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DRESSING TRANSFORMATION ON QUANTUM COMPACT GROUPS+

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0. INTRODUCTION

Recently, a remarkable attention was paid to the simple complex quantum groups considered over reals [9] as well as to the corresponding enveloping algebras [1]. This means that one takes into account not only "holomorphic" but also "antiholomorphic" quantum functions on a complex group. Trying to understand this structure one is naturally lead to recalling the notion of quantum double [5]. This is in full accordance with the classical case where the Iwasawa decomposition plays an important role . A similar situation takes place also on the dual level. The first goal of this paper is to make clear the structure of the dual quantum double concealed in the construction presented in [1]. We also note that in the literature one can find deep applications of methods envolving double group to a quantum analogue of Gauss decomposition [3,12].

On the other hand, quantum double is intimely related to quantum dressing transformation [11]. This transformation can be described in terms of algebras of quantum functions as well as in terms of deformed enveloping algebras. Both descriptions are naturally dual. Dressing orbits on quantum compact groups were considered in detail in [5]. Some necessary facts are briefly recalled in Sec. 3. But here we are concentrating on the dual form of the dressing transformation. So the second goal of this paper is to present a simple formula for the dual dressing transformation as well as for

+ This paper is in final form and no version of it will be submitted for publication elsewhere.

its restriction to dressing orbits. In fact, this is possible owing to a more profound formula for multiplication in the dual quantum double presented here in a coordinate - independent form. The coordinate - dependent form is given for example in [12, (2.24)].

1. QUANTUM DOUBLE AND ITS DUAL

The definition of the quantum double goes back to Drinfeld [2]. In the case of compact groups, an alternative description exploiting the representation theory was given in [10]. Let $\mathcal{A}_c$ be a $*$ - Hopf algebra, $\mathcal{U}_c$ be the dual $*$ - Hopf algebra and $\mathcal{A}_d$ be another $*$ - Hopf algebra identical to $\mathcal{U}_c$ as an algebra and opposite to $\mathcal{U}_c$ as a coalgebra. It is worth of noting that here and everywhere in what follows the word "identical" should be thought as being put into inverted commas for all the vector spaces are infinite-dimensional and the topological background is not satisfactorily established yet. If $\mathcal{U}_d$ is the $*$ - Hopf algebra dual to $\mathcal{A}_d$ than it is opposite to $\mathcal{A}_c$ as an algebra and identical to $\mathcal{A}_c$ as a coalgebra. Thus the pairing $\langle .,. \rangle : \mathcal{A}_d \otimes \mathcal{A}_c \longrightarrow \mathbb{C}$ fulfills

$$\langle uv, c \rangle = \langle u \otimes v, \Delta(c) \rangle \ , \ \langle \Delta(u), c \otimes d \rangle = \langle u, dc \rangle$$

and the pairing $\langle .,. \rangle : \mathcal{U}_c \otimes \mathcal{U}_d \longrightarrow \mathbb{C}$ fulfills

$$\langle X, Y_1 Y_2 \rangle = \langle \Delta X, Y_2 \otimes Y_1 \rangle \ , \ \langle X_1 \otimes X_2, \Delta Y \rangle = \langle X_1 X_2, Y \rangle \ .$$

We use the standard notation m (resp. "."), Δ , S and ε for multiplication, comultiplication, antipode and counite both in the original space as well as in the dual one.

Let

$$\varrho = \sum x_s \otimes a_s$$

be the canonical element in $\mathcal{A}_d \otimes \mathcal{A}_c$ with $\{x_s\}$ and $\{a_s\}$ being dual basis in $\mathcal{A}_d$ and $\mathcal{A}_c$, respectively. The quantum double $\mathcal{D}$ is a $*$ - Hopf algbera which coincides with $\mathcal{A}_c \otimes \mathcal{A}_d$ as a $*$ - algebra but the comultiplication is defined by

$$\Delta_{\mathcal{D}} = (\mathrm{id} \otimes \Phi \otimes \mathrm{id})(\Delta_c \otimes \Delta_d) \ ,$$

where

$$\Phi : \mathcal{A}_c \otimes \mathcal{A}_d \longrightarrow \mathcal{A}_d \otimes \mathcal{A}_c \ , \ \Phi(c \otimes u) = \varrho (u \otimes c) \varrho^* \ ,$$

