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INTRODUCTION

In [6] the author studied various global properties of maps between compacta using notions and techniques of Borsuk's shape theory. In particular, we extended from compacta to maps between compacta shape invariant properties like C-movability [11], (weak) C_p-movability [7], (C, D)-tameness [11], C-calmness [3], and C-triviality [11]. However, we also introduced interesting new properties of maps like domain C-movability, domain (C, D)-tameness, C-surjectivity, and C-injectivity.

Earlier, in [8], [9], and [10], the author in a similar fashion extended to maps between compacta eC-movability [5], eC_p--movability [4], eC-calmness [4], and e(C, D)-tameness [5] which are the invariants of a part of geometric topology that we call the equicontinuous shape theory. We use this name whenever there occurs some sort of control on the size of maps or homotopies between them expressed in terms of an arbitrary real number $\varepsilon > 0$. For example, Clapp's approximate absolute neighborhood retracts [12], Ferry's ε-homotopy equivalences [14], Coram-Duvall's approximate fibrations [13], and Mardešić-Rushing's shape fibrations [17] belong to the equicontinuous shape theory.

The point of view of the present paper is to look at the properties which in a way combine these two types of thinking. The idea is to use maps of pairs (K, K_0) into neighborhoods of the domain and/or the codomain and to require control only on the subset K_0.

In this note we shall illustrate this approach with the three examples of the new kind of properties which we call medial. The main emphasis is on the class of e_0C_p-surjections obtained in this way by modifying the notions of a C-surjection [6] and an eC-surjection [5]. The e_0C_p-surjections include F. Cathey's S-embeddings [1] as a special case and are easily described in terms of inverse limits similarly to the way in which shape fibrations are described in [17]. This description suggests how to define e_0C_p-surjections between arbitrary topological spaces (using Mardešić's resolutions [16] and by replacing the role of an ε by normal covers) and thus introduce strong shape category with arbitrary topological spaces as objects following formally the procedure in [1] which is based on S-embeddings.

PRELIMINARIES

Throughout the paper C will be an arbitrary class of topological spaces while C_p and D_p will be arbitrary classes of pairs (K, K_0) of metrizable spaces with K_0 a closed subset of K. We put $C_p' = \{K \mid \exists\ (L, L_0) \in C_p,\ K = L\}$ and $C_p'' = \{K_0 \mid \exists\ (L, L_0) \in C_p,\ K_0 = L_0\}$. We reserve P_p for the class of all pairs of compact ANR's.

A map $f:X' \longrightarrow X$ is called a K-<u>map</u> (an ANR-<u>map</u>, a Q-<u>map</u>) provided both X' and X are compact metric spaces (ANR's, copies of the Hilbert cube Q, respectively).

We shall say that maps f and g of a space X' into a metric space (X, ρ) are ε-<u>close</u> provided $\rho(f(z), g(z)) < \varepsilon$ for every $z \in X'$.

Let $(K, K_0) \in C_p$. Maps $f, g:K \longrightarrow X$ of K into a metric space (X, ρ) are ε_0-<u>homotopic</u> [ε-<u>homotopic</u>] (and we write $f \simeq^{\varepsilon}_{0} g$

[$f \simeq^{\varepsilon} g$]) if there is a homotopy $h_t: K \longrightarrow X$, $0 \leq t \leq 1$, between f and g such that $h_0|K_0$ and $h_t|K_0$ [h_0 and h_t] are ε-close for all $t \in I = [0, 1]$.

For a compact ANR M and an $\varepsilon > 0$, let $\Gamma(M, \varepsilon)$ denote the set of all $\delta > 0$ such that every two δ-close maps $f, g: M' \longrightarrow M$ defined on a metrizable space M' are ε-homotopic.

For a map $f: X' \longrightarrow X$ between metric spaces, let $\Lambda(f, \varepsilon)$ be the set of all $\delta > 0$ with the property that $\varrho(x, y) < \delta$ in X' implies $\varrho(f(x), f(y)) < \varepsilon$ in X.

We shall say that a map $f: X' \longrightarrow X$ is <u>embedded</u> into a map $F: M' \longrightarrow M$ provided $X' \subset M'$, $X \subset M$, and $f = F|X'$.

