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HEAT EXPOSURE OPTIMIZATION APPLIED TO MOULDING PROCESS IN THE AUTOMOTIVE INDUSTRY

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Abstract

This contribution contains a description and comparison of two methods applied to exposure optimization applied to moulding process in the automotive industry.

1. Introduction

Consider an aluminium shape weighting approximately 300 kg. This shape should be uniformly warmed to $270^o C$ by approximately 100 heating lamps of the same power. Every lamp is defined by the coordinates of its endpoints A, B and the lighting direction u (9 parameters). All the lamps have the same length d. The shape surface is defined by using approximately 10000 plane elements. Every plane element is represented by the coordinates of its center T and its outer normal v (6 parameters). The initial coordinates of the lamps are given. To obtain a uniform exposure of the surface to the heat radiation, we optimize the lamp coordinates.

2. Formulation of a constrained optimization problem

2.1. Equations for the exposure of a plane element by a lamp

Let $x^T = (x_1^T, x_2^T, x_3^T)$ be the center of a plane element, $x^N = (x_1^N, x_2^N, x_3^N)$ be its outer normal, $x^A = (x_1^A, x_2^A, x_3^A)$, $x^B = (x_1^B, x_2^B, x_3^B)$ be the endpoints of the lamp and $x^S = (x_1^S, x_2^S, x_3^S)$ be the lighting direction of the lamp. We also denote $v = -x^N$, $u = x^S$ and use the following constraints

$$\sum_{i=1}^{3}(x_i^S)^2 = 1, \qquad \sum_{i=1}^{3} x_i^S(x_i^B - x_i^A) = 0, \qquad \sum_{i=1}^{3}(x_i^B - x_i^A)^2 = d^2, \tag{1}$$

where d is the length of the lamp. The first constraint ensures the unit length of vector x^S, the second its orthogonality to the axis of the lamp, and the third stabilizes the length of the lamp.

The lamp is a linear body of the length d, consisting of p lighting elements of lengths $d_k = d/p$, $1 \le k \le p$. The connecting line between the center of the lighting element and the center of the plane element is expressed as

$$w_k = x^T - (1-\lambda_k)x^A - \lambda_k x^B, \quad \lambda_k = \frac{2k-1}{2p}, \tag{2}$$

where $1 \le k \le p$. The exposure I of the selected plane element by the particular lamp is given by the formula

$$I = \sum_{k=1}^{p} I_k, \quad I_k = \left(3\alpha_k + \frac{1}{2}\sqrt{1-\alpha_k^2}\right)\frac{\beta_k}{\|w_k\|^2} d_k, \tag{3}$$

where

$$\alpha_k = \frac{u^T w_k}{\|u\|\|w_k\|} = \tilde{u}^T \tilde{w}_k, \quad \beta_k = \frac{v^T w_k}{\|v\|\|w_k\|} = \tilde{v}^T \tilde{w}_k,$$

and

$$\tilde{u} = u/\|u\|, \quad \tilde{v} = v/\|v\|, \quad \tilde{w_k} = w_k/\|w_k\|$$

(the expression for I_k has been obtained by measurements). Analytical expressions for the derivatives of the exposure I with respect to the elements of vectors x^A, x^B, x^S (elements of the vectors x^T, x^N are constants, since the heated surface is fixed) have the form

$$\begin{aligned}
\frac{\partial I}{\partial x_i^A} &= \sum_{k=1}^{p} \frac{\partial I_k}{\partial x_i^A} = -\sum_{k=1}^{p} (1-\lambda_k)\frac{\partial I_k}{\partial w_{ik}}, \\
\frac{\partial I}{\partial x_i^B} &= \sum_{k=1}^{p} \frac{\partial I_k}{\partial x_i^B} = -\sum_{k=1}^{p} \lambda_k \frac{\partial I_k}{\partial w_{ik}} \\
\frac{\partial I}{\partial x_i^S} &= \sum_{k=1}^{p} \frac{\partial I_k}{\partial x_i^S} = \sum_{k=1}^{p} \frac{\partial I_k}{\partial u_i},
\end{aligned}$$