(the subscripts will be omitted if not necessary). Denote by $\mathcal{D}^*$

the * - Hopf algebra dual to $\mathcal{D}$. Then $\mathcal{D}^*$ coincides with $\mathcal{U}_c\otimes\mathcal{U}_d$ as a coalgebra but the multiplication is given by

$$m_{\mathcal{D}^*} = (m_c\otimes m_d)(id\otimes\Phi^{\vee}\otimes id) ,$$

where $\Phi^{\vee}\colon \mathcal{U}_d\otimes\mathcal{U}_c \longrightarrow \mathcal{U}_c\otimes\mathcal{U}_d$ is the mapping dual to Φ .

Lemma 1. *It holds*

$$\Phi^{\vee}(Y\otimes X) = (\langle.,.\rangle\otimes id\otimes\langle S.,.\rangle)(id\otimes\Delta)\Delta(X\otimes Y) , \qquad (1)$$

(the pairing $\langle.,.\rangle$ *is defined on* $\mathcal{U}_c\otimes\mathcal{U}_d$ *).*

Proof. To get the proof it is enough to take into account that $\varrho^* = \varrho^{-1} = (id\otimes S)\varrho$ and the relations

$$\langle Y\otimes X,\varrho(u\otimes c)\rangle = (\langle.,.\rangle\otimes\langle.,c\otimes u\rangle)\Delta(X\otimes Y) ,$$

$$\langle Y\otimes X,(u\otimes c)\varrho^*\rangle = (\langle.,c\otimes u\rangle\otimes\langle S.,.\rangle)\Delta(X\otimes Y) . \qquad \square$$

The multiplication is then determined by

$$m((X\otimes 1)\otimes(1\otimes Y)) = X\otimes Y ,$$

$$m((1\otimes Y)\otimes(X\otimes 1)) = \Phi^{\vee}(Y\otimes X) .$$

The left dressing transformation of $\mathcal{A}_d$ on $\mathcal{A}_c$ is the * - algebra morphism

$$L := \Phi\iota_c : \mathcal{A}_c \longrightarrow \mathcal{A}_d\otimes\mathcal{A}_c , \qquad (2)$$

where $\iota_c : \mathcal{A}_c \longrightarrow \mathcal{A}_c\otimes\mathcal{A}_d$ is the natural embedding. L fulfills

$$(\varepsilon_d\otimes id)L = id ,$$

$$(id\otimes L)L = (\Delta_d\otimes id)L .$$

Besides, every two-sided ideal $\mathcal{I}$ in $\mathcal{A}_c$ is L-invariant, i.e.,

$$L(\mathcal{I}) \subset \mathcal{A}_d\otimes\mathcal{I} ,$$

and thus one can define the factor action

$$L_{\mathcal{I}} : \mathcal{A}_c/\mathcal{I} \longrightarrow \mathcal{A}_d\otimes(\mathcal{A}_c/\mathcal{I}) .$$

The mapping dual to L is called here the dual left dressing transfromation and it is a coalgebra morphism,

$$L^{\vee} := (id\otimes\varepsilon_d)\Phi^{\vee}\colon \mathcal{U}_d\otimes\mathcal{U}_c \longrightarrow \mathcal{U}_c . \qquad (3)$$

It fulfills

$$L^{\vee}(1\otimes X) = X ,$$

$$L^{\vee}(Y_1\otimes L^{\vee}(Y_2\otimes X)) = L^{\vee}(Y_1Y_2\otimes X) .$$

As an immediate consequence of Lemma 1 one gets

Proposition 2. *It holds*

$$L^{\vee}(Y\otimes X)) = \Sigma_{\nu} \langle X^1_{\nu}S(X^3_{\nu}),Y\rangle X^2_{\nu} , \qquad (4)$$

where

$$(\mathrm{id}\otimes\Delta)\Delta X = \Sigma_{\nu}\ X^1_{\nu}\otimes X^2_{\nu}\otimes X^3_{\nu}\ .$$

