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ON THE MECHANICS WITH NONCONTINUOUS HAMILTONIANS

W.Kondracki, M.Kozak

INTRODUCTION

In physical applications we have sometimes to deal with mechanical systems with nondifferentiable, noncontinuous or even numerical hamiltonians. Such systems appear, for example, when we consider a reflection from a perfectly hard wall, a passage of a particle through a potential barier or in a control theory with noncontinuous controls. It appears a natural necessity to extend Hamilton mechanics on a bigger class of hamiltonians - formulas until existing were demanding a differentialability of hamiltonians. A purpose of this paper is a formulation of mechanics with nondifferentiable or noncontinuous hamiltonians. In order to do it we have introduced a notion of an overflow, which is more common as the notion of a flow and more adequate to describe a dynamics of the mechanical system.

§ 1. HAMILTONIAN MECHANICS IN SYMPLECTIC FORMALISM.

Let ω be a nondegenerated and closed differential two-form on 2n-dimensional smooth manifold S. Let us consider a mechanical system (S,ω,H) with hamiltonian H being a smooth function on a phase-space S,A dynamics of the system (S,ω,H) is described by a flow $\varphi^H: S\times\mathbb{R}^1 \supset D \ni (s,t) \to \varphi^H(s,t) \equiv \varphi^H_t(s) \in S$. The flow φ^H_t is unicely determined by a following equation: $\frac{d}{dt}\big|_0 \varphi^H_t \lrcorner\, \omega = X_H \lrcorner\, \omega = -dH$ where X_H denotes a vector field of the hamiltonian H. From a physical point-of view the flow ought to satisfy following postulates:

1.1. <u>Axiom of symmetry</u>.

If ξ is a symplectomorphism and $H^1 = H\circ\xi$ then $\varphi^{H^1}_t = \xi\circ\varphi^H_t\circ\xi^{-1}$

1.2. <u>Principle of locality</u>.

If Θ is an open subset of S and $H_1|_\Theta = H_2|_\Theta$ then $\varphi^{H^1}_t|_\Theta = \varphi^{H_2}_t|_\Theta$

1.3. <u>Calibration of energy</u>

This paper is in final form and no version of it will be submitted for publication elsewhere.

If $H_1 = H + e$, $e = \text{constant}$ then $\varphi_t^{H_1} = \varphi_t^{H_2}$

1.4. Connection beetween time - unit and energy

If $H_1 = cH$, $c = \text{constant}$ then $\varphi_{ct}^{H_1} = \varphi_t^{H}$

1.5. Principle of conservation of energy and laws of mechanics

φ_t^H keeps H and ω : $\forall s \in S \ \frac{d}{dt} H({}_t^H\varphi(s)) = 0 \quad \varphi_t^*\omega = \omega$, where

$\varphi_t^* : \Gamma^\infty(\Lambda^2 T^* S) \to \Gamma^\infty(\Lambda^2 T^* S)$ is a transport of skewsymmetric two-forms. Sometimes in physical applications it is very useful to distinguish a n-dimensional configuration space M of the system (S,ω,H) in it's 2-n dimensional phase-space S. We can do it on a following way:

1.6. The phase-space S of the mecanical system is the space of a cotangent bundle (T^*M,π,M). Each fibre of the bundle is a vector space of momenta p of the mechanical system.

1.7. The hamiltonian H is a smooth function on T^*M $H:T^*M \to \mathbb{R}^1$. In each of fibres we can define a Riemannian structure $\langle\cdot,\cdot\rangle$ and the hamiltonian H as follows: $H(x,p) = \langle p|p\rangle + \tilde{V}(x)$, where $\tilde{V}(x) = V(\pi(x))$ is a lifting by a mapping π of the potential V with a domain on the configuration space M_o. $\langle p|p\rangle$ has a sens of a kinetic energy, $\tilde{V}(x)$ a potential energy of the system (S,ω,H).

1.8. The skew-symmetric two-form ω is defined as follows: Let α be any smooth section of the bundle T^*M. $(\alpha \in \Gamma^\infty(T^*M))$. Two form ω satisfies a formula: $\alpha^*\omega = d\alpha$, $\alpha^* \in \Gamma^\infty(\Lambda^2 T^* S) \to \Gamma^\infty(\Lambda^2 T^* M)$ $S = T^*M$. It appears that so defined form ω is closed and nondegenerated.

