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Locally uniformly non-$l_n^{(1)}$ Orlicz spaces

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LOCALLY UNIFORMLY NON-$l_n^{(1)}$ ORLICZ SPACES

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Summary. There are given some criteria for non-$l_n^{(1)}$ and local uniform non-$l_n^{(1)}$ properties of Orlicz spaces in the case of an atomless infinite (but σ-finite) as well as in the case of a purely atomic measure. In the case of an atomless finite measure there is given only a criterion for non-$l_n^{(1)}$ property of Orlicz spaces.

INTRODUCTION

In the following (T,Σ,μ) denotes a space of positive and σ-finite measure. F denotes the space of all Σ-measurable functions from T into the real line $\mathbb{R}$. Ofcourse, two functions which differ only on a set of measure zero will be regarded as equal. Define $e_k=(0,\ldots,0,1,0,\ldots)$, where 1 is on kth place for $k=1,2,\ldots$.

By an Orlicz function we mean a map $\Phi:\mathbb{R}\longrightarrow[0,\infty]$ which is convex, even, vanishing and continuous at zero and not identically equal zero. Let Φ be an Orlicz function. Define the modular $I: F\longrightarrow[0,\infty]$ by

$$I(x)=\int_T \Phi(x(t))d\mu.$$

The Orlicz space generated by Φ and μ is the B-space $(L^{\Phi}(\mu),\|\ \|_{\Phi})$, where

$$L^{\Phi}(\mu)=\{x\in F: I(\lambda x)<\infty \text{ for some } \lambda>0\}$$

and the norm $\|\ \|_{\Phi}$ is defined by

$$\|x\|_{\Phi}=\inf\{r>0: I(x/r)\le 1\}.$$

In the case of a purely atomic measure we write tradicionally $l^{\Phi}(\mu)$ in place of $L^{\Phi}(\mu)$.

We say an Orlicz function Φ satisfies the condition Δ_2 for all u (at infinity) [at zero] if there exist constants $K,a>0$ such that the inequality $\Phi(2u)\le K\Phi(u)$ holds for all u (for u satisfying $\Phi(u)\ge a$) [for u satisfying $\Phi(u)\le a$].

A normed space $(X,\|\ \|)$ is called non-$l_n^{(1)}$ ($n\in\mathbb{N}$, $n\ge 2$) if for any norm-one elements $x_1,\ldots,x_n$ in X, we have $\|x_1\pm\ldots\pm x_n\|<n$ for some choice of signs.

We say that a normed space $(X,\|\ \|)$ is locally uniformly non-$l_n^{(1)}$ if for every $x_1 \in X$ with $\|x\| = 1$ there exists $\delta(x_1) \in (0,1)$ such that for every norm-one elements $x_2, \ldots, x_n$ in X there holds $\|x_1 \pm \ldots \pm x_n\| \leq n(1 - \delta(x_1))$ for some choice of signs.

Locally uniformly non-$l_2^{(1)}$ spaces are called locally uniformly non-square (see [9] , p. 131).

A normed space $(X, \|\ \|)$ is called locally uniformly rotund if for any $x \in X$ with $\|x\| = 1$ and for every $\varepsilon > 0$ there exists $\delta(x, \varepsilon) \in (0,1)$ such that $\|x + y\| \leq 2(1 - \delta(x,\varepsilon))$ whenever $y \in X$, $\|y\| = 1$ and $\|x - y\| \geq \varepsilon$.

We say a normed space $(X, \|\ \|)$ is strictly convex (rotund) if for any norm-one elements x, y in X, $x \neq y$, we have $\|x + y\| < 2$.

RESULTS

Every strictly convex normed space $(X, \|\ \|)$ is non-$l_n^{(1)}$. It is sufficient to show that X is non-$l_2^{(1)}$. Let $\|x_1\| = \|x_2\| = 1$. If $x_1 = x_2$, then $\|x_1 - x_2\| = 0$. If $x_1 \neq x_2$, then by rotundity of X, we get $\|x_1 + x_2\| < 2$.

Every locally uniformly rotund normed space $(X, \|\ \|)$ is locally uniformly non-square and so it is locally uniformly non-$l_n^{(1)}$ for any $n \in \mathbb{N}$, $n \geq 2$.