2. COMPLEX QUANTUM GROUPS OVER REALS

Let $\mathcal{A}_c = \mathcal{A}_q(K)$ and $\mathcal{A}_d = \mathcal{A}_q(AN)$ where $G = K.AN$ is the Iwasawa decomposition of a complex simple Lie group G into its compact and solvable parts. Assuming that $R = R_{12}$ is the underlying quantum R - matrix [4,12] satisfying the Yang-Baxter equation and U and Λ are the vector representations of the groups K_q and AN_q, respectively, we have the relations [5]

$$\langle\Lambda_1;U_2\rangle = R_{21}{}^{-1}\ ,\ \langle\Lambda^*_1;U_2\rangle = R_{12}{}^{-1}\ .$$

$$RU_1U_2 = U_2U_1R\ ,\quad U^* = U^{-1}\ ,$$

$$R\Lambda_1\Lambda_2 = \Lambda_2\Lambda_1R\ ,\ \Lambda^*_1R^{-1}\Lambda_2 = \Lambda_2R^{-1}\Lambda^*_1\ ,$$

Λ is upper triangle, $\prod \Lambda_{ii} = 1$.

The quantum double $\mathcal{D}$ can be identified with the * - Hopf algebra consisting of quantum functions on the complex group G . The fundamental representation $T = U\dot{\otimes}\Lambda$ satisfies

$$RT_1T_2 = T_2T_1R\ ,\ T_1{}^*R^{-1}T_2 = T_2R^{-1}T_1{}^*\ . \tag{5}$$

The second relation in (5) was derived by Podles [9]. On the other hand, the authors of [1] took (5) as the starting point in their description of the corresponding enveloping algebra. They managed to rewrite (5) into one huge RTT - equation and then made use of the L - matrices following the general ideas of F-R-T [3,12]. Though this approach makes it possible to write down all formulas in a single compact form the structure of the quantum double remains concealed. The dual quantum double $\mathcal{D}^*$ should be identified with the deformed enveloping algebra $\mathcal{U}_h(g^{\mathbb{R}})$ where $g^{\mathbb{R}}$ means g taken over reals and then again complexified. The * - Hopf algebra $\mathcal{U}_c$ is in fact the deformed enveloping algebra $\mathcal{U}_h(g)$ for the complexification of the compact subalgebra $\mathfrak{k}_{\mathbb{C}} = g$ and, similarly, $\mathcal{U}_d = \mathcal{U}_h(\mathfrak{an}_{\mathbb{C}})$ is the deformed enveloping algebra for the complexified solvable part $\mathfrak{an}_{\mathbb{C}}$. Let us supply this scheme with some details.

The * - algebra $\mathcal{U}_h(g)$ is generated by the entries of matrices

L_c , L_c^* which fulfill

$$\langle L_{c1}, U_2\rangle = R_{21} \ , \ \langle L_{c1}^*, U_2\rangle = R_{12} \ ,$$

$$\Delta L_c = L_c \dot{\otimes} L_c \ , \ \varepsilon(L_c) = 1 \ , \ SL_c = L_c^{-1} \ ,$$

$$R_{21}L_{c1}L_{c2} = L_{c2}L_{c1}R_{21} \ , \ L_{c2}R_{12}L_{c1}^* = L_{c1}^* R_{12}L_{c2} \ .$$

In fact, L_C and L_c^* coincide with the matrices $L^{(+)}$ and $L^{(-)}$ described in [3,12]. Identifying $\mathcal{U}_c$ with $\mathcal{A}_d$ as a vector space dual to $\mathcal{A}_c$ we have $L_c \equiv \Lambda^{-1}$.