Let us notice that a procedure of describing the dynamics of the mechanical system by a notion of the flow concerns situations when hamiltonian is of class C^1. In a practice we deal often with systems with nondifferentiable or even noncontinuous hamiltonians. In order to describe the dynamics of such systems let us introduce a notion of a more general object - an overflow which could globally describe a history of the mechanical system, also after a reaching hamiltonian nondifferentialability points.

1.9. DEFINITION

Let S be a manifold, D an opened subset of $R^1 \times S$. A mapping: $D \ni (t,x) \to \varphi_t(x) \in S$ is an overflow if are satisfied the following conditions:

1° Let $(x,s) \in D$. There exists an open subset $\Theta \subset S$ such that $s \in \Theta$ and $\varphi_t(\cdot)$ is a smooth diffeomorphism Θ on $\varphi_t(\Theta) \subset S$.

2° $\forall s \in S$ $\varphi_{(\cdot)}(s)$ is a smooth curve in S, parametrised by opened subset of such t for wchich $(t,s) \in D$

3° If $(t,s),(l,s),(t+l,s) \in D$ then $(l,\varphi_t(s)) \in D$ and $\varphi_l(\varphi_t(s)) = \varphi_{l+t}(s)$

The notion of overflow allows us to consider integral curves $\varphi_{(\cdot)}(s)$ defined on nonconnected subsets of R^1. Two following theorems give connections beetween flows and overflows:

1.10. Theorem

If M is a manifold, N it's submanifold and $\varphi_t^o(x)$ a flow on domain $d_o \subset M$ then $\varphi_t(x) \overset{df}{=} \varphi_t^o(x)|_d$ where d is a set of such points belonging to d_o for which $x \in N$ and $\varphi_t^o(x) \in N$ is an overflow.

1.11. Theorem

Let $\varphi_t(s)$ be an overflow defined on $D \subset \mathbb{R}^1 \times S$. We denote D_o as a set of such points $(t,s) \in D$ for which $(\alpha t,s) \in D$ for every $\alpha \in [0,1]$. Then D_o is opened and connected subset of $\mathbb{R}^1 \times S$ and an overflow $\varphi_t(s)|_D$ is a flow.

Let us notice, that overflow determines unicely a vector field but a vector field gives in general a lot of different overflows.

1.12. Example

Let $M = \mathbb{R}^1$. Let us consider a flow $\varphi_t^o(x) = x + t$ with domain $D_o = \mathbb{R}^1 \times \mathbb{R}^1$. If we remove a point 0 from M, then D will be a set of such points (t,x) for which $x \neq 0$ and $t + x \neq 0$. Then $\varphi_t^o(x)$ is an overflow on $\mathbb{R}^1 - \{0\}$, because integral curves $\varphi_{(\cdot)}^o(\cdot)$ aren't connected.

§ 2. THE EVOLUTION OF A MECHANICAL SYSTEM WITH A NONCONTINUOUS HAMILTONIAN.

We consider cases in which a configuration space $M = \mathbb{R}^1$:

2.1. A reflection on an ideally hard wall.

Let us consider a hamiltonian given by a formula:

$$\mathbb{R}^1 \ni (x,p) \to H(x,p) = \frac{p^2}{2m} + V(x) \in \mathbb{R}^1, \text{ where}$$

$$V(x) : \mathbb{R}^1 \ni x \to V(x) = \begin{cases} 0 & \text{for } x \in \mathbb{R}^1 - \{0\} \\ \infty & \text{for } x = 0 \end{cases}$$

We approximate the potential $v(x)$ by a sequence of smooth functions

$V_n(x)$ satisfying following conditions:

2.1.1. $V_n(x)$ converges pointwisely to $V(x)$ for $x \in \mathbb{R}^1$

2.1.2. $V_n(x)$ converges almost uniformly to $V(x)$ for $x \in \mathbb{R}^1 - \{0\}$

2.1.3 $\forall E > 0 \ \exists N_1 \in \mathbb{N} \ \forall n > N_1 \ V_n'(x) > 0$ for $x \in]-\infty, x_0[$