Let $x_1, x_2 \in X$, $\|x_1\| = \|x_2\| = 1$. Then $\|x_1 - x_2\| \leq 2^{-1}$ or $\|x_1 + x_2\| \leq 2(1 - \delta(x_1, 2^{-1}))$, i.e. X is locally uniformly non-square.

LEMMA 1. The space l^∞ is not non-$l_n^{(1)}$.

Proof. Let $\varepsilon_i = (\varepsilon_i^1, \ldots, \varepsilon_i^n)$ be all choices of signs ± 1 with $\varepsilon_i^1 = 1$ for $i = 1, \ldots, 2^{n-1}$. Putting

$$x_j = \sum_{i=1}^{2^{n-1}} \varepsilon_i^j e_i$$

for $j = 1, \ldots, n$, we have $\|x_1 \pm \ldots \pm x_n\|_\infty = n$ for any choice of signs.

LEMMA 2. The space $L^1(\mu)$ is not non-$l_n^{(1)}$.

Proof. Let $A_1, \ldots, A_n$ be pairwise disjoint sets of positive and finite measure. Let $a_1, \ldots, a_n$ be positive numbers such that $a_i = (\mu(A_i))^{-1}$ for $i = 1, \ldots, n$. Define $x_i = a_i \chi_{A_i}$ for $i = 1, \ldots, n$. We have $\|x_i\|_\Phi = 1$ and $\|x_1 \pm \ldots \pm x_n\|_1 = \sum_{i=1}^n a_i \mu(A_i) = n$ for any choice of signs.

LEMMA 3.(i) . If μ is an atomless infinite (finite) measure and Φ is an Orlicz function satisfying condition Δ_2 for all u (at infinity), then for every $\varepsilon \in (0,1)$ there exists $\delta(\varepsilon) \in (0,1)$ such that $\|x\|_\Phi \leq 1 - \delta(\varepsilon)$ whenever $I(x) \leq 1 - \varepsilon$.

(ii) If $\mu = (b_k)$ is a purely atomic measure with $\inf b_k = \liminf b_k = b > 0$, Φ is an Orlicz function satisfying condition Δ_2 at zero and $\Phi(u_1) = b^{-1}$ for some $u_1 > 0$, then for any $\varepsilon \in (0,1)$ there exists $\delta(\varepsilon) \in$

(0,1) such that $\|x\|_\Phi \le 1-\delta(\varepsilon)$ whenever $I(x)\le 1-\varepsilon$.

For the proof see [2], [7], [8].

THEOREM 1. If μ is a purely atomic measure as in LEMMA 3 (ii) and Φ is an Orlicz function such that $\Phi(u_1)=b^{-1}$ for some $u_1>0$, then the following assertions are equivalent:

1°. $l^\Phi(\mu)$ is locally uniformly non-$l_n^{(1)}$,

2°. $l^\Phi(\mu)$ is non-$l_n^{(1)}$,

3°. Φ satisfies condition Δ_2 at zero and

(i) $\Phi(u/n) < \Phi(u)/n$ for any $u>0$,

4°. $l^\Phi(\mu)$ is non-$l_2^{(1)}$.

Proof. 3°$\Longrightarrow$1°. Let $\|x_1\|_\Phi=\ldots=\|x_n\|_\Phi=1$. By virtue of condition Δ_2 at zero, we have $I(x_1)=\ldots=I(x_n)=1$ (see [1]). Let $k\in\mathbb{N}$ be such that $|x_1(k)|=\|x_1\|_\infty$ and denote $|x_1(k)|\,b_k=d$. There exists $\sigma\in(0,1)$ such that

(1) $\Phi(u/n)\le \sigma\Phi(u)/n$

for any $u\in[|x_1(k)|,u_1]$. Let Σ^o denote the operator of summation over all 2^{n-1} possible choice of signs. We have

$$(2)\qquad 2^{n-1}-\Sigma^o I((x_1\pm\ldots\pm x_n)/n)=n^{-1}2^{n-1}\Sigma_{i=1}^n I(x_i)-\Sigma^o I((x_1\pm\ldots\pm x_n)/n)$$

$$\ge n^{-1}2^{n-1}\Sigma_{i=1}^n\Phi(x_i(k))\,b_k-\Sigma^o\Phi((x_1(k)\pm\ldots\pm x_n(k))/n)\,b_k.$$

