2 y_2, t) f(y_1, y_2)\, \mathrm{d}y_1\, \mathrm{d}y_2. \tag{5}$$

The equation (3) must be accompanied by appropriate initial and boundary conditions, which depend on the option type.

As an example, we consider a European put option on the maximum of two assets. This option gives its holder the right, but not the obligation, to sell the maximum of two underlying assets at the strike price K at expiry T. In this case, the initial condition representing the value of the option at maturity is $U(S_1, S_2, 0) = \max(K - \max(S_1, S_2), 0)$. Note that this function does not have a first-order derivative on some parts of its domain.

We localize the equation (3) to a bounded domain $\Omega = (0, S) \times (0, S)$, where S is a sufficiently large value, and we denote the parts of the boundary $\partial\Omega$ of the domain Ω as

$$\Gamma_1 = \{(S_1, 0), S_1 \in (0, S)\},\ \Gamma_2 = \{(0, S_2), S_2 \in (0, S)\},\ \Gamma_3 = \partial\Omega \backslash (\Gamma_1 \cup \Gamma_2). \tag{6}$$

Since the differential operator is degenerate on Γ_1 and Γ_2, no boundary conditions are prescribed there, see [1].

Below, we consider only European put options, because the values for call options can be computed using the put-call parity, see [1]. Since the value of a put option is negligible when the price S_1 or S_2 is very large, we set $U(S_1, S_2, t) = 0$ for $(S_1, S_2, t) \in \Gamma_3 \times (0, T)$.

Furthermore, in alignment with [2, 7], we define a region of interest

$$\mathrm{ROI} = \left[\frac{K}{2}, \frac{3K}{2}\right] \times \left[\frac{K}{2}, \frac{3K}{2}\right], \tag{7}$$

and we focus on the approximation of the option value in this region.

As mentioned above, the differential operator $\mathcal{L}_D$ is degenerate. This degeneracy is found in many other differential and integro-differential equations representing option pricing problems. The mathematical literature presents two ways to deal with this issue. The first method consists of using a variational formulation directly for an equation with a degenerate differential operator, making it necessary to use weighted Sobolev spaces and complicating the mathematical analysis of the problem and the process used to find its numerical solution. However, in this case, it is often possible to impose relatively simple boundary conditions in a manner similar to that used above. This approach was studied in [1], but only for some models represented by differential equations and has not yet been used for the multi-asset Merton model.

The second method employs substitution into logarithmic prices, which typically has a significant advantage that the transformed differential operator is elliptic and contains constant coefficients. Thus, standard Sobolev spaces are used for the discretization, and the analysis is quite standard. The disadvantage is that the substitution leads to an unbounded domain, which has to be approximated by a bounded domain, and the prescription of appropriate boundary conditions is a delicate task. This approach has been studied in various papers but has been used mostly for PDE models. It has been used for PIDEs in, e.g. [7].

This paper studies the first approach. Let $\langle \cdot, \cdot \rangle$ and $\|\cdot\|$ denote the inner product and the norm of the space $L^2(\Omega)$, respectively. Define the Hilbert space

$$V = \left\{ v \in L^2(\Omega) : S_1 \frac{\partial v}{\partial S_1} \in L^2(\Omega), S_2 \frac{\partial v}{\partial S_2} \in L^2(\Omega) \right\} \tag{8}$$

endowed with the inner product

$$\langle u, v \rangle_V = \langle u, v \rangle + \left\langle S_1 \frac{\partial u}{\partial S_1}, S_1 \frac{\partial v}{\partial S_1} \right\rangle + \left\langle S_2 \frac{\partial u}{\partial S_2}, S_2 \frac{\partial v}{\partial S_2} \right\rangle. \tag{9}$$

The seminorm is defined as

$$|v|_V = \sqrt{\left\langle S_1 \frac{\partial v}{\partial S_1}, S_1 \frac{\partial v}{\partial S_1} \right\rangle + \left\langle S_2 \frac{\partial v}{\partial S_2}, S_2 \frac{\partial v}{\partial S_2} \right\rangle}, \tag{10}$$

and this seminorm is, in fact, a norm, see [1]. For $u, v \in V$, the bilinear form a_D corresponding to the differential term is defined as

$$\begin{aligned} a_D(u, v) =& \frac{\sigma_1^2}{2} \left\langle S_1 \frac{\partial u}{\partial S_1}, S_1 \frac{\partial v}{\partial S_1} \right\rangle - \rho \sigma_1 \sigma_2 \left\langle S_1 \frac{\partial u}{\partial S_1}, S_2 \frac{\partial v}{\partial S_2} \right\rangle \\ &+ \frac{\sigma_2^2}{2} \left\langle S_2 \frac{\partial u}{\partial S_2}, S_2 \frac{\partial v}{\partial S_2} \right\rangle + \left(\sigma_1^2 - r + \lambda \kappa_1\right) \left\langle S_1 \frac{\partial u}{\partial S_1}, v \right\rangle \\ &+ \left(\sigma_2^2 - r + \lambda \kappa_2\right) \left\langle S_2 \frac{\partial u}{\partial S_2}, v \right\rangle + (r + \lambda) \langle u, v \rangle . \end{aligned} \tag{11}$$