$V_n'(x) < 0$ for $x \in]x_1, \infty[$

2.1.4. $\forall E > 0 \ \exists N_2 \in \mathbb{N} \ \forall n > N_2 \ V_n''(x) \geq 0$ for $x \in]-\infty, x_0[\cup]x_1, \infty[$

where $x_0, x_1 : V(x_0) = V(x_1) = E$, E is an energy of a system

2.1.5. <u>Theorem</u>

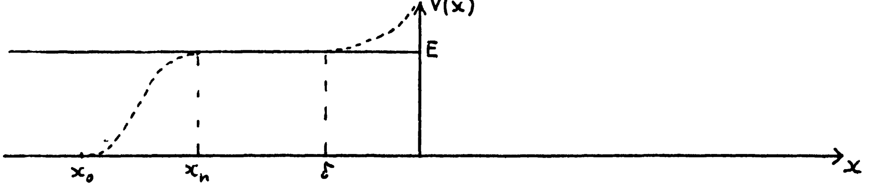
If a sequence of functions V_n satisfies the above conditions, then a sequence of flows φ_k corresponding to hamiltonians $H = \frac{p^2}{2m} + V_n(x)$ converges pointwisely to an overflow

$$\varphi_t(x,p) = \left(-\left|x + \frac{pt}{m}\right|, p \,\mathrm{sgn}\left(\frac{xm}{p} - t\right)\right), \quad p > 0, \ x < 0, \ t \neq \frac{xm}{p} \quad \text{on}$$

$$\mathbb{R}^2 - (\{0\} \times \mathbb{R}^1)$$

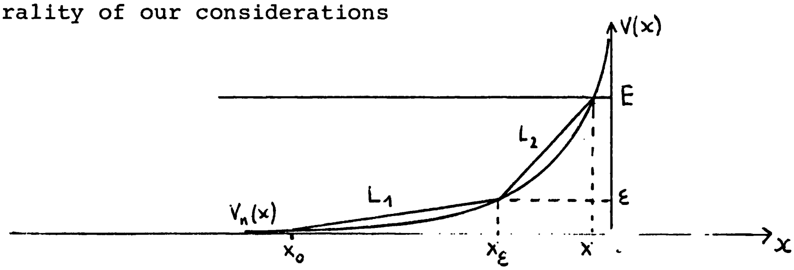
we say that the overflow corresponds to hamiltonian H with a potential V.

From a theorem, that if coefficients of a vector field converge almost uniformly, then flows corresponding to them converge at least pointwisely, we obtain a convergence of flows for $x \in]-\infty, 0[$ and $t < \frac{-xm}{p}$. To prove the convergence for the whole flow, it means for $t > \frac{-xm}{p}$ it is enough to show that a time of passing from a state (x_0, p), $x_0 < 0, p > 0$ to a state (x_0, p) with a reflection on a wall converges to $|2x_0|\frac{m}{p}$. Conditions 2.1.3, 2.1.4 ensure that a sequence of times of passing from a state (x_0, p) to a state $(x_0, -p)$ corresponding to a sequence of hamiltonians $H_n(x,p) = \frac{p^2}{2m} + V_n(x)$ will be convergent. One should, for example, eliminate cases in which $V_n(x)$ satisfies conditions 2.1.1. and 2.1.2 and has a saddle (it means $V_n(x) = 0$) with vale E on interval $]x_n, \delta[$



In such a case a passing from a state (x_0, p) $x_0 < x_n < 0$, $p < 0$ $p = \sqrt{2mE}$ to a state $(x_0, -p)$ is not possible in a finite time because when a saddle is achieved by a system, it's state is not changing. We call such behaviour of mechanical system as a waste of time.

Let us consider a state (x_o,p) with an energy E; we will realize our argumentation for a left half-axis, whatever doesn't brake a generality of our considerations



Let us introduce a following notation:

$t_{x_o x_1}$ - a time of passing a system from a state (x_o,p) to a state (x_1,p)

$t_{x_o O x_o}$ - a time of passing a system from a state (x_o,p) to a state $(x_o,-p)$

Let $\varepsilon: O < \varepsilon < E$, $x_\varepsilon : V_n(x_\varepsilon) = \varepsilon$, $x_E : V_n(x_E) = E$

We have a common formula $t_{x_o x_1} = \sqrt{\frac{m}{2}} \int_{x_o}^{x_1} \frac{dx'}{\sqrt{E-V(x')}}$.