We have $|x_1(k)\pm\ldots\pm x_n(k)|\le\max_i|x_i(k)|$ for some choice of signs. Applying (1), we get for this choice of signs

$$\Phi((x_1(k)\pm\ldots\pm x_n(k))/n)\le\sigma n^{-1}\,\Phi(\max_i|x_i(k)|)\le\sigma n^{-1}\Sigma_{i=1}^n\Phi(x_i(k)).$$

Hence we obtain

$$\Sigma^o\Phi((x_1(k)\pm\ldots\pm x_n(k))/n)\le n^{-1}(2^{n-1}-1+\sigma)\Sigma_{i=1}^n\Phi(x_i(k)).$$

Applying this inequality and (2), we get

$$2^{n-1}-\Sigma^o I((x_1\pm\ldots\pm x_n)/n)\ge n^{-1}(1-\sigma)\Sigma_{i=1}^n\Phi(x_i(k))\,b_k$$

$$\ge n^{-1}(1-\sigma)\,d\,,\ \text{i.e.}\ I((x_1\pm\ldots\pm x_n)/n)\le 2^{n-1}(1-\eta),$$

where $\eta=(1-\sigma)\,d/n\,2^{n-1}$. Hence, we obtain $I((x_1\pm\ldots\pm x_n)/n)\le 1-\eta$ for some choice of signs. The proof of the implication 3°$\Longrightarrow$1° may be finished by application of LEMMA 3 (ii).

$2^\circ \Longrightarrow 3^\circ$. If Φ does not satisfy condition Δ_2 at zero, then $l^\Phi(\mu)$ contains an isometric copy of l^∞ (see [5]) and so, by LEMMA 1, $l^\Phi(\mu)$ is not non-$l^{(1)}_n$. Assume that condition 3°(i) is not satisfied. We may assume that Φ satisfies condition Δ_2 at zero. Hence it follows that Φ vanishes only at zero. There exists $u > 0$ such that $\Phi(u/n) = \Phi(u)/n$. Hence it follows that $\Phi(v/n) = \Phi(v)/n$ for any $v \in [0,u]$, i.e. Φ is a linear function on the interval $[0,u]$. Let $l = k/n \in \mathbb{N}$ and $k\Phi(u) \geq n$. There exists a number $v \in (0,u]$ such that $k\Phi(v) = n$. Define

$$x_j = \sum_{i=1}^{l} v\, e_{i+(j-1)l}$$

for $j = 1,\ldots,n$. We have $l\,\Phi(v) = 1$ and so $I(x_j) = \|x_j\|_\Phi = 1$ for $j = 1,\ldots,n$. Moreover, we get for any choice of signs

$$I((x_1 \pm \ldots \pm x_n)/n) = k\,\Phi(v/n) = k\,\Phi(v)/n = l\,\Phi(v) = 1,$$

i.e. $\|x_1 \pm \ldots \pm x_n\|_\Phi = n$. So, $l^\Phi(\mu)$ is not non-$l^{(1)}_n$.

The implication $1^\circ \Longrightarrow 2^\circ$ is obvious, so the equivalence of conditions 1°, 2° and 3° is proved. The equivalence $2^\circ \Longleftrightarrow 4^\circ$ follows by the equivalence of condition 3°(i) for any two $n \in \mathbb{N}$, $n \geq 2$ (see [2], Lemma 1.7). The proof is finished.

THEOREM 2. Let Φ be an Orlicz function and μ be an atomless infinite measure. The following conditions are equivalent:

1°. $L^\Phi(\mu)$ is locally uniformly non-$l^{(1)}_n$,

2°. $L^\Phi(\mu)$ is non-$l^{(1)}_n$,

3°. Φ satisfies condition Δ_2 for all u and

(i) $\Phi(u/n) < \Phi(u)/n$ for any $u > 0$,

4°. $L^\Phi(\mu)$ is non-$l^{(1)}_2$.