The * - algebra $\mathcal{U}_h(\mathfrak{an}_{\mathbb{C}})$ is generated by entries of the matrix L_d which fulfill

$$\langle L_{d1}, \Lambda_2\rangle = R_{12}^{-1} \ , \ \langle L_{d1}, \Lambda_2^*\rangle = R_{21}^{-1} \ ,$$

$$\Delta L_d = L_d \dot{\otimes} L_d \ , \ \varepsilon(L_d) = 1 \ , \ SL_d = L_d^{-1} \ ,$$

$$R_{21}L_{d1}L_{d2} = L_{d2}L_{d1}R_{21} \ .$$

Identifying $\mathcal{U}_d$ and $\mathcal{A}_c$ as vector spaces dual to $\mathcal{A}_d$ we have $L_d \equiv U$.

The pairing between $\mathcal{U}_c$ and $\mathcal{U}_d$ follows directly from the the pairing between $\mathcal{A}_d$ and $\mathcal{A}_c$ and so

$$\langle L_{c1}, L_{d2}\rangle = R_{21} \ , \ \langle L_{c1}^*, L_{d2}\rangle = R_{12} \ . \tag{6}$$

Proposition 3. *It holds*

$$R_{21}L_{c1}L_{d2} = L_{d2}L_{c1}R_{21} \ . \tag{7}$$

Proof. The relation (7) is equivalent to

$$\Phi^{\vee}(L_{d2}\dot{\otimes}L_{c1}) = R_{21}(L_{c1}\dot{\otimes}L_{d2})R_{21}^{-1} \ .$$

As the pairing (6) is known explicitly this equality can be shown with the help of Lemma 1. □

3. QUANTUM DRESSING ORBITS ON COMPACT GROUPS

Before considering the dual dressing transformation we recall some necessary facts about quantum dressing orbits on compact groups. Below we are using now more or less standard notation: $X_i^{\pm}$, H_i for quantum Chevalley generators corresponding to simple roots and satisfying deformed commutation relations as well as q-Serre relations (we retain notation from [5]) . Besides, we assume that

$q = e^{-h} \in (0,1)$, the quantum integer $[k] = (q^k - q^{-k})/(q - q^{-1})$ and set $\lambda = q - q^{-1}$. Further, $\overline{\mathcal{U}_h(g)}$ designates a completion of $\mathcal{U}_h(g)$ by the quantum Weyl elements $\check{w}_\iota$ introduced in [6,7] and also by elements of the maximal torus $M \subset K$ in accordance with the rules

$$\langle t,U\rangle = t \ , \ \Delta t = t\otimes t \ .$$

Note that to each $t \in M$ there corresponds a one-dimensional representation τ of $\mathcal{A}_q(K)$, $\tau(f) := \langle t,f\rangle$.

Let $\mathcal{L}$ be a $*$ - algebra generated by the elements z and z^* which obey the commutation relation

$$1 + zz^* = q^{-2}(1 + z^*z) \ .$$

It follows that each element $f \in \mathcal{L}$ can be normally ordered,

$$f = \sum \sum f_{jk} \, (z^*)^j z^k \ , \ f_{jk} \in \mathbb{C} \ ,$$

(we allow also infinite formal power series). Besides, one can define a (vacuum value) functional ε_v on $\mathcal{L}$ by

$$\varepsilon_v(f) = f_{00} \ .$$

The dual space $\mathcal{X}$ becomes a coalgebra and is spanned by quantum differential operators on $\mathcal{L}$. The differential calculus follows unambiguously from the rules

$$\partial_z z = 1 + q^2 z\partial_z \ , \ \partial_z z^* = q^2 z^*\partial_z \ ,$$

$$\partial_{\bar z} z = q^{-2} z\partial_{\bar z} \ , \ \partial_{\bar z} z^* = 1 + q^{-2} z^*\partial_{\bar z} \ ,$$

and consequently $\partial_{\bar z}\partial_z = q^2\partial_z\partial_{\bar z}$. The pairing $\langle .,.\rangle\colon \mathcal{X}\otimes\mathcal{L} \longrightarrow \mathbb{C}$ is then defined by