Let us notice that $t_{x_o O x_o}$ is limited below by $t_{\min\, x_o O x_o}$ -a time of passing from a state (x_o,p) to a state $(x_o,-p)$ with a potential $V(x) = O$, for $x \in [x_o, O[$ and $t_{\min\, x_o O x_o} = -x_o \sqrt{\frac{2m}{E}}$ $x_o < O$. It is easy to see that we haven't any waste of time because from assumptions 2.1.3 and 2.1.4 there exists for $t_{x_o O x_o}$ a top estimation $t_{\sup\, x_o O x_o}$ defined by an approximation of $V_n(x)$ by linear functions L_1 and L_2 on intervals $[x_o,x_\varepsilon],[x_\varepsilon,x_E]$:

$$L_1(x) = \frac{\varepsilon - V_n(x_o)}{x_\varepsilon - x_o}(x-x_o) + V_n(x_o), \quad L_2(x) = \frac{E-\varepsilon}{x_E - x_\varepsilon}(x-x_\varepsilon) + \varepsilon$$

We can write $t_{\sup\, x_o O x_o}$ explicite

$$t_{\sup\, x_o O x_o} = 2\sqrt{\frac{m}{2}} \lim_{\substack{n\to\infty \\ \varepsilon \to O}} \left(\int_{x_o}^{x_\varepsilon} \frac{dx'}{\sqrt{E-L_1(x')}} + \int_{x_\varepsilon}^{x_E} \frac{dx'}{\sqrt{E-L_2(x')}} \right) = -x_o\sqrt{\frac{2m}{E}}$$

Let us notice that $t_{\sup\, x_o O x_o} \geq t_{x_o O x_o} \geq t_{\min\, x_o O x_o}$ and

$t_{\sup x_o 0 x_o} = t_{\min x_o 0 x_o} = -x_o\sqrt{\frac{2m}{E}}$, so we obtain $t_{x_o 0 x_o} = -x_o\sqrt{\frac{2m}{E}}$ what we wanted to show.

2.2. Now we consider a hamiltonian $\mathbb{R}^2 \ni (x,p) \to H(x,p) = \frac{p^2}{2m} + V(x)$

$$\mathbb{R}^1 \ni x \to V(x) = \begin{cases} 0 & x \in]-\infty,0[\\ E & x \in [0,\infty[\end{cases}$$

a) let us consider a state with energy $E_o < E$. We approximate a potential $V(x)$ by a family of functions $V_n(x)$ satisfying following conditions:

2.2.1. $V_n(x)$ converges almost uniformly with a first derivative to $V(x)$ for $x \in \mathbb{R}^1 - \{0\}$

2.2.2. $V_n(x)$ converges pointwisely to $V(x)$ for $x = 0$

2.2.3. $V_n'(x) \geq 0$ for $x \in]-\infty,\infty[$

2.2.4. $\exists N \in \mathbb{N}\ \forall n > N\ V''(x) \geq 0$ on interval $]-\infty,x_o[$

Using the same procedure as in the case 2.1 we obtain that a time of passing from a state (x_1,p) to a state $(x_1,-p)$ is equal $-2x_1\frac{m}{p}$.

An overflow has a form:

$$\varphi_t(x,p) = (-|x + \frac{pt}{m}|, p\ \mathrm{sgn}(\frac{xm}{p} - t)),\quad t \neq \frac{xm}{p}$$

b) States with energy $E_o = E$. An evolution of a mechanical system has a physical sense only to a moment of achievement of a point $x = 0$ by the system.

c) States with energy $E_o > E$. Overflow has a form $\varphi_t(x,p) = (x + \frac{pt}{m}, p - \Theta(x)\sqrt{2mE})$, where we have chosen $x \in]-\infty,0[$, $p > 0$.

Let us notice that a time of passing from a state (x_1,p) to a state (x_2,p), where $x_1x_2 > 0$, $p > 0$ is equal $|x_1-x_2|\frac{p}{m}$.

