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# On some convexity properties of Musielak-Orlicz spaces

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# On some convexity properties of Musielak-Orlicz spaces

by

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Abstract. It is shown here that geometrical properties such as rotundity, local uniform rotundity, uniform rotundity in every direction, are equivalent in the Musielak-Orlicz spaces equipped with Luxemburg norm, if the measure is atomless.

Introduction. This paper is a continuation of the investigations concerning the geometrical properties in the space of Orlicz type (e.g. [2], [3], [4], [6], [7], [8]). Here we are interested in such properties as uniform rotundity in every direction and local uniform rotundity in the generalized Orlicz spaces, called Musielak-Orlicz spaces. We are finding tests for these properties. The problem concerning the local uniform rotundity of the Orlicz space was solved in [8], either in the case of atomless measure or in the case of a sequence space. Now, we recall the needed definitions and notations.

We say that a Banach space X is locally uniformly rotund (LUR), [10], if for each $\varepsilon > 0$ and each $y \in X$ with $\|y\| = 1$ there is a $\delta(y,\varepsilon) > 0$ such that if $x \in X$ with $\|x\| = 1$ and $\|x - y\| \geq \varepsilon$, then $\|(x + y)/2\| \leq 1 - \delta(x,\varepsilon)$.

A Banach space X is uniformly rotund in every direction (URED), [1], [10], if for each $\varepsilon > 0$ and nonzero $z \in X$ there exists $\delta(z,\varepsilon) > 0$ such that if x and y belong to X with $\|x\| = \|y\| = 1$. $\|x - y\| \geq \varepsilon$ and $x - y = \alpha z$ for some $\alpha \in \mathbb{R}$, then $\|(x + y)/2\| \leq 1 - \delta(z,\varepsilon)$.
It is known, by the paper [1], that the property URED is equivalent to the following one:

For each nonzero z in X there is a positive number $\delta(z)$ such that if $x \in X$ with $\|x\| \leq 1$ and $\|x + z\| \leq 1$ then $\|x + \frac{1}{2}z\| \leq 1 - \delta(z)$.
In the sequel we shall use this definition. The above mentioned and

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This paper is in final form and no version of it will be submitted for publication elsewhere.

other convexity properties e.g. midpoint local uniform rotundity (MLUR) are given and exactly examined in [10]. Here, let us note that LUR $\rightarrow$ MLUR $\rightarrow$ R and URED $\rightarrow$ R. Now, we introduce some notions joined with Musielak-Orlicz spaces (for details see [9]). Let $T, \Sigma, \mu$ be a measure space, where $T$ is an arbitrary set, $\Sigma$ a $\sigma$-algebra of subset of $T$ and $\mu$ - a nonnegative, complete, atomless measure defined on $\Sigma$. All subsets of $T$ appearing in this note are measurable, i.e. they belong to $\Sigma$. By $\mathcal{M}$ denote a set of all $\mu$-measurable functions $x: T \rightarrow \mathbb{R}$. The functions different only on a null set are considered as identical Let $\varphi: \mathbb{R}\times T \rightarrow [0,+\infty)$ be a convex, even function of $u$, $\varphi(0,t) = 0$ outside of some null set and let it be a $\mu$-measurable function of $t$ for all $u \in \mathbb{R}$. For fixed $t \in T$, such functions are usually called Young or Orlicz functions. The Musielak-Orlicz space $L_\varphi$ is the subset of $\mathcal{M}$ such that $I_\varphi(\lambda x) = \int_T \varphi(\lambda x(t),t)\,d\mu < \infty$ for some $\lambda>0$ dependent on $x$. The functional $\|x\|_\varphi = \inf\{\varepsilon>0: I_\varphi(x/\varepsilon) \leqslant 1\}$ is a norm in this space, usually called Luxemburg norm. We say that $\varphi$ satisfies the condition $\Delta_2$, if there are a constant $k>0$ and a nonnegative function $h$, such that $\int_T h(t)\,d\mu < \infty$ and $\varphi(2u,t) \leqslant k\varphi(u,t) + h(t)$ for a.e. $t \in T$. Let us note that in this condition, if $\varphi(u,t)>0$ for $u \neq 0$ then the function $h$ may be chosen in such a way that the integral $\int_T h(t)\,d\mu$ is arbitrarily small [4]. Recall that the function $\varphi$ is strictly convex a.e. in $T$ if for all $u, v, \alpha, \beta \in \mathbb{R}$ such that $\alpha,\beta \geqslant 0$ and $\alpha + \beta = 1$ we have $\varphi(\alpha u + \beta v, t) < \alpha\varphi(u,t) + \beta\varphi(v,t)$ for each $t$ outside of some null set. We formulate the notion of LUR and URED for modular $I_\varphi$ in the space $L_\varphi$, replacing the space $X$ by $L_\varphi$ and the norm by the modular, in suitable definitions. For instance, we say that $I_\varphi$ is uniformly rotund in every direction in the space $L_\varphi$, if for each nonzero $z \in L_\varphi$ there exists $\delta(z)>0$ such that if $x \in L_\varphi$ and $I_\varphi(x) \leqslant 1$ and $I_\varphi(x+z) \leqslant 1$ then $I_\varphi(x+\frac{1}{2}z) \leqslant 1-\delta(z)$.