Since the integral term is defined over the unbounded domain, it is convenient to approximate it in the following way

$$\begin{aligned}
\mathcal{L}_I(U) =&\lambda \int_0^\infty \int_0^\infty U(S_1 y_1, S_2 y_2, t)\, f(y_1, y_2)\, \mathrm{d}y_1\, \mathrm{d}y_2 \\
=&\lambda \int_0^\infty \int_0^\infty \frac{U(z_1, z_2, t)}{S_1 S_2} f\left(\frac{z_1}{S_1}, \frac{z_2}{S_2}\right) \mathrm{d}z_1\, \mathrm{d}z_2 \\
\approx&\lambda \int_0^S \int_0^S \frac{U(z_1, z_2, t)}{S_1 S_2} f\left(\frac{z_1}{S_1}, \frac{z_2}{S_2}\right) \mathrm{d}z_1\, \mathrm{d}z_2.
\end{aligned} \tag{12}$$

Then the bilinear form a_I corresponding to the integral term is defined as

$$a_I(u, v) = \lambda \int_0^S \int_0^S \int_0^S \int_0^S \frac{u(z_1, z_2)}{S_1 S_2} f\left(\frac{z_1}{S_1}, \frac{z_2}{S_2}\right) v(S_1, S_2)\, \mathrm{d}z_1\, \mathrm{d}z_2\, \mathrm{d}S_1\, \mathrm{d}S_2. \tag{13}$$

Lemma 1. *The bilinear form $a : V \times V \to \mathbb{R}$ defined as $a = a_D - a_I$ is continuous and satisfies the Gårding inequality, i.e. there exist constants $C_1 > 0$ and $C_2 \geq 0$ such that, for any $v \in V$,*

$$a(v, v) \geq C_1 |v|_V^2 - C_2 \|v\|^2.$$

Proof. The proof of the continuity and the Gårding inequality for the bilinear form a_D follows the lines of the proofs of Lemma 2.9 and Lemma 2.10 in [1]. Due to the fact that the kernel of the integral operator in (13) is nonnegative, it holds that $a_I(v, v) \leq C \|v\|^2$, which implies the continuity and the validity of the Gårding inequality for the bilinear form a. □

Let the symbol V' denote the dual space of V. The variational formulation of the equation (3) reads as: For given $U_0 \in L^2(\Omega)$, find $U \in L^2(0, T; V)$ such that $\frac{\partial U}{\partial t} \in L^2(0, T; V')$ and

$$\left\langle \frac{\partial U}{\partial t}, v \right\rangle + a(U, v) = 0 \quad \forall v \in V, \text{ a.e. in } (0, T);\ U(\cdot, \cdot, 0) = U_0. \tag{14}$$

Theorem 2. *There exists a unique solution U of the variational problem (14).*

Proof. This theorem is a direct consequence of Lemma 1, for details on the theory concerning the Gårding inequality and the existence and uniqueness of variational problems, see, e.g. [1, 7]. □

3. Wavelet basis

Since the spatial discretization is based on using wavelets as basis functions, we briefly review the definition of a wavelet basis and provide a concrete example.

Let $\mathcal{J}$ be an index set such that $\lambda \in \mathcal{J}$ takes the form $\lambda = (j, k)$ and let $|\lambda| = j$ denote the level.

Definition 3. *A wavelet basis of a Hilbert space H is defined as the family $\Psi = \{\psi_\lambda, \lambda \in \mathcal{J}\}$ satisfying the following conditions:*

(i) *The family Ψ is a Riesz basis for H.*

(ii) *The functions are local, i.e. diam supp $\psi_\lambda \leq C2^{-|\lambda|}$ for all $\lambda \in \mathcal{J}$.*

(iii) *A wavelet basis has the hierarchical structure*

$$\Psi = \Phi_{j_0} \cup \bigcup_{j=j_0}^{\infty} \Psi_j, \quad \Phi_{j_0} = \{\phi_{j_0,k}, k \in \mathcal{I}_{j_0}\}, \quad \Psi_j = \{\psi_{j,k}, k \in \mathcal{J}_j\}. \tag{15}$$

(iv) *Wavelets have vanishing moments, i.e.*

$$\int_{supp\, \psi_{j,k}} p(x)\ \psi_{j,k}(x)\, \mathrm{d}x = 0, \quad k \in \mathcal{J}_j, \tag{16}$$

for any polynomial p of degree less than L, where $L \geq 1$ is dependent on the type of wavelet.