Proof. $3^\circ \Longrightarrow 1^\circ$. Let $\|x_1\|_\Phi = \ldots = \|x_n\|_\Phi = 1$. By condition Δ_2 for all u, we have $I(x_1) = \ldots = I(x_n) = 1$ (see [1]). Let $c > 0$ be such that the set

$$A_1 = \{t \in T:\ c^{-1} \leq |x_1(t)| \leq c\}$$

satisfies the condition $I(x\chi_{A_1}) \geq 7/8$. Let $d > 0$ be such that $\Phi(c)/\Phi(d) \leq 1/8(n-1)$ and let

$$A_i = \{t \in T:\ |x_i(t)| \leq d\},\quad i = 2,\ldots,n.$$

We have $\Phi(d)\mu(T \setminus A_i) < I(x_i\chi_{T\setminus A_i}) \leq 1$, i.e. $\mu(T\setminus A_i) < 1/\Phi(d)$ for $i = 2,\ldots,n$. Hence, we get

$$I(x_1\chi_{A_1\setminus A_i}) \leq \Phi(c)\mu(A_1\setminus A_i) \leq \Phi(c)/\Phi(d) \leq 1/8(n-1).$$

Denoting $D = \bigcap_{i=1}^n A_i$, we have

$$7/8 \leq I(x_1\chi_{\bigcup_{i=1}^n (A_1\setminus A_i)}) + I(x_1\chi_D)$$

$$\leq \sum_{i=1}^{n} I(x_1\chi_{(A_1\setminus A_i)}) + I(x_1\chi_D) \leq 1/8 + I(x_1\chi_D) .$$

Hence, we obtain

(3) $$I(x_1\chi_D) \geq 3/4.$$

Moreover,

(4) $$2^{n-1} - \sum^{o} I((x_1 \pm \ldots \pm x_n)/n) = n^{-1}2^{n-1}\sum_{i=1}^{n} I(x_i) - \sum^{o} I((x_1 \pm \ldots \pm x_n)/n)$$

$$\geq n^{-1}2^{n-1}\sum_{i=1}^{n} I(x_i\chi_D) - \sum^{o} I((x_1 \pm \ldots \pm x_n)\chi_D/n).$$

Since $|x_1(t) \pm \ldots \pm x_n(t)| \leq \max_i |x_i(t)|$ for some choice of signs depending on t, so

(5) $$I((x_1 \pm \ldots \pm x_n)\chi_D/n) \leq n^{-1}(2^{n-1}-1+\sigma)\sum_{i=1}^{n} I(x_i\chi_D),$$

where $\sigma = \sup\{ n\Phi(u/n)/\Phi(u) : \Phi(u) \in [c^{-1},d]$. Obviously, $\sigma \in (0,1)$. Combining (3),(4) and (5), we get

$$2^{n-1} - \sum^{o} I((x_1 \pm \ldots \pm x_n/n) \geq n^{-1}(1-\sigma)\sum_{i=1}^{n} I(x_i\chi_D) \geq 3(1-\sigma)/4n = \eta .$$

The number η belongs to (0,1) and depends only on x_1. The last inequality is equivalent to the following one

$$\sum^{o} I((x_1 \pm \ldots \pm x_n)/n) \leq 2^{n-1}(1-q),$$

where $q = \eta/2^{n-1}$. Hence, we have $I((x_1 \pm \ldots \pm x_n)/n) \leq 1-q$ for some choce of signs. Applying LEMMA 3 (i), we get $\|x_1 \pm \ldots \pm x_n\|_\Phi \leq n(1-\delta(q))$, where $\delta(q) \in (0,1)$, for the same choice of signs as in the previous inequality. This finishs the proof of the implication $3^\circ \Longrightarrow 1^\circ$.

The implication $1^\circ \Longrightarrow 2^\circ$ is obvious. Now, we shall prove the implication $2^\circ \Longrightarrow 3^\circ$. If Φ does not satisfy condition Δ_2 for all u, then $L^\Phi(\mu)$ contains an isometric copy of l^∞ (see [2], [3]), so by LEMMA 1, $L^\Phi(\mu)$ is not non-$l_n^{(1)}$. Assume that Φ satisfy condition Δ_2 for all u and does not satisfy condition 3°(i). Then there exists $u > 0$ such that $\Phi(u/n) = \Phi(u)/n$ and $\Phi(u) > 0$. Let B_i, $i=1,\ldots,n$, be pairwise disjoint and Σ-measurable subsets of T such that $\mu(B_i) = 1/\Phi(u)$ for $i=1,\ldots,n$. Defining $x_i = u\chi_{B_i}$, we have $I(x_i) = \|x_i\|_\Phi = 1$ for $i=1,\ldots,n$. Moreover

$$I((x_1 \pm \ldots \pm x_n)/n) = 1, \text{ i.e. } \|x_1 \pm \ldots \pm x_n\|_\Phi = n$$

for any choice of signs. So, $L^\Phi(\mu)$ is not non-$l_n^{(1)}$. The implication $2^\circ \Longrightarrow 3^\circ$ is proved.