**0.1.Theorem** [2],[3]. The space $L_\varphi$ is rotund iff $\varphi$ is strictly convex a.e. in T and satisfies the condition $\Delta_2$.

**0.2.Theorem** [5] .The modular convergence is equivalent to the norm convergence in $L_\varphi$ (i.e. $I_\varphi(x) \to 0 \Leftrightarrow \|x\|_\varphi \to 0$) iff $\varphi$ satisfies the condition $\Delta_2$ and $\varphi(u,t) > 0$ for $u \neq 0$ outside of some null set.

Instead of the last condition in this theorem,we often write that $\varphi$ vanishes only at zero.The proofs of the next two lemmas will be omitted,because applying Theorem 0.2,they are similar to that of Lemma 1 in [6] (see also th.1.11 in [4]) and Lemma 0.2 in [8] .

**0.3.Lemma.** The space $L_\varphi$ is locally uniformly rotund [uniformly rotund in every direction] iff the modular $I_\varphi$ is locally uniformly rotund [uniformly rotund in every direction] , $\varphi$ satisfies the condition $\Delta_2$ and $\varphi$ vanishes only at zero.

**0.4.Lemma.** If $\varphi$ satisfies the condition $\Delta_2$ and $\varphi$ vanishes only at zero then for every $\varepsilon > 0$ there is a $\delta > 0$ such that for all $x \in L_\varphi$ and $y \in \{z \in L_\varphi : \|z\|_\varphi \leqslant 1\}$ the condition $I_\varphi(x - y) < \delta$ implies $|I_\varphi(x) - I_\varphi(y)| < \varepsilon$ .

Results.

**1.Lemma.** If $\varphi$ is strictly convex a.e. in T,then for every $\varepsilon > 0$ and $d_1, d_2 \in (0,\infty)$, $d_1 < d_2$, there exists a measurable function $p: T \to (0,1)$ such that

$$\varphi((u + v)/2\ ,t) \leqslant (1 - p(t))\,(\varphi(u,t) + \varphi(v,t))/2$$

for a.e. $t \in T$, if $|u - v| \geqslant \varepsilon \max\{|u|,|v|\}$ and $\max\{\varphi(u,t)\ ,\varphi(v,t)\} \in [d_1, d_2]$ .

**Proof.** By Lemma 0.5 in [8], for all t outside of some null set there is a number $p(t) \in (0,1)$ satisfying the inequality from the thesis So,it is enough to show the measurability of the function p. Let

$$A_{u,v} = \{t \in T : \max\{\varphi(u,t)\ ,\varphi(v,t)\} \in [d_1, d_2]\} .$$

It is evident that this set is measurable.Let us consider the following function

$$q(t) = \sup_{u,v\in\mathbb{R}} \left\{ \frac{2\varphi((u+v)/2,t)}{\varphi(u,t)+\varphi(v,t)} : |u-v| \geqslant \varepsilon \max\{|u|,|v|\} \wedge \max\{\varphi(u,t),\varphi(v,t)\} \in [d_1,d_2] \right\}$$

Denoting by Q the set of all rational numbers we get

$$q(t) = \sup_{u,v\in Q} \left\{ \frac{2\varphi((u+v)/2\,\chi_{A_{u,v}}(t),t)}{\varphi(u,t)+\varphi(v,t)} : |u-v| \geqslant \varepsilon \max\{|u|,|v|\} \right\}$$

by the definition of $A_{u,v}$. Therefore q is measurable as the supremum of a countable family of measurable functions, which ends the proof, since $p = 1 - q$.