so they can be easily computed from gradients

$$\begin{aligned}
\nabla_u I_k &= \left(3 - \frac{1}{2}\frac{\alpha_k}{\sqrt{1-\alpha_k^2}}\right)\frac{\beta_k d_k}{\|w_k\|^2}\nabla_u \alpha_k, \\
\nabla_{w_k} I_k &= \left(3 - \frac{1}{2}\frac{\alpha_k}{\sqrt{1-\alpha_k^2}}\right)\frac{\beta_k d_k}{\|w_k\|^2}\nabla_{w_k} \alpha_k \\
&+ \left(3\alpha_k + \frac{1}{2}\sqrt{1-\alpha_k^2}\right)\left(\frac{d_k}{\|w_k\|^2}\nabla_{w_k}\beta_k - 2\frac{\beta_k d_k}{\|w_k\|^4} w_k\right).
\end{aligned}$$

Furthermore, one has

$$
\begin{aligned}
\nabla_u \alpha_k &= \frac{w_k}{\|u\|\|w_k\|} - \frac{u^T w_k}{\|u\|\|w_k\|}\frac{u}{\|u\|^2} = \frac{1}{\|u\|}(\tilde{w}_k - \alpha_k \tilde{u}), \\
\nabla_{w_k} \alpha_k &= \frac{u}{\|u\|\|w_k\|} - \frac{u^T w_k}{\|u\|\|w_k\|}\frac{w_k}{\|w_k\|^2} = \frac{1}{\|w_k\|}(\tilde{u} - \alpha_k \tilde{w}_k), \\
\nabla_{w_k} \beta_k &= \frac{v}{\|v\|\|w_k\|} - \frac{v^T w_k}{\|v\|\|w_k\|}\frac{w_k}{\|w_k\|^2} = \frac{1}{\|w_k\|}(\tilde{v} - \beta_k \tilde{w}_k),
\end{aligned}
$$

and after substitution we obtain

$$
\nabla_u I_k = \left(3 - \frac{1}{2}\frac{\alpha_k}{\sqrt{1-\alpha_k^2}}\right)\frac{\beta_k d_k}{\|u\|\|w_k\|^2}(\tilde{w}_k - \alpha_k \tilde{u}) \tag{4}
$$

$$
\begin{aligned}
\nabla_{w_k} I_k &= \left(3 - \frac{1}{2}\frac{\alpha_k}{\sqrt{1-\alpha_k^2}}\right)\frac{\beta_k d_k}{\|w_k\|^3}(\tilde{u} - \alpha_k \tilde{w}_k) \\
&+ \left(3\alpha_k + \frac{1}{2}\sqrt{1-\alpha_k^2}\right)\frac{d_k}{\|w_k\|^3}(\tilde{v} - 3\beta_k \tilde{w}_k).
\end{aligned} \tag{5}
$$

It is not necessary to known the elements of vectors u, v and w_k, $1 \le k \le p$. We use only their Euclidean norms and the elements of normalized vectors $\tilde{u}$, $\tilde{v}$ and $\tilde{w}_k$, $1 \le k \le p$, in our numerical algorithm.

2.2. Objective function and constraints for the uniform exposure

We have n_e plane elements and n_l lamps. Every plane element can be exposed by several lamps. Let L_j be a set of indices of the lamps that expose the jth plane element. Choose $1 \le j \le n_e$ and $l \in L_j$. If we denote I_{jl} the exposure of the jth element by the lth lamp, (this value corresponds to the value I from the previous subsection), then the total exposure I_j of the jth element is given by the formula

$$
I_j = \sum_{l \in L_j} I_{jl}.
$$

The derivatives of I_j are computed by the formulas

$$
\begin{aligned}
&\frac{\partial I_j}{\partial x_{il}^A} = \frac{\partial I_{jl}}{\partial x_{il}^A}, \quad \frac{\partial I_j}{\partial x_{il}^B} = \frac{\partial I_{jl}}{\partial x_{il}^B}, \quad \frac{\partial I_j}{\partial x_{il}^S} = \frac{\partial I_{jl}}{\partial x_{il}^S}, \quad l \in L_j, \\
&\frac{\partial I_j}{\partial x_{il}^A} = 0, \qquad \frac{\partial I_j}{\partial x_{il}^A} = 0, \qquad \frac{\partial I_j}{\partial x_{il}^A} = 0, \qquad l \notin L_j,
\end{aligned}
$$

where we substitute the previously defined quantities. Let $\overline{I}$ be the prescribed value of the exposure (the same for all elements of the shape surface). Then