THE FIRST EXAMPLE: (*)e_0C_p-MOVABLE MAPS

We start with the medial properties of e_0C_p-movability and domain e_0C_p-movability (or $*e_0C_p$-movability).

(3.1) DEFINITION. A K-map $f: X' \longrightarrow X$ is (*)e_0C_p-<u>movable</u> provided when embedded into some, and hence into every, ANR-map $F: M' \longrightarrow M$ for every $\varepsilon > 0$ and a neighborhood U of X in M there is a neighborhood U' of X' in M' such that the following condition holds.

(*)$\varepsilon_0 C_p^{mo}(U, U', f; F)$: For every neighborhood V of X in M (V' of X' in M'), every pair $(K, K_0) \in C_p$, and every map $\varphi': K \longrightarrow U'$ there is $\psi: K \longrightarrow V$ ($\psi': K \longrightarrow V'$) with $F \circ \varphi' \simeq_0^{\varepsilon} \psi$ ($F \circ \varphi' \simeq_0^{\varepsilon} F \circ \psi'$) in U.

(3.2) REMARKS. (i) A $*e_0C_p$-movable map is e_0C_p-movable.

(ii) A (*)e_0C_p-movable map is (*)C_p'-movable [6].

(iii) A (*)eC_p'-movable map [8] is (*)e_0C_p-movable.

(iv) A C_p-movable K-map [6] defined on an eC_p''-movable compactum [5] is e_0C_p-movable.

The following results about (*)e_0C_p-movable maps can be proved modifying the proofs of the corresponding results for the (*)eC-movable maps [8].

(3.3) If a K-map $f:X' \longrightarrow X$ is (*)approximately dominated [8] by a class of $(*)e_0C_p$-movable maps, then f is also $(*)e_0C_p$-movable.

(3.4) Let $f':X'' \longrightarrow X'$ and $f:X' \longrightarrow X$ be K-maps.

(a) If either f' or f is e_0C_p-movable, then the composition $f\circ f'$ is also e_0C_p-movable.

(b) If f' is $*e_0C_p$-movable, then $f\circ f'$ is also $*e_0C_p$-movable.

(3.5) The Cartesian product $f = \prod_{i=1}^{\infty} f_i$ of K-maps f_i is (*) e_0C_p-movable iff each f_i is $(*)e_0C_p$-movable.

(3.6) The cone $Cf:CX' \longrightarrow CX$ [the suspension $Sf:SX' \longrightarrow SX$] of a K-map $f:X' \longrightarrow X$ is $(*)e_0C_p$-movable iff f is $(*)e_0C_p$-movable.

A class C_p e_0-<u>dominates</u> a class D_p provided for every pair (L, L_0) in D_p and for every open cover $\underline{U}$ of L_0 there is a (K, K_0) in C_p and maps $u:(L, L_0) \longrightarrow (K, K_0)$ and $d:(K, K_0) \longrightarrow (L, L_0)$ such that $d\circ u \simeq^{\underline{U}}_{0} id_{(L, L_0)}$ (i. e., there is a homotopy $H:L\times I \longrightarrow L$ between $d\circ u$ and id with $H(\{x\}\times I)$ contained in some member of $\underline{U}$ for each $x \in L_0$).

(3.7) If a class C_p e_0-dominates a class D_p and a K-map $f:X' \longrightarrow X$ is $(*)e_0C_p$-movable, then f is also $(*)e_0D_p$-movable.

Recall that an ANR-<u>sequence</u> is an inverse sequence $\underline{X} = (X_i, p_{ij})$ with each X_i a compact ANR. A level preserving map of sequences (abbreviated as <u>level map</u>) $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ is a sequence of maps $\underline{f} = (f_i)$, where $f_i:X_i' \longrightarrow X_i$ and $f_i\circ p_{ij}' = p_{ij}\circ f_j$ for all i and j, $j \geq i$.