The equivalence of conditions 2° and 4° may be deduced in the same way as in THEOREM 1. The proof is completion.

THEOREM 3. Let μ be an atomless finite measure and let Φ be an

Orlicz function. $L^{\Phi}(\mu)$ is non-$l_n^{(1)}$ if and only if:

(i) Φ satisfies condition Δ_2 at infinity and it is finite, and

(ii) $\Phi(u/n) < \Phi(u)/n$ for all u satisfying $\Phi(u) \geq n/\mu(T)$.

Proof. Sufficiency. Let $\|x_1\|_{\Phi} = \ldots = \|x_n\|_{\Phi} = 1$. Taking into account condition (i), we get $I(x_1) = \ldots = I(x_n) = 1$ (see [1], [8] and [12]) and Φ is continuous. So, there exists a number $\theta \in (0,1)$ such that the inequality $\Phi(u/n) < \Phi(u)/n$ holds for all u satisfying $\Phi(u) \geq n\theta/\mu(T)$. Denote $\varepsilon = \sqrt{\theta}$ and define

$$A = \{t \in T : \sum_{i=1}^{n} \Phi(x_i(t)) \geq n\varepsilon/\mu(T)\}.$$

Now, we shall show that for every $t \in A$, we have

$$(6) \qquad \Phi((x_1(t) \pm \ldots \pm x_n(t))/n) < n^{-1} \sum_{i=1}^{n} \Phi(x_i(t))$$

for some choice of signs. For this purpose we shall consider two cases

1°. $\max_i \Phi(x_i(t)) \geq n\theta/\mu(T)$. We have for some choice of signs $|x_1(t) \pm \ldots \pm x_n(t)| \leq \max_i |x_i(t)|$. Hence, we get by (ii)

$$\Phi(x_1(t) + \ldots + x_n(t))/n) \leq \Phi(\max_i |x_i(t)|/n) < \max_i \Phi(x_i(t))/n$$

$$\leq n^{-1} \sum_{i=1}^{n} \Phi(x_i(t)).$$

2°. $\max_i \Phi(x_i(t)) < n\theta/\mu(T)$. Then at least two from the numbers $\Phi(x_i(t))$ must be positive. So, we have for such choice of signs that $|x_1(t) \pm \ldots \pm x_n(t)| \leq \max_i |x_i(t)|$,

$$\Phi(x_1(t) \pm \ldots \pm x_n(t))/n) \leq n^{-1} \Phi(\max_i |x_i(t)|) = n^{-1} \max_i \Phi(x_i(t))$$

$$< n^{-1} \sum_{i=1}^{n} \Phi(x_i(t)).$$

Thus, inequality (6) for $t \in A$ is proved. Denoting by Σ° the operator of summation over all 2^{n-1} choices of signs, we have for all $t \in A$

$$\Sigma^{\circ} \Phi(x_1(t) \pm \ldots \pm x_n(t))/n) < n^{-1} 2^{n-1} \sum_{i=1}^{n} \Phi(x_i(t)).$$

Applying this inequality and taking into account that $I(x_i) = 1$, we get

$$2^{n-1} - \Sigma^{\circ} I((x_1 \pm \ldots \pm x_n)/n) = n^{-1} 2^{n-1} \sum_{i=1}^{n} I(x_i) - \Sigma^{\circ} I((x_1 \pm \ldots \pm x_n)/n)$$

$$\geq n^{-1} 2^{n-1} \sum_{i=1}^{n} I(x_i \chi_A) - \Sigma^{\circ} I((x_1 \pm \ldots \pm x_n)\chi_A/n) > 0.$$

This means that $I((x_1 \pm \ldots \pm x_n)/n) < 1$ for some choice of signs. Applying condition (i), we get $\|x_1 \pm \ldots \pm x_n\|_{\Phi} < n$ for some choice of signs (see [1], [8] and [12]). The proof of sufficiency is finished.