Furthermore, the functions $\phi_{j_0,k}$ are called scaling functions, and the functions $\psi_{j,k}$ are called wavelets.

The concept of a wavelet basis is not unified in the literature, and some of the conditions $i) - iv)$ are generalized or omitted in other papers.

As an example, we present a cubic spline-wavelet basis from [5], which will be also used in the numerical experiments presented in Section 5.

The scaling basis is formed by the standard quadratic B-splines. Let ϕ, ϕ_{b1}, and ϕ_{b2} be quadratic B-splines defined on knots $[0, 1, 2, 3]$, $[0, 0, 0, 1]$, and $[0, 0, 1, 2]$, respectively. For the explicit forms of these functions see [5]. For $j \geq 2$ and $x \in [0, S]$, we set

$$\begin{aligned}
\phi_{j,1}(x) =& 2^{j/2}\phi_{b1}(2^j x/S), \quad \phi_{j,2}(x) = 2^{j/2}\phi_{b2}(2^j x/S), \\
\phi_{j,k}(x) =& 2^{j/2}\phi(2^j x/S - k + 3), k = 3, ..., 2^j, \\
\phi_{j,2^j+1}(x) =& 2^{j/2}\phi_{b2}(2^j(1 - x/S)).
\end{aligned} \tag{17}$$

Then we define a wavelet ψ and boundary wavelets ψ_b and ψ_b^D as

$$\begin{aligned}\psi(x) =& -\frac{1}{4}\phi(2x)+\frac{3}{4}\phi(2x-1)-\frac{3}{4}\phi(2x-2)+\frac{1}{4}\phi(2x-3), \\ \psi_b(x) =& -\phi_{b1}(2x)+\frac{13\phi_{b2}(2x)}{12}-\frac{37\phi(2x)}{72}+\frac{\phi(2x-1)}{8}, \\ \psi_b^D(x) =& -\frac{\phi_{b2}(2x)}{4}+\frac{47\phi(2x)}{120}-\frac{13\phi(2x-1)}{40}+\frac{\phi(2x-2)}{10}.\end{aligned} \tag{18}$$

The graphs of these wavelets are displayed in Figure 1.

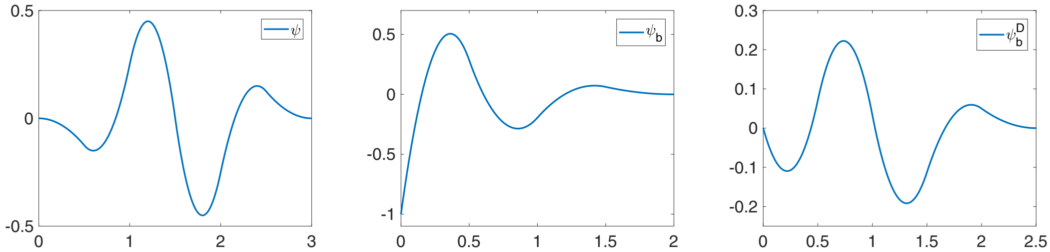

Figure 1: The wavelet ψ, and the boundary wavelets ψ_b and ψ_b^D.

It was proved in [5] that these wavelets have three vanishing moments and the shortest possible support among all quadratic B-spline wavelets of the same type. For $j \geq 2$ and $x \in [0, S]$, we define

$$\begin{aligned}\psi_{j,k}(x) =& 2^{j/2}\psi(2^j x/S - k + 2), k = 2, \ldots, 2^j - 1, \\ \psi_{j,1}(x) =& 2^{j/2}\psi_b(2^j x/S), \quad \psi_{j,2^j}(x) = 2^{j/2}\psi_b^D(2^j(1 - x/S)).\end{aligned} \tag{19}$$

We set $\Phi_j = \{\phi_{j,k}, k = 1, \ldots, 2^j + 1\}$ and $\Psi_j = \{\psi_{j,k}, k = 1, \ldots, 2^j\}$.

