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ON REGULARITY OF SOLUTIONS TO SOME PROBLEMS OF MATHEMATICAL PHYSICS

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Let $\Omega \subset \mathbb{R}^m$ ($m \geq 2$) be a bounded domain with C^1 boundary $\partial\Omega$. Consider a boundary value problem for the elliptic system

$$(1) \quad Lu \equiv D_i a_i(x, Du) = 0, \quad x \in \Omega,$$

$$(2) \quad u(x) = g(x), \quad x \in \partial\Omega, \quad g \in W_2^1(\Omega).$$

Here $u(x) = \{u^1(x), \dots, u^N(x)\}$ is an unknown vector function, $Du = \{D_0 u, D_1 u, \dots, D_m u\}$, $D_0 u = u$, $D_i = \partial / \partial x_i$ ($i = 1, \dots, m$).

Until now we considered mostly the case when the coefficients $a_i(x, p) = \{a_i^1(x, p), \dots, a_i^N(x, p)\}$ were measurable with respect to x , continuously differentiable with respect to $p = \{p_0, \dots, p_m\}$ and for the matrix

$$(3) \quad A = \frac{\partial a_i}{\partial p_k}, \quad (i, k = 0, \dots, m)$$

the inequalities

$$(4) \quad \mu \|\xi\|^2 \leq A \xi \cdot \xi \leq \nu \|\xi\|^2, \quad \|A\| < C$$

took place with some positive constants μ , ν and C . Suppose that A is symmetric. Denote as $\{\lambda_1(x, p)\}$ the set of all eigenvalues of A at the point $\{x, p\}$ and put $\lambda = \inf \lambda_1$ and $\Lambda = \sup \lambda_1$. It was proved in [1] that if the inequality

$$(5) \quad \frac{\Lambda - \lambda}{\Lambda + \lambda} \left[1 + \frac{(m-2)^2}{m-1} \right]^{1/2} < 1$$

holds then the weak solution u of (1), (2) is locally Hölder-continuous in Ω . It was also proved that the condition (5) is sharp.

In some important problems the differentiability of the coefficients a_i with respect to p doesn't hold. It happens e.g. in case of the systems of equations for elasto-plastic media with hardening, the Maxwell systems for materials with ferromagnetic intrudings etc.

Suppose now that, instead of (4), the inequalities

$$(6) \quad \begin{aligned} (a(x, p) - a(x, q)) \cdot (p - q) &\geq \mu \|p - q\|^2 \\ \|a(x, p) - a(x, q)\| &\leq \nu \|p - q\| \end{aligned}$$

take place with some positive constants μ, ν for almost all $x \in \Omega$ and all $p, q \in R^{(m+1)N}$. (The norms are calculated in $R^{(m+1)N}$.)

Denote

$$(7) \quad K = \inf_{\varepsilon > 0} \sup \frac{\|p - q - \varepsilon[a(x, p) - a(x, q)]\|}{\|p - q\|} .$$

THEOREM 1. If

$$(8) \quad K \left[1 + \frac{(m-2)^2}{m-1} \right]^{1/2} < 1,$$

then u is locally Hölder continuous on Ω . The condition (8) is sharp.

Consider now the equilibrium system of the theory of small deformations for a material with strenght condition (so called Hencky theory). Let $S = \{\sigma_{ij}\}$ and $D = \{\varepsilon_{ij}\}$ are the stress and strain tensors, respectively. Suppose

$$\sigma_{ik} = a_1^k(x; \varepsilon_{j1}), \quad \text{where} \quad j1 = \frac{u^j}{x_1} + \frac{u^1}{x_j}$$

and u is the displacement vector.

Then the system corresponding to (1) has the form

$$(1') \quad D_1 a_1(x; \varepsilon_{j1}) - f = 0,$$

where f is a vector of mass forces.

$$T^2 = \inf_{\varepsilon > 0} \sup_{x, p, q} \frac{\sum_{i, k=1}^m \left| (p_1^k + p_k^1) - (q_1^k + q_k^1) - \varepsilon \left[a_1^k(x; p_j^1 + p_1^j) - a_1^k(x; q_j^1 + q_1^j) \right] \right|^2}{\sum_{i, k=1}^m \left| (p_1^k + p_k^1) - (q_1^k + q_k^1) \right|^2}.$$

THEOREM 2. If

$$2T \left[1 + \frac{(m-2)^2}{m-1} \right]^{1/2} \left[1 + \frac{m}{2} \left(1 + \frac{m-2}{m+1} \right)^{1/2} \right] < 1,$$

then the displacement u (the weak solution of the problem (1')-(2)) is locally Hölder-continuous on Ω .

As we have seen above, the conditions (5) and (8) are sharp. Nevertheless, there are the cases when the solution u has some additional properties. Suppose that Γ_s ($s=1, \dots, m-2$) is a smooth s -dimensional manifold in Ω .

THEOREM 3. Let the conditions (4) be satisfied and let

$$\frac{\Lambda - \lambda}{\Lambda + \lambda} \left[1 + \frac{(m-2-s)^2}{m-1-s} \right]^{1/2} < 1.$$

Then $u \varphi \in L_1(\Gamma_s)$ for all C_0^∞ and $\|u \varphi\|_{L_1(\Gamma_s)}$ is Hölder continuous with respect to the coordinates orthogonal to Γ_s .

Consider now the parabolic system

$$P(u) \equiv \dot{u} - L(u) = 0$$

in the cylinder $Q = \Omega \times (0, T)$ with the initial condition $u|_{t=0} = 0$ and the condition (2). Suppose that the conditions (4) are satisfied.

THEOREM 4. There exists such a constant $C(m) > 0$ that if the inequality

$$\frac{\Lambda - \alpha}{\Lambda + \alpha} C(m) < 1$$

holds, then the weak solution u of the problem

$$P(u) = 0, u|_{t=0} = 0, u|_{\partial\Omega} = g(x)$$

is for almost all $t \in [0, T]$ locally Hölder-continuous with respect to the variable x in Ω .

This theorem has some corollaries concerning the systems with hysteresis coefficients.

[1] A.I. Koshchev, Regularity of the solutions of elliptic equations and systems, Nauka, Moscow, 1986.