$$\langle \xi,f\rangle = \varepsilon_v(\xi . f) \ .$$

It holds

$$\Delta(\partial_z{}^k\partial_{\bar z}{}^j) = \sum_{s=0}^{\infty} q^{-\frac{1}{2}s(s+1)} \frac{(-\lambda)^s}{[s]!} \sum_{\sigma=0}^{j} \sum_{\nu=0}^{k} \begin{bmatrix} j \\ \sigma \end{bmatrix} \begin{bmatrix} k \\ \nu \end{bmatrix}$$

$$\times\, q^{\nu(k-\nu)-\sigma(j-\sigma)-2(j-\sigma+s)\nu} \, \partial_z{}^{\nu+s}\partial_{\bar z}{}^{\sigma}\otimes\partial_z{}^{k-\nu}\partial_{\bar z}{}^{j-\sigma+s} \ ,$$

$\mathcal{L}$ should be viewed as the $*$ - algbera of quantum functions living on that dressing orbit in $SU_q(2)$ corresponding to the Weyl element w . The "restriction" morphism of * - algebras $\psi_v: \mathcal{A}_q(SU(2)) \longrightarrow \mathcal{L}$ is defined by

$$\psi_v(U) = \begin{pmatrix} (1 + zz^*)^{-1/2}z & (1 + zz^*)^{-1/2} \\ -q^{-1}(1 + zz^*)^{-1/2} & z^*(1 + zz^*)^{-1/2} \end{pmatrix} .$$

The dual coalgebra morphism $\check{\psi}_v: \mathcal{X} \longrightarrow \overline{\mathcal{U}_h(sl(2))}$ was described explicitly in [5]. Particularly,

$$\check{\psi}_v(1) = \check{w} .$$

In the general case, to each Weyl element with a reduced decomposition

$$w = w_{i_1} \dots w_{i_k}$$

and to each element $t \in M$ there corresponds a quantum dressing orbit

$$\mathcal{L}_{vt} = \mathcal{L}_{i_1} \otimes \dots \otimes \mathcal{L}_{i_k}$$

with the "restriction" homomorphism

$$\psi_{vt} := (\psi_{i_1} \otimes \dots \otimes \psi_{i_k} \otimes \tau)\Delta_{k+1}: \mathcal{A}_q(K) \longrightarrow \mathcal{L}_{vt} ,$$

where $\Delta_{k+1}: \mathcal{A}_q(K) \longrightarrow \mathcal{A}_q(K)\otimes \dots \otimes \mathcal{A}_q(K)$ is iterated comultiplication, $\psi_i := \psi_{v_i} \circ \varphi_i: \mathcal{A}_q(K) \longrightarrow \mathcal{L}_i$ and the mapping $\varphi_i: \mathcal{A}_q(K) \longrightarrow \mathcal{A}_q(SU(2))$ is dual to the embedding $\check{\varphi}_i: \mathcal{U}_h(sl(2)) \longrightarrow \mathcal{U}_h(g)$ corresponding to the i^{th} simple root.

Let us make a short digression noting that knowledge of the morphisms ψ_{vt} enables one a very simple construction of all irreducible * - representations of $\mathcal{A}_q(K)$ [5] . Relate to each element $f \in \mathcal{L}_{vt}$ the holomorphic part of its symbol,

$$F_f(\eta) = \sum f_{j0}\, \eta^j ,$$

where $\eta = (\eta_1, \dots, \eta_k) \in \mathbb{C}^k$, $j = (j_1, \dots, j_k) \in \mathbb{N}_0^{\,k}$. Provided this power series is convergent then it can be regarded as a holomorphic function on the classical dressing orbit $\mathcal{L}^{cl}_{vt} \cong \mathbb{C}^k$ The representation π_{vt} is then defined by

$$\pi_{vt}(c)\ F_f = F_{\psi_{vt}(c)f}\ .$$

One can also introduce a scalar product to get a * - representation,

$$\langle F_f(\eta)\,|F_g(\eta)\rangle = \langle 1, f^* g\rangle\ .$$

Of course, one has to restrict π_{vt} to the Hilbert space consisting of holomorphic functions on $\mathcal{X}^{cl}_{vt}$ with finite norms.