2. Lemma. For all $u,v \in \mathbb{R}$, $t \in T$, the following inequality

$$\max\{\varphi(u+v,t), \varphi(u,t)\} \geqslant \varphi(v/2,t)$$

holds.

Proof. In the case when u,v are of the same signs, the inequality is evident. So, let $u \geqslant 0$ and $v < 0$. If $v \geqslant -u$ then

$\max\{\varphi(u+v,t), \varphi(u,t)\} = \varphi(u,t) \geqslant \varphi(-v,t) = \varphi(v,t)$ .

Now, let $v \leqslant -u$. If $v \in [-2u,-u]$ then $-(u+v) \leqslant u$ and $u \geqslant -v/2$ .

So $\max\{\varphi(u+v,t), \varphi(u,t)\} = \varphi(u,t) \geqslant \varphi(-v/2,t) = \varphi(v/2,t)$ .

If $v < -2u$ then $-(u+v) > u$ and $-(u+v) > -v/2$ . Therefore the required inequality is also satisfied. Thus we proved the lemma, because the remaining case is similar to the above one.

3. Lemma. Let $f_\tau : T \to \mathbb{R}$ be a family of functions with the following properties:

$1^o$ the set functions $v_\tau(A) = \int_A |f_\tau(t)|\,d\mu$ are equicontinuous with respect to the measure $\mu$, i.e. for each $\varepsilon > 0$ there exist a set $T_\varepsilon \in \Sigma$ of finite measure $\mu$ and $\delta > 0$ such that $v_\tau(T \setminus T_\varepsilon) \leqslant \varepsilon$ and $v_\tau(A) \leqslant \varepsilon$ for $A \subset T_\varepsilon$ with $\mu A \leqslant \delta$ for each index $\tau$.

$2^o$ $v_\tau(T) = \int_T |f_\tau(t)|\,d\mu \geqslant \alpha$ for some $\alpha > 0$ and each $\tau$.

Then for an arbitrary measurable function $q : T \to (0,\infty)$ and $\varepsilon \in (0,\alpha)$

there exists a constant $q>0$ such that

$$\int_Q |f_\tau(t)|\,d\mu \geqslant \alpha - \varepsilon$$

for each $\tau$, where $Q = \{t \in T : q(t) > q\}$ .

Proof. Let $T_{\varepsilon/2}$ be the set from $1^\circ$ chosen for $\varepsilon/2$ in place of $\varepsilon$. Also let $Q_n = \{t \in T : q(t) \geqslant 1/n\}$. Since $\mu T_{\varepsilon/2} < \infty$ and $\bigcap_{n\in\mathbb{N}}[T_{\varepsilon/2} \cap (T \setminus Q_n)] = \emptyset$ then $\lim_{n\to\infty} \mu[T_{\varepsilon/2} \cap (T\setminus Q_n)] = 0$. So, by $1^\circ$, there is $n_0 \in \mathbb{N}$ such that $v_\tau[T_{\varepsilon/2} \cap (T \setminus Q_{n_0})] < \varepsilon/2$ for each $\tau$. Putting $q = 1/n_0$ we obtain

$$\int_Q |f_\tau(t)|\,d\mu = v_\tau(T) - v_\tau[T_{\varepsilon/2} \cap (T\setminus Q_{n_0})] - v_\tau[(T\setminus Q_{n_0}) \setminus T_{\varepsilon/2}]$$
$$\geqslant \alpha - \varepsilon ,$$

because $v_\tau[(T\setminus Q_{n_0}) \setminus T_{\varepsilon/2}] \leqslant v_\tau(T \setminus T_{\varepsilon/2}) \leqslant \varepsilon/2$ by $1^\circ$ and $v_\tau(T) \geqslant \alpha$ by $2^\circ$.