We ought to examnine if it appears a waste of time by passing from a state (x_1,p) to a state $(x_2,p-\sqrt{2mE})$ $x_1 < 0$, $x_2 > 0$, $p > 0$. For this purpose let us calculate a time of passing from a state $(-\varepsilon,p)$ to a state $(\varepsilon,p-\sqrt{2mE})$,

$$\varepsilon > 0 : t_{-\varepsilon\varepsilon} = \frac{m}{2}\int_{-\varepsilon}^{\varepsilon} \frac{dx'}{\sqrt{E_o - V(x')}} .$$

Finding $\lim_{\varepsilon \to 0} t_{-\varepsilon\varepsilon} = 0$ we see that we haven't any waste of time.

2.3. Let us consider a hamiltonian

$\mathbb{R}^2 \ni (x,p) \to H(x,p) = \frac{p^2}{2m} + V(x)$ where

$$V(x) = \begin{cases} 0 & \text{for } x \in \mathbb{R}^1 - \{0\} \\ E & \text{for } x = 0 \end{cases}$$

a) States with energy $E_o < E$. On the same way as in the case 2.2 we will approximate a potential $V(x)$ by a family of smooth functions keeping following conditions:

$$x_o : V_n(x_o) = E_o \quad x_o = \begin{cases} x_{o1}, & x_o < 0 \\ x_{o2}, & x_o > 0 \end{cases}$$

2.3.1. $V_n(x)$ converges almost uniformly with the first derivative to $V(X)$ on a set $\mathbb{R}^1 - \{0\}$.

2.3.2. $V_n(x)$ converges pointwisely to $V(x)$ for $x = 0$

2.3.3. $\forall E_o < E \exists N_1 \in N \ \forall n > N_1 \ V_n'(x) \geq 0$ for $x \in]-\infty, x_{o1}[$

$V_n'(x) \leq 0$ for $x \in]x_{o2}, \infty[$

2.3.4. $\forall E_o < E \ \exists N_2 \in N \ \forall n > N_2 \ V_n''(x) \geq 0$ for $x \in]-\infty, x_{o1}[\cup]x_{o2}, \infty[$

In this case an overflow has a form

$$\varphi_t(x,p) = (-|x + \frac{pt}{m}|, p \ \mathrm{sgn}(-\frac{xm}{p} - t)), \quad t \neq \frac{xm}{p}$$

(b) States with energy $E_o = E$. If a initial state is (x,p) $x<0, p>0$ then an evolution of a system we can characterize only to a moment, when a system achieves a point $x = 0$

(c) States with energy $E_o > E$. Overflow has a from:

$$\varphi_t(x,p) = \begin{cases} (x + \frac{pt}{m}, p - \sqrt{2mE}), & x = 0 \\ (x + \frac{pt}{m}, p), & x \in \mathbb{R}^1 - \{0\} \end{cases}$$

where we have chosen $x \in]-\infty, 0[, p > 0$.

A time of passing from a state (x_1,p) to a state (x_2,p) for $x_1 x_2 > 0, p > 0$ is equal $t_{x_1 x_2} = |x_1 - x_2|\frac{p}{m}$. $t_{x_1 x_2}$ is also equal

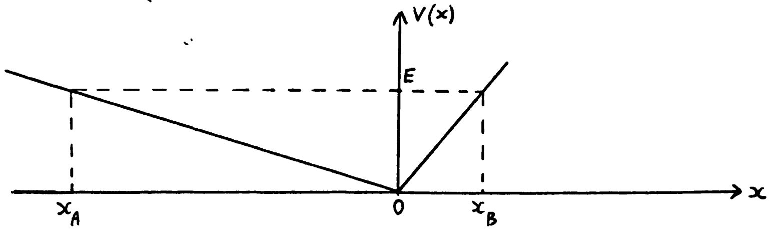
$|x_1 - x_2|\frac{p}{m}$ for $p > 0$, $x_1 x_2 < 0$ because there is no waste of time by passing from a state (x_1,p) to a state $(x_2,p), x_1 x_2 < 0$:

we have $\lim_{\varepsilon \to 0} t_{-\varepsilon\varepsilon} = \lim_{\varepsilon \to 0} (\sqrt{\frac{m}{2}} \int_{-\varepsilon}^{\varepsilon} \frac{dx'}{\sqrt{E_o - V(x')}}) = 0$ on the same way as in case 2.2.