The wavelet basis Ψ on Ω satisfying the homogeneous Dirichlet boundary conditions on Γ_3 is obtained via the (isotropic) tensor product, i.e.

$$\Psi = (\Phi_2 \otimes \Phi_2) \cup \bigcup_{j=2}^{\infty} (\Phi_j \otimes \Psi_j \cup \Psi_j \otimes \Phi_j \cup \Psi_j \otimes \Psi_j). \tag{20}$$

The following theorem was proved in [5].

Theorem 4. *The set Ψ is a wavelet basis of the space $L^2(\Omega)$, and $\psi_\lambda(x) = 0$ for all $x \in \Gamma_3$ and for all $\psi_\lambda \in \Psi$.*

4. Wavelet-Galerkin method

Let Ψ be a wavelet basis for the space $L^2(\Omega)$ containing wavelets with $L \geq 1$ vanishing moments and let Ψ be a basis for V. Let $\Psi^k \subset \Psi$ be a multiscale basis containing scaling functions at the coarsest level j_0 and wavelets up to level k. Furthermore, set $X_k = \operatorname{span} \Psi^k$ and denote the dual of X_k as X_k'.

Let $U_0 \in L^2(\Omega)$, $U_{k,0}$ be an approximation of U_0 in X_k, and let X_k' be the dual space for X_k. Then, the wavelet-Galerkin formulation reads as: Find $U^k \in L^2(0,T;X_k)$ such that $\frac{\partial U}{\partial t} \in L^2(0,T;X_k')$ and for all $v_k \in X_k$ and a.e. in $(0,T)$

$$\left\langle \frac{\partial U_k}{\partial t}, v_k \right\rangle + a(U_k, v_k) = 0, \quad U_k(\cdot,\cdot,0) = U_{k,0}. \tag{21}$$

Theorem 5. *There exists a unique solution of the semidiscrete equation (21).*

Proof. This theorem is also a consequence of Lemma 1. □

For the temporal discretization, we use the Crank-Nicolson scheme. Let $M \in \mathbb{N}$, $\tau = T/M$, $t_l = l\tau$, $l = 0,\ldots,M$, $U_k^l(S_1,S_2) = U_k(S_1,S_2,t_l)$. The scheme has the following form: for all $v_k \in X_k$,

$$\frac{\left\langle U_k^{l+1}, v_k \right\rangle}{\tau} - \frac{\left\langle U_k^{l}, v_k \right\rangle}{\tau} + \frac{a\left(U_k^{l+1}, v_k\right)}{2} + \frac{a\left(U_k^{l}, v_k\right)}{2} = 0, \quad U_k^0 = U_{k,0}. \tag{22}$$

Writing $U_k^l = \sum_{\psi_\lambda \in \Psi^k} \left(\mathbf{c}_k^l\right)_\lambda \psi_\lambda$ and setting $v_k = \psi_\mu$ for $l = 0,\ldots,M-1$, we get the system $\mathbf{A}^k \mathbf{c}_k^{l+1} = \mathbf{f}_k^l$, where $\mathbf{A}^k = \mathbf{G}^k - \mathbf{K}^k$,

$$\mathbf{G}^k_{\mu,\lambda} = \frac{\langle \psi_\lambda, \psi_\mu \rangle}{\tau} + a_D(\psi_\lambda, \psi_\mu), \quad \psi_\lambda, \psi_\mu \in \Psi^k, \tag{23}$$

$$\mathbf{K}^k_{\mu,\lambda} = a_I(\psi_\lambda, \psi_\mu), \quad \left(\mathbf{f}_k^l\right)_\mu = \frac{\left\langle U_k^l, \psi_\mu \right\rangle}{\tau} - \frac{a\left(U_k^l, \psi_\mu\right)}{2}. \tag{24}$$

Since the matrix $\mathbf{A}^k$ is not symmetrical, we use the GMRES method with a Jacobi preconditioner for the numerical solution of the system.

Next, we focus on the structure of the discretization matrices. Let $N_k \times N_k$ be the size of the matrix $\mathbf{G}^k$. Due to the locality of the differential operators, hierarchical structure of the wavelet basis, and local support of the wavelets, the matrix $\mathbf{G}^k$ has $\mathcal{O}(N_k \log N_k)$ nonzero entries. Moreover, the matrix-vector product $\mathbf{G}^k\mathbf{x}$ can be computed by $\mathcal{O}(N_k)$ flops employing the Kronecker product and the discrete wavelet transform.

The situation with the matrix $\mathbf{K}^k$ is not so straightforward, because for many methods, matrices arising from the discretization of the integral term are full. However, in the case of the wavelet-Galerkin method, due to the vanishing moments, there is decay in the matrix entries. For the sake of simplicity, the following theorem concerning decay estimates is formulated for the wavelet basis presented in Section 3.