4. Lemma. Let z be a function with properties $0 < I_\varphi(z/2) < I_\varphi(2z) < \infty$. Then there exist positive numbers $c, d, \delta$ such that

$$I_\varphi(z\chi_{W_0(x)}) > \delta$$

for all x satisfying $I_\varphi(2x) \leqslant K$ for some $K>0$, where $W_0(x) = W_1 \cap W_x$ and

$$W_1 = \{t \in T : \quad 1/c \leqslant \varphi((1/2)z(t),t) \wedge \varphi(2z(t),t) \leqslant c\}$$
$$W_x = \{t \in T : \quad \varphi(2x(t),t) \leqslant d\}.$$

Remark: If $\varphi$ satisfies the condition $\Delta_2$ and vanishes only at zero then the assumptions of this Lemma may reduced to $0 < I_\varphi(z) < \infty$ and $I_\varphi(x) \leqslant 1$.

Proof. Let us choose a measurable set B of positive measure such that $\varphi(z(t)/2, t) > 0$ for each $t \in B$. Then, by the well known property of the integral, for each $\varepsilon > 0$ there exists $\delta > 0$ such that $I_\varphi(z\chi_A) < \delta$ implies $\mu A < \varepsilon$ for each measurable $A \subset B$. So, if $\mu A \geqslant \varepsilon$ then $I_\varphi(z\chi_A) \geqslant \delta$ for $A \subset B$. By the assumptions and by the choice of B, one can find $c>0$ such that

(4.1) $\qquad \mu(B \setminus W_1) \leqslant (1/4)\mu B$ .

Let d be greater or equal than $4K/\mu B$. Thus, since we have $\mu(B\setminus W_x)d \leqslant K$, so

(4.2) $\mu(B\setminus W_x) \leq (1/4)\mu B$

for each x satisfying $I_\varphi(2x) \leq K$. Therefore, $\mu(B\setminus(W_1\cap W_x)) \leq (1/2)\mu B$, by (4.1) and (4.2). Hence $\mu(W_1\cap W_x\cap B) \geq (1/2)\mu B$ for all considered x. Then one can find a $\delta > 0$ dependent only on z, chosen for $(1/2)\mu B$ in place of $\varepsilon$, such that $I_\varphi(z\chi_{W_1\cap W_x\cap B}) \geq \delta$. But this means the thesis, because $W_1\cap W_x\cap B\subset W_0(x)$.

Now we may formulate and prove the main theorem.

Theorem. The following conditions are equivalent

(i) the function $\varphi$ satisfies the condition $\Delta_2$ and is strictly convex a.e. in T,

(ii) the space $L_\varphi$ is rotund,

(iii) the space $L_\varphi$ is midpoint locally uniformly rotund,

(iv) the space $L_\varphi$ is locally uniformly rotund,

(v) the space $L_\varphi$ is uniformly rotund in every direction.

Proof. In virtue of Theorem 0.1 and general relations between properties R, LUR, MLUR, and URED it is enough to show the implications (i) $\to$ (iv) and (i) $\to$ (v).

(i)$\to$(iv). Let $\varepsilon > 0$ and $y\in L_\varphi$ be given such that $I_\varphi(y) = 1$. Consider the set of all x for which $I_\varphi(x) = 1$ and $I_\varphi(x - y) \geq \varepsilon$. Since every strictly convex function $\varphi$ vanishes only at zero, so by the supposed $\Delta_2$-condition, there exist a constant k and a non-negative function h such that

(1) $\int_T h(t)d\mu < (1/16)\varepsilon$ and $\varphi(2u,t) \leq k\varphi(u,t) + h(t)$

for a.e. $t\in T$. Next, we find constants $c_1, c_2$ such that $c_2 > c_1 > 1$ and

(2) $\int_{T_1}\varphi(y(t),t)d\mu < (1/64k)\varepsilon$ and

$$T_1 = \{t\in T: \varphi(y(t),t) < 1/c_1 \vee \varphi(y(t),t) > c_1\},$$

(3) $c_1/c_2 \leq (1/32k)\varepsilon$.