$$
F(x) = \frac{1}{2}\sum_{j=1}^{n_e}(I_j - \overline{I})^2, \tag{6}
$$

where vector x has elements x_{1l}^A, x_{2l}^A, x_{3l}^A, x_{1l}^B, x_{2l}^B, x_{3l}^B, x_{1l}^S, x_{2l}^S, x_{3l}^S, $1 \le l \le n_l$ (nine for every lamp). One has

$$\frac{\partial F(x)}{\partial x_{il}^A} = \sum_{j=1}^{n_e} (I_j - \overline{I}) \frac{\partial I_j}{\partial x_{il}^A}, \quad \frac{\partial F(x)}{\partial x_{il}^B} = \sum_{j=1}^{n_e} (I_j - \overline{I}) \frac{\partial I_j}{\partial x_{il}^B}, \quad \frac{\partial F(x)}{\partial x_{il}^S} = \sum_{j=1}^{n_e} (I_j - \overline{I}) \frac{\partial I_j}{\partial x_{il}^S},$$

where we substitute quantities computed in the previous relations. The prescribed value of the exposure is determined by the initial positions of the lamps through the formula

$$\overline{I} = \frac{1}{n_e} \sum_{j=1}^{n_e} I_j.$$

The objective function $F(x)$ is minimized in the feasible region given by the equality constraints (1) (three constraints for every lamp). Computation of derivatives of these constraints with respect to the elements of vector x is easy. All constraints are sparse, so the memory size and the number of arithmetic operations are not large.

The described problem consists in the minimization of a sum of squares with respect to nonlinear equality constraints. The number of partial functions in the sum of squares is $n_e \sim 10000$ (the number of the plane elements). The number of variables is $9n_l \sim 900$ (nine for every lamp). The Hessian matrix of the objective function is not sparse. The number of nonlinear equality constraints is $3n_l \sim 300$ (three for every lamp). The Jacobian matrix of nonlinear equality constraints is sparse. These facts have an influence on the choice of the numerical method. We have used the recursive quadratic programming method with iterative solution of linear KKT system by indefinitely preconditioned conjugate gradient method (see [3]). This method uses partial derivatives derived above.

3. Formulation of an unconstrained optimization problem

In this section, we use constraints (1) to eliminate vector $u = x^S$ from the formula (3). For this purpose we assume that the basis of the warmed shape lies in the horizontal plane, the lamps are placed over the heated surface and the lighting directions of the lamps are mostly perpendicular to the basis of the shape. This assumption is not very restrictive and results obtained in this way are comparable with those obtained by approach used in the previous section.

Let y be a vector parallel to vector $x^B - x^A$. Then we can write $x^B - x^A = (y/\|y\|)d$ and $w_k = x^T - x^A - \lambda_k (y/\|y\|)d$, $1 \le k \le p$, where $d = \|x^B - x^A\|$ (see (2)). By our assumption, the angle between vector $u = x^S$, which is perpendicular to vector y, and the normal $e = (0, 0, -1)$ is minimal. If the norm of vector u is unit, it can be uniquely determined from vectors y and e.

Theorem 1 *Vector*

$$u = \frac{e + \lambda y}{\sqrt{e^T(e + \lambda y)}}, \qquad \lambda = -\frac{e^T y}{y^T y}.$$

is the solution of the optimization problem

$$\text{Maximize} \quad e^T u \quad \text{subject to} \quad y^T u = 0, \quad u^T u = 1.$$

Since the length of vector u can be arbitrary, we put

$$u = e - \frac{e^T y}{y^T y} y = e - e^T \tilde{y} \tilde{y},$$

where $\tilde{y} = y/\|y\|$ (vector $e = (0, 0, -1)$ has the unit norm). To compute the gradient of the objective function, we need the transposed Jacobian matrices of vectors u and w_k (with respect to y), which we denote $\nabla_y u$ and $\nabla_y w_k$.