Let $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ be a level map of ANR-sequences. Let $\varprojlim \underline{X}' = (X', p_i')$ and $\varprojlim \underline{X} = (X, p_i)$, where $p_i':X' \longrightarrow X_i'$ and $p_i:X \longrightarrow X_i$ are the natural projections. The unique map $f:X' \longrightarrow X$ such that $f_i\circ p_i' = p_i\circ f$ for every index i is said to be <u>induced</u> by $\underline{f}$.

A level map $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ is $(*)e_0C_p$-<u>movable</u> provided for every $\varepsilon > 0$ and each i there is a $j \geq i$ such that the following holds.

$(*)\varepsilon_0C_p^{mo}(i, j; \underline{f})$: For every pair $(K, K_0) \in C_p$, a map $\varphi':K \longrightarrow X_j'$, and a $k \geq j$ there is a map $\psi:K \longrightarrow X_k$ ($\psi':K \longrightarrow X_k'$) with $p_{ij}\circ f_j\circ \varphi' \simeq^{\varepsilon}_{0} p_{ik}\circ \psi$ ($p_{ij}\circ f_j\circ \varphi' \simeq^{\varepsilon}_{0} p_{ik}\circ f_k\circ \psi'$).

(3.8) A K-map $f:X' \longrightarrow X$ is $(*)e_0C_p$-movable iff every level map $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ of ANR-sequences which induces f is $(*)e_0C_p$-movable.

THE SECOND EXAMPLE: $(*)e_0(C_p, D_p)$-TAME MAPS

In this section we shall briefly consider a medial version of the notion of $(*)(C, D)$-tameness [6].

(4.1) DEFINITION. A K-map $f:X' \longrightarrow X$ is $(*)e_0(C_p, D_p)$-<u>tame</u> provided when embedded into some, and hence into every, ANR-map $F:M' \longrightarrow M$ for every $\varepsilon > 0$ and a neighborhood U of X in M (and a neighborhood U' of X' in M' with $F(U') \subset U$) there is a neighborhood V' of X' in M' such that the following condition holds.

$\mathcal{E}_0(C_p, D_p)^{ta}(U, V', f; F)$: For every pair $(K, K_0) \in C_p$ and a map $\varphi:K \longrightarrow V'$ there is an $(L, L_0) \in D_p$ and maps $\alpha:(K, K_0) \longrightarrow (L, L_0)$ and $\beta:L \longrightarrow U$ with $(F|V')\circ\varphi \simeq_0^{\varepsilon} \beta\circ\alpha$ in U.

$(*\mathcal{E}_0(C_p, D_p)^{ta}(U, U', V', f; F)$: For every pair $(K, K_0) \in C_p$ and a map $\varphi:K \longrightarrow V'$ there is an $(L, L_0) \in D_p$ and maps $\alpha:(K, K_0) \longrightarrow (L, L_0)$ and $\beta:L \longrightarrow U'$ with $(F|V')\circ\varphi \simeq_0^{\varepsilon} (F|U')\circ\beta\circ\alpha$ in U.)

(4.2) REMARKS. (i) A $*e_0(C_p, D_p)$-tame map is $e_0(C_p, D_p)$-tame.

(ii) A $(*)e_0(C_p, D_p)$-tame map is $(*)(C_p', D_p')$-tame [6].

The following results about $(*)e_0(C_p, D_p)$-tame maps can be proved by modifying the corresponding arguments in [9] and [6].

(4.3) If a K-map $f:X' \longrightarrow X$ is (*)approximately dominated by a class of $(*)e_0(C_p, D_p)$-tame maps, then f is also $(*)e_0(C_p, D_p)$-tame.

(4.4) Let $f':X'' \longrightarrow X'$ and $f:X' \longrightarrow X$ be K-maps.

(a) If either f' or f is $e_0(C_p, D_p)$-tame, then $f\circ f'$ is also $e_0(C_p, D_p)$-tame.

(b) If f' is $*e_0(C_p, D_p)$-tame, then $f\circ f'$ is also $*e_0(C_p, D_p)$--tame.

(4.5) If a class $\overline{C}_p$ e_0-dominates a class C_p, a class D_p e_0-dominates a class $\overline{D}_p$, and a K-map $f:X' \longrightarrow X$ is $(*)e_0(\overline{C}_p, \overline{D}_p)$-tame, then it is also $(*)e_0(C_p, D_p)$-tame.