Let $\delta$ be from Lemma 0.4 chosen for $(1/4k)\varepsilon$ in place of $\varepsilon$. Moreover, let p be the function from Lemma 1 for $\delta/4$, $1/c_1$, $c_2$ in place of $\varepsilon, d_1, d_2$. There exists $c_3 > 0$ such that

(4) $$\int_{T_2}\varphi(y(t),t)\,d\mu < (1/64k)\,\varepsilon ,$$

where $T_2 = \{t\in T : p(t) < c_3\}$, putting in Lemma 3, $f_\tau(t) = \varphi(y(t),t)$. Let $T_x = \{t\in T : \varphi(x(t),t) > c_2\}$. Denote $T_0(x)$ as $T\setminus(T_1\cup T_2\cup T_x)$. It means that

$$T_0(x) = \{t\in T : 1/c_1 \leqslant \varphi(y(t),t) \leqslant c_1\}\cap\{t\in T : p(t)\geqslant c_3\}$$
$$\cap\{t\in T : \varphi(x(t),t)\leqslant c_2\} .$$

It will be shown that

(5) $$I_\varphi((x-y)\chi_{T_0(x)}) \geqslant \delta$$

for all considered x. In order to do this, it is enough to study a subset of such x for which $I_\varphi((x-y)\chi_{T_0(x)}) < (3/4)\,\varepsilon$. Then, in virtue of the assumption $I_\varphi(x-y)\geqslant\varepsilon$, we have $I_\varphi((x-y)\chi_{T_1\cup T_2\cup T_x}) > (1/4)\,\varepsilon$. We have also $\int_{T_x\setminus(T_1\cup T_2)}\varphi(y(t),t)\,d\mu \leqslant c_1\,\mu(T_x\setminus(T_1\cup T_2)) \leqslant c_1/c_2 \leqslant$ $\leqslant (1/32k)\,\varepsilon$, by (3) and facts such as $c_2\,\mu T_x < 1$ and $I_\varphi(x) = 1$. However $\int_{T_1\cup T_2}\varphi(y(t),t)\,d\mu \leqslant (1/32k)\,\varepsilon$, by (2) and (4), so

(6) $$I_\varphi(y\chi_{T_1\cup T_2\cup T_x}) \leqslant (1/16k)\,\varepsilon .$$

Hence

$$(1/4)\,\varepsilon < I_\varphi((x-y)\chi_{T_1\cup T_2\cup T_x}) \leqslant (k/2) I_\varphi(x\chi_{T_1\cup T_2\cup T_x}) + (3/32)\,\varepsilon .$$

Therefore

(7) $$I_\varphi(x\chi_{T_1\cup T_2\cup T_x}) \geqslant (5/16k)\,\varepsilon .$$

Then $I_\varphi(y\chi_{T_0(x)}) - I_\varphi(x\chi_{T_0(x)}) > (1/4k)\,\varepsilon$, in virtue of the definition of $T_0(x)$ and (6) and (7). Now, applying Lemma 0.4 we get (5).

Let

$$T_3(x) = \{t\in T_0(x) : |x(t)-y(t)| \geqslant (\delta/4)\max(|x(t)|,|y(t)|)\}.$$

Since $1/c_1 \leqslant \max\{\varphi(x(t),t), \varphi(y(t),t)\} \leqslant c_2$ for $t\in T_0(x)$, then

$$\varphi((x(t)+y(t))/2,t) \leqslant (1-p(t))(\varphi(x(t),t)+\varphi(y(t),t))/2$$

for $t\in T_0(x)$, by Lemma 1 and the choice of the function p. However, $p(t)\geqslant c_3$ for $t\in T_3(x)$, so

(8) $$I_\varphi((x+y)/2) \leqslant 1 - (c_3/2)(I_\varphi(x\chi_{T_3(x)}) + I_\varphi(y\chi_{T_3(x)})) .$$