Theorem 6. *Let ψ_λ and ψ_μ be wavelets with $L = 3$ vanishing moments from the wavelet basis Ψ defined by (20). Let Ω_λ and Ω_μ be the supports of ψ_λ and ψ_μ, respectively. Then*

$$|a_I(\psi_\lambda, \psi_\mu)| \le C_{\lambda,\mu} 2^{-(L+1)(|\lambda|+|\mu|)}, \tag{25}$$

where

$$C_{\lambda,\mu} = \max_{l,k=1,2,3} \frac{4}{l!\,(L-l)!k!\,(L-k)!} C_{l,k} \tag{26}$$

and

$$C_{l,k} = \max_{\substack{(S_1,S_2)\in\Omega_\mu \\ (z_1,z_2)\in\Omega_\lambda}} \left| \frac{\partial^{2L}}{\partial S_1^l \partial S_2^{L-1} \partial z_1^k \partial z_2^{L-k}} \frac{1}{S_1 S_2} f\left(\frac{z_1}{S_1}, \frac{z_2}{S_2}\right) \right|. \tag{27}$$

Proof. The proof is not presented here, because it is long and technical. However, it follows the lines of the proofs of Theorem 15 in [5] and Theorem 5 in [4]. The constant 4 in (26) is generally dependent on the chosen wavelet basis, namely on the length of the support of the scaling functions and wavelets and on the L^1-norm of the wavelets. □

Based on the decay estimate obtained in Theorem 6, many entries of the matrix $\mathbf{K}^k$ are very small and can be thresholded. Thus, the matrix can be efficiently approximated by a sparse matrix.

5. Numerical example

We consider a European put option on the maximum of two assets, because in this special case, the analytical solution is known [2]. We model the value of the option with parameters [2]: $K = 100$, $T = 1$, $r = 0.05$, $\sigma_1 = 0.12$, $\sigma_2 = 0.15$, $\rho = 0.3$, $\lambda = 0.6$, $\gamma_1 = -0.1$, $\gamma_2 = 0.1$, $\hat{\rho} = -0.20$, $\delta_1 = 0.17$, and $\delta_2 = 0.13$. We set $S = 5K$ and $\tau = 1/730$.

In Figure 2, the convergence history is presented. The symbol ρ_∞ denotes the $L^\infty(\Omega)$ norm of the error, $\rho_\infty^{\mathrm{ROI}}$ denotes the $L^\infty(\mathrm{ROI})$ norm of the error, ρ_2 is the relative error with respect to the $L^2(\Omega)$ norm, and ρ_2^{ROI} is the relative error with respect to the $L^2(\mathrm{ROI})$ norm. The quadratic spline approximation of the smooth functions yields $\rho_2, \rho_\infty \approx CN^{-3/2}$, where N is the number of basis functions. The slope for this rate of convergence is represented by the triangle in Figure 2.

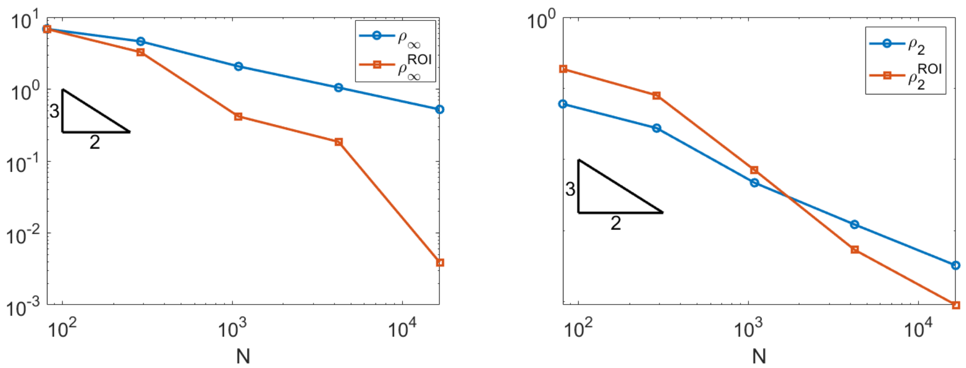

Figure 2: Errors in the L^∞ (left) and relative L^2 (right) norms.

6. Conclusions

We presented the wavelet-Galerkin method combined with the Crank-Nicolson scheme for the numerical valuation of two-asset European options under the Merton jump-diffusion model. Due to the vanishing moments of the wavelets, the matrix corresponding to the integral term was efficiently approximated by a sparse matrix. In the region of interest, the optimal rate of convergence was realized in the numerical experiments.

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