2.4. As the last, the us consider hamiltonian:

$$\mathbb{R}^1 \ni (x,p) \to H(x,p) = \frac{p^2}{2m} + V(x) \quad \text{where}$$

$$\mathbb{R}^1 \ni x \to V(x) = \begin{cases} -\alpha x & \text{for } x \in]-\infty,0[\,, \quad \alpha > 0 \\ \beta x & \text{for } x \in]0,\infty[, \quad \beta > 0 \end{cases}$$



Let us consider a state with energy $E > 0$. As in above cases we approximate a potential $V(x)$ by a family of smooth functions $V_n(x)$ satisfying the following condition:

$$x_o : V_n(x_o) = E, \qquad x_o = \begin{cases} x_{o1}, & x_o < 0 \\ x_{o2}, & x_o > 0 \end{cases}$$

2.4.1. $V_n(x)$ converges almost uniformly with the first derivative to $V(x)$ on a set $\mathbb{R}^1 - \{0\}$.

2.4.2. $V_n(X)$ converges pointwisely to $V(x)$ on $\mathbb{R}^1$.

2.4.3 $\forall E > 0 \quad \exists N_1 \in N \forall n > N_1 \begin{cases} V_n'(x) < 0, & x \in]x_{o1}-\varepsilon,0[, \ \varepsilon>0 \\ V_n'(x) > 0, & x \in]0,x_{o2}+\varepsilon[, \ \varepsilon>0 \end{cases}$

2.4.4. $\forall E > 0 \exists N_2 \in N \ \forall n > N_2 \quad V_n''(x) \geq 0, \ x \in]x_{o1} - \varepsilon, x_{o2} + \varepsilon[$.

we have, of course $t_{x_1x_2} = \sqrt{\frac{m}{2}} \cdot \int_{x_1}^{x_2} \frac{dx'}{\sqrt{E-V(x')}}$

Let $x_A, x_B : V(x_A) = V(x_B) = E$

It is easy to see that a system has not any waste of time. It could be only in neighbourhoods of the points x_A and x_B, but the condition 2.4.3 dispels our doubts. The overflow has a form: $\varphi_t(-t,p) = (x(t);p(t))$ where we have chosen a state $(-x,p)$, where $x > 0$, $p > 0$ as an initial state.

$$x(t) = \begin{cases} -x + \frac{p}{m}t + \frac{\alpha t^2}{2}, & t \in A \\ \sqrt{\frac{p^2}{m^2} + 2x\alpha}\,[t + \frac{p}{m\alpha} - \frac{1}{\alpha}\sqrt{\frac{p^2}{m^2} + 2x\alpha}] - \frac{\beta}{2}[t + \frac{p}{m\alpha} - \frac{1}{\alpha}\sqrt{\frac{p^2}{m^2} + 2x\alpha}]^2, & t \in B \\ \frac{1}{2\beta}(\frac{p^2}{m^2} + 2x\alpha) - \frac{\beta}{2}[t - \frac{1}{\beta}\sqrt{\frac{p^2}{m^2} + 2x\alpha}\,(1 + \frac{\beta}{\alpha}) + \frac{p}{m\alpha}]^2, & t \in C \\ -\sqrt{\frac{p^2}{m^2} + 2x\alpha}\,[t - \frac{1}{\beta}\ \frac{p^2}{m^2} + 2x\alpha(2 + \frac{\beta}{\alpha}) + \frac{p}{m\alpha}] + & \\ + \frac{\alpha}{2}[t - \frac{1}{\beta}\ \frac{p^2}{m^2} + 2x\alpha(2 + \frac{\beta}{\alpha}) + \frac{p}{m\alpha}]^2 & t \in D \end{cases}$$