Using the definition of $T_3(x)$ and the inequality (5) it is easily obtained that $I_\varphi((x - y)\chi_{T_3(x)}) \geq \delta/2$. Now, let us choose a new constant $k_1$ and a nonnegative function $h_1$ such that

$\int_T h_1(t)d\mu \leq \delta/4$ and $\varphi(2u,t) \leq k_1\varphi(u,t) + h_1(t)$

for a.e. $t \in T$. Then

$$I_\varphi(x\chi_{T_3(x)}) + I_\varphi(y\chi_{T_3(x)}) \geq (2/k_1)(I_\varphi((x - y)\chi_{T_3(x)}) - \int_T h_1(t)d\mu \geq \delta/2k_1 .$$

Therefore $I_\varphi((x + y)/2) \leq 1 - c_3\delta/2k_1$, by (8), where the constant $c_3\delta/2k_1$ is dependent only on y and $\varepsilon$. This proves, in virtue of Lemma 0.3, the local uniform rotundity of $L_\varphi$.

(i)→(v). Let $z \in L_\varphi$, $z \neq 0$ and x be such that $I_\varphi(x) \leq 1$ and $I_\varphi(x + z) \leq 1$. The functions z, x satisfy the assumptions of Lemma 4 (see also Remark). Then, there are constants $c, d > 0$ and $\delta \in (0,1)$ such that

(9) $\qquad I_\varphi(z\chi_{W_0(x)}) > \delta$

for arbitrary x satisfying $I_\varphi(x) \leq 1$, where $W_0(x)$ is the same set as in Lemma 4. There exists a function $p : T \to (0,1)$ chosen by Lemma 1 for $\delta/4$, $1/c$, $(c + d)/2$ in place of $\varepsilon, d_1, d_2$. The family of functions $\{\varphi(z(\cdot)\chi_{W_0(x)}(\cdot), \cdot) : I_\varphi(x) \leq 1\}$ satisfies the assumptions of Lemma 3, because (9) holds, $W_0(x) \subset W_1$ and $\mu W_1 < \infty$. Then, there is a positive number p such that

(10) $\qquad I_\varphi(z\chi_{W_0(x)\cap P}) \geq (3/4)\delta$

for all x fulfilling $I_\varphi(x) \leq 1$, where $P = \{t \in T : p(t) \geq p\}$. Putting $W_3(x) = \{t \in W_0(x) \cap P : |z(t)| \geq (\delta/4)\max\{|z(t) + x(t)|, |x(t)|\}\}$ we have

$$1/c \leq \varphi(z(t)/2,t) \leq \max\{\varphi(z(t) + x(t),t), \varphi(x(t),t)\}$$
$$\leq (1/2)\varphi(2z(t),t) + (1/2)\varphi(2x(t),t) \leq (c + d)/2$$

for all $t \in W_0(x)$, by Lemma 2 and definitions of $W_1$ and $W_x$. So, in virtue of Lemma 1 and the choice of the function p, there holds

$$\varphi((z(t)/2) + x(t),t) \leq (1 - p)(\varphi(z(t) + x(t),t) + \varphi(x(t),t))/2$$

for all $t \in W_3(x)$ . Hence

(11) $I_\varphi((z/2) + x) \leq 1 - (p/2)[I_\varphi((z + x)\chi_{W_3(x)}) + I_\varphi(x\chi_{W_3(x)})]$

Let the condition $\Delta_2$ be satisfied with $k_2 > 0$ and $h_2 : T \to (0,\infty)$ such that $\int_T h_2(t)d\mu \leq \delta/8$ . Now, it is enough to note that the inequalities (10) , (11) play a similar role as (5) , (8) ,respectively. Therefore, by the same technique we get $I_\varphi((z/2) + x) \leq 1 - p\delta/8k_2$ for all x satisfying $I_\varphi(x) \leq 1$ and $I_\varphi(z + x) \leq 1$, where the constant $p\delta/8k_2$ is dependent only on z.

Remark. This theorem is a generalization of Th. 1 in [8], where the equivalence of the first four conditions in the case of Orlicz spaces was proved. But the implication (i) → (v) is new, even for Orlicz spaces.

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