4. DUAL DRESSING TRANSFORMATION

In the case of $\mathfrak{g} = \mathfrak{sl}(2,\mathbb{C})$ set $\check{\psi}_v(\xi) = \tilde{\psi}(\xi)\check{w}$. The linear mapping $\tilde{\psi}$: $\mathcal{X} \longrightarrow \mathcal{U}_h(\mathfrak{sl}(2))$ was calculated in [5]. Letting

$$Y^{\pm} = \exp(\pm hH/2)\ X^{\pm}$$

we have

$$\tilde{\psi}(\partial_z{}^k \partial_{\bar{z}}{}^j) = (-1)^k\, q^{\frac{1}{2}k(2k+1)+\frac{1}{2}j} \sum_{s=0}^{\min(j,k)} q^{s(s-j-k+1)} \frac{[j]!}{[j-s]!}\,\frac{[k]!}{[k-s]!}$$

$$\times \prod_{r=1}^{s} \left(\frac{q^{2(k-r)}\, e^{hH} - 1}{q^{2r} - 1} \right) (Y^+)^{k-s}(Y^-)^{j-s}\ ,$$

Clearly, $\tilde{\psi}$ is injective. Let now T_i be the automorphism of the algebra $\mathcal{U}_h(\mathfrak{g})$ given by

$$T_i(X) = \check{w}_i\ X\ \check{w}_i^{-1}\ .$$

These automorphisms were introduced in [8] and investigated in detail in [6]. In the general case the morphism $\check{\psi}_{vt}$ can be written as

$$\check{\psi}_{vt}(\xi) = \tilde{\psi}_{i_1}(\xi_1)\ T_{i_1}\tilde{\psi}_{i_2}(\xi_2)\ \ldots\ T_{i_1}T_{i_2}\ldots T_{i_{k-1}}\tilde{\psi}_{i_k}(\xi_k)\ \check{w}t\ .$$

The results of [6] imply that the elements

$$H_1^{m_1}\ldots H_n^{m_n}(Y^+_{i_1})^{l_1}(Y^-_{i_1})^{j_1}T_{i_1}((Y^+_{i_2})^{l_2}(Y^-_{i_2})^{j_2})\ldots$$

$$\ldots\ T_{i_1}T_{i_2}\ldots T_{i_{k-1}}((Y^+_{i_k})^{l_k}(Y^-_{i_k})^{j_k})$$

are linearly independent, $m \in \mathbb{N}_0{}^n$ ($n = \operatorname{rank} \mathfrak{g}$) , $l, j \in \mathbb{N}_0{}^k$. As a corollary of this assertion we get

Proposition 4. *The morphism* $\check{\psi}_{vt}$ *is injective.*

If L_{vt} is the factor-action corresponding to the two-sided ideal $\mathcal{I}_{vt} := \mathrm{Ker}\ \psi_{vt}$ in $\mathcal{A}_q(K)$ and $\check{L}_{vt}$ is the dual action then we have a commutative diagram

$$\begin{tikzcd} \mathcal{U}_d \otimes \mathcal{X}_{vt} \arrow[r, "\check{L}_{vt}"] \arrow[d, "\mathrm{id}\otimes\check{\psi}_{vt}"'] & \mathcal{X}_{vt} \arrow[d, "\check{\psi}_{vt}"] \\ \mathcal{U}_d \otimes \mathcal{U}_c \arrow[r, "\check{L}"'] & \mathcal{U}_c \end{tikzcd} .$$

Propositions 2 and 4 combined with this diagram lead immediately to

Proposition 5. *It holds*

$$\check{L}_{vt}(Z\otimes\xi) = \Sigma_{\nu} \langle\check{\psi}_{vt}(\xi^{1}_{\nu})\ S\check{\psi}_{vt}(\xi^{3}_{\nu}), Z\rangle\ \xi^{2}_{\nu} \quad , \tag{8}$$

where

$$(\mathrm{id}\otimes\Delta)\Delta\xi = \Sigma_{\nu}\ \xi^{1}_{\nu}\otimes\xi^{2}_{\nu}\otimes\xi^{3}_{\nu} \ .$$

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