$$p(t) = \begin{cases} p + \alpha t m, & t \in A \\ m\sqrt{\frac{p^2}{m^2} + 2x\alpha} - m\beta(t + \frac{p}{m\alpha} - \frac{1}{\alpha}\sqrt{\frac{p^2}{m^2} + 2x\alpha}), & t \in B \\ -m\beta[t - \frac{1}{\beta}\ \frac{p^2}{m^2} + 2x\alpha(1 + \frac{\beta}{\alpha}) + \frac{p}{m\alpha}], & t \in C \\ -m\sqrt{\frac{p^2}{m^2} + 2x\alpha} + m\alpha[t - \frac{1}{\beta}\sqrt{\frac{p^2}{m^2} + 2x\alpha}\,(2 + \frac{\beta}{\alpha}) + \frac{p}{m\alpha}], & t \in D \end{cases}$$

$$A = [nT, -\frac{p}{m\alpha} + \sqrt{\frac{p^2}{m^2} + 2x\alpha}\ \frac{1}{\alpha} + nT[$$

$$B =]nT + \frac{1}{\alpha}\sqrt{\frac{p^2}{m^2} + 2x\alpha} - \frac{p}{m\alpha},\ \frac{1}{\beta}\sqrt{\frac{p^2}{m^2} + 2x\alpha}\,(1 + \frac{\beta}{\alpha}) - \frac{p}{m\alpha} + nT[$$

$$C =]nT + \frac{1}{\beta}\sqrt{\frac{p^2}{m^2} + 2x\alpha}\,(1 + \frac{\beta}{\alpha}) - \frac{p}{m\alpha},\ \frac{1}{\beta}\sqrt{\frac{p^2}{m^2} + 2x\alpha}\,(2 + \frac{\beta}{\alpha}) - \frac{p}{m\alpha} + nT[$$

$$D =]nT + \frac{1}{\beta}\sqrt{\frac{p^2}{m^2} + 2x\alpha}\,(2 + \frac{\beta}{\alpha}) - \frac{p}{m\alpha},\ (\frac{2}{\beta} + \frac{2}{\alpha})\sqrt{\frac{p^2}{m^2} + 2x\alpha} - \frac{p}{m\alpha} + nT[$$

where $T = (\frac{2}{\beta} + \frac{2}{\alpha})\sqrt{\frac{p^2}{m^2} + 2x\alpha}$ is a period of a movement of system

It is easy to generalize all above cases on a case of n-dimensional configuration space. We have the following theorem.

2.5. <u>Theorem</u>.

Assumptions: Let a differentiable manifold M be a n-dimensional configuration space of a mechanical system. Let M_o be a (n-1)-dimensional submanifold in M. Let in a vector bundle T^*M be a

Riemannian structure <.,.>. Iet us assume that on T^*M we have a hamiltonian $H = \langle p|p\rangle + \tilde{V}$, where $\tilde{V}$ is given by lifting by π of V being a function defined on M, smooth in points of $M - M_o$. V has besides a following property: $\forall x \in M_o$ there exists such mapping that open neighbourhood $U \ni x$ is mapped on $\kappa(U) \subset \mathbb{R}^n$ and $\kappa(U \cap M_o) \to \kappa(U) \cap \mathbb{R}^{n-1}$, $V|_U = \tilde{V}(\kappa(U))$ where $\tilde{V}(x^1,\ldots,x^n) = \hat{V}(x^1)$ and $(x^2,\ldots,x^n)$ are coordinates of $\mathbb{R}^{n-1}$ and $\hat{V}(x^1)$ has in point $x^1 = 0$ a germ equal germs in 0 of potentials considered in case 1,2,3,4.

Thesis: There exists the unique overflow corresponding on a natural way to hamiltonian H. The physical postulates from a chapter one are fulfilled.

Outline of a proof: if we write a problem in the map as in the assumption of the theorem 2.5 we obtain one of the cases presented above. Smooth hamiltonians considered in those cases are satisfying our postulates and it is easy to see that they are also fulfilled by a limit of flows φ_n corresponding to hamiltonians H_n.

As we see the introduced notion of an overflow enable us to describe a dynamics of mechanic systems with nondifferentiable or even non--continuous hamiltonians. A history of a mechanical system described in terms of a flow was coming to the end in singularity-points of a hamiltonian-using the notion of an overflow we overcome these difficulties and we are able to describe the dynamics of a mechanical system also after reaching the hamiltonian-singularity points.

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