Necessity. If condition (i) is not satisfied, then $L^{\Phi}(\mu)$ contains an isometric copy of l^{∞} (see [12]) . Thus, by LEMMA 1, $L^{\Phi}(\mu)$ is not non-$l_n^{(1)}$. Now, assume that Φ satisfy condition (i) and does not satisfy condition (ii) . Then, there exists u such that $\Phi(u) \geq n/\mu(T)$ and $\Phi(u/n) = \Phi(u)/n$. Let $A_1,\ldots,A_n$ be pairwise disjoint and Σ-measurable subsets of T such that $\mu(A_i) = 1/\Phi(u)$ for $i=1,\ldots,n$. We have $\sum_{i=1}^{n} \mu(A_i) = n/\Phi(u) \leq \mu(T)$, so such sets A_i exist. Defining $x_i = u\chi_{A_i}$ for $i=1,\ldots,n$, we get

$$I(x_i) = I((x_1 \pm \ldots \pm x_n)/n) = n\, I(x_1/n) = I(x_1) = 1$$

for any choice of signs and for $i=1,\ldots,n$. So, $\| x_1 \pm \ldots \pm x_n \|_{\Phi} = n$ for any choice of signs and $\|x_i\|_{\Phi} = 1$ for $i = 1,\ldots,n$, i.e. $L^{\Phi}(\mu)$ is not non-$l_n^{(1)}$.

Define that the modular I is non-$l_n^{(1)}$ if for every $x_1,\ldots,x_n \in L^{\Phi}(\mu)$ with $I(x_1) = \ldots = I(x_n) = 1$, we have $I((x_1 \pm \ldots \pm x_n)/n) < 1$ for some choice of signs.

The modular I is locally uniformly non-$l_n^{(1)}$ if for any $x_1,\ldots,x_n$ in $L^{\Phi}(\mu)$ with $I(x_1) = \ldots = I(x_n) = 1$, there exists $\delta(x_1) \in (0,1)$ (depending only on x_1) such that $I((x_1 \pm \ldots \pm x_n)/n) \leq (1 - \delta(x_1))$ for some choice of signs

COROLLARY 1. Our theorems for I instead of $\| \ \|_{\Phi}$ are true without suitable condition Δ_2.

Indeed, suitable condition Δ_2 was used only to the implications $\|x\|_{\Phi} = 1 \Longrightarrow I(x) = 1$ and I is non-$l_n^{(1)}$ (locally uniformly non-$l_n^{(1)}$) implies that $\| \ \|_{\Phi}$ is non-$l_n^{(1)}$ (locally uniformly non-$l_n^{(1)}$).

COROLLARY 2. $L^{\Phi}(\mu)$ is locally uniformly non-$l_n^{(1)}$ whenever it is rotund.

Proof. If $L^{\Phi}(\mu)$ is rotund, then Φ is strictly convex on the whole $\mathbb{R}$ in the case of an atomless measure and on the an interval $[0,a]$ in the case of a purely atomic measure as in THEOREM 1 (see [1],[6], [10] and [12]) . Hence it follows that $\Phi(u/n) < \Phi(u)/n$ for any $u > 0$.

REMARK 1. The converse statement to COROLLARY 2 does not hold.

Proof. Note that $L^{\Phi}(\mu)$ may be locally uniformly non-$l_n^{(1)}$ even if Φ is linear on an interval $[a,\infty)$, when μ is as in THEOREMS 1 and 2 and on the interval $[0,n\theta/\mu(T)]$, where $\theta \in (0,1)$, when μ is as in THEOREM 3.

REMARK 2. Condition Δ_2 at infinity and the condition $\Phi(u/n) < \Phi(u)/n$ are sufficient in order that $L^{\Phi}(\mu)$ be locally uniformly non-$l_n^{(1)}$

in the case of an atomless finite measure.

REMARK 3. The definitions of non-$l_n^{(1)}$ property and local uniform non-$l_n^{(1)}$ property remain the same if we replace $\|x_i\|=1$ by $\|x_i\|\leq 1$ for $i=1,\dots,n$ (in the case $n=2$ see [11]).

COROLLARY 3. If Φ is an Orlicz function vanishing only at zero and satisfying the condition $\lim_{u\to 0}(\Phi(u)/u)=0$, then $L^{\Phi}(\mu)$ is locally uniformly non-$l_n^{(1)}$ iff it is non-$l_n^{(1)}$ and iff Φ satisfies suitable (to the measure μ) condition Δ_2.

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