Theorem 2 *One has*

$$\begin{aligned} \nabla_y u &= \left(2 \frac{y\, y^T}{y^T y} - I \right) \frac{e^T y}{y^T y} - \frac{e\, y^T}{y^T y} = \frac{1}{\|y\|} \left((2\, \tilde{y} \tilde{y}^T - I)\, e^T \tilde{y} - e\, \tilde{y}^T \right) \\ \nabla_y w_k &= \frac{\lambda_k d}{\|y\|} \left(\frac{y\, y^T}{y^T y} - I \right) = \frac{\lambda_k d}{\|y\|} \left(\tilde{y} \tilde{y}^T - I \right) \end{aligned}$$

The exposure (3) now depends on vectors $x = x^A$ and y (then $x^B = x^A + (y/\|y\|)d$ and vector $x^S = u$ is obtained by Theorem 1). Analytical expressions for gradients of the exposure I have the form

$$\nabla_x I = \sum_{k=1}^{p} \nabla_x I_k = -\sum_{k=1}^{p} \nabla_{w_k} I_k, \qquad \nabla_y I = \sum_{k=1}^{p} \nabla_y I_k = \sum_{k=1}^{p} (\nabla_y u \nabla_u I_k + \nabla_y w_k \nabla_{w_k} I_k),$$

where gradients $\nabla_u I_k$ and $\nabla_{w_k} I_k$ are computed by formulas (4) and (5). Note that using Theorem 2 we can write

$$\nabla_y u \nabla_u I_k + \nabla_y w_k \nabla_{w_k} I_k = -\frac{1}{\|y\|} \left(\gamma_e (\nabla_u I_k - 2\gamma_u \tilde{y}) + \gamma_u e + \lambda_k d (\nabla_{w_k} I_k - \gamma_{w_k} \tilde{y}) \right),$$

where $\gamma_e = \tilde{y}^T e$, $\gamma_u = \tilde{y}^T \nabla_u I_k$ and $\gamma_{w_k} = \tilde{y}^T \nabla_{w_k} I_k$.

Analogously to the previous section, we minimize the sum of squares (6), but now without constraints. The number of variables is $6n_l \sim 600$ (six for every lamp). The Hessian matrix of the objective function is not sparse. This fact have an influence to the choice of the numerical method. We have used the combination of the Gauss-Newton method and the BFGS variable metric method, which is described in [2]. This combination uses partial derivatives derived above.

4. Numerical comparison

The purpose if this section is to show that the elimination of constraints and the solution of the unconstrained optimization problem significantly increase the efficiency of the computation. To demonstrate this fact, we have used four test problems L1–L4 introduced in [1]. The following table contains the results corresponding to the two approaches described in the previous sections. Here NIT and NFV are the numbers of iterations and function evaluations, F_0 and F are the initial and the final values of the objective function. Computational time is given in seconds. The $*$ symbol means that 10000 function evaluations did not suffice for obtaining the solution. The results were obtained by the interactive system for universal functional optimization UFO described in [4].

The following figure demonstrates the solution of problem L1.

		Method with constraints				Method without constraints			
Problem	F_0	NIT	NFV	Time	F	NIT	NFV	Time	F
L1	169.53	1125	4653	396.14	27.68	74	165	18.67	29.16
L2	198.14	712	2456	218.68	31.22	83	186	21.22	32.75
L3	22.50	382	812	118.79	14.25	57	126	20.50	12.02
L4	11.86	1094	10007	742.15	2.03 $*$	43	98	9.71	1.27

Table 1: Comparison of two approaches for the heat exposure optimization.

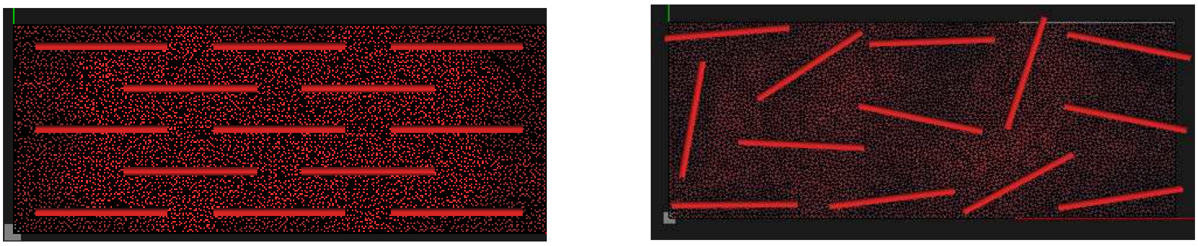

Figure 1: Initial (left) and final (right) positions of the lamps.

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