(4.6) A K-map $f:X' \longrightarrow X$ is $(*)e_0(C_p, D_p)$-tame iff every level map $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ of ANR-sequences which induces f is $(*)e_0(C_p, D_p)$-tame (i. e., iff for every $\varepsilon > 0$ and an index i there is a $j \geq i$ such that given a $(K, K_0) \in C_p$ and a map $\varphi':K \longrightarrow X_j'$ there is an $(L, L_0) \in D_p$ and maps $\alpha:(K, K_0) \longrightarrow (L, L_0)$ and $\beta:L \longrightarrow X_i$ ($\beta':L \longrightarrow X_i'$) with $p_{ij} \circ f_j \circ \varphi' \simeq_0^{\varepsilon} \beta \circ \alpha$ ($p_{ij} \circ f_j \circ \varphi' \simeq_0^{\varepsilon} f_i \circ \beta' \circ \alpha$)).

(4.7) If a K-map $f':X'' \longrightarrow X'$ is $e_0(C_p, D_p)$-tame and a K-map $f:X' \longrightarrow X$ is e_0D_p-movable, then $f \circ f'$ is e_0C_p-movable.

(4.8) If a K-map $f:X' \longrightarrow X$ is $*e_0(C_p, D_p)$-tame and $(*)e_0D_p$-movable, then it is also $(*)e_0C_p$-movable.

THE THIRD EXAMPLE: e_0C_p-SURJECTIONS

Our last example provide e_0C_p-surjections which combine properties of both C-surjections [6] and eC-surjections [5]. This class of maps is related to Cathey's S-embeddings [1] (see Theorem (5.2)).

(5.1) DEFINITION. A K-map $f:X' \longrightarrow X$ is an e_0C_p-<u>surjection</u> provided when embedded into some, and hence into every, ANR-map $F:M' \longrightarrow M$ for every $\varepsilon > 0$, every neighborhood U of X in M, and every neighborhood U' of X' in M' with $F(U') \subset U$ there exist a $\delta > 0$, a neighborhood V of X in M, and a neighborhood V' of X' in M' such that the following condition holds.

• $\varepsilon_0^{\delta}C_p^{su}(U, V, U', V', f; F)$: For every pair $(K, K_0) \in C_p$ and maps $\varphi:K \longrightarrow V$ and $\varphi_0':K_0 \longrightarrow V'$ with $d(F \circ \varphi_0', \varphi|K_0) < \delta$ there is a map $\varphi':K \longrightarrow U'$ such that $d(\varphi_0', \varphi'|K_0) < \varepsilon$ and $F \circ \varphi' \simeq_0^{\varepsilon} \varphi$ in U.

Observe that an eC_p-bundle [4] is an e_0C_p-surjection. In particular, a hereditary shape equivalence [15] is an e_0P_p-surjection. However, e_0C_p-surjections need not be onto as the following theorem shows.

Recall [1] that a closed subset A of a metrizable space X is a

<u>shape</u> <u>strong</u> <u>deformation</u> <u>retract</u> (SSDR) of X provided when embedded as a closed subset into an ANR M the following condition holds: for every pair of neighborhoods (U, V) of (X, A) in M there exists a homotopy $H:X \times I \longrightarrow U$ such that $H_0 = id$, $H_1(X) \subset V$, and $H_t|A = id$ for all $t \in I$.

(5.2) THEOREM. Let A and X, $A \subset X$, be compacta. The inclusion $i:A \longrightarrow X$ is an e_0P_p-surjection iff A is an SSDR of X.

PROOF. Suppose first that i is an e_0P_p-surjection, X lies in the Hilbert cube Q, U is a neighborhood of X in Q, and U' is a compact ANR neighborhood of A in U. Pick an $\varepsilon > 0$, a compact ANR neighborhood V of X in U, a neighborhood V' of A in U', a compact ANR K_0, and a δ, $\varepsilon > \delta > 0$, such that $N_{3\varepsilon}(A) \subset U'$, $A \subset K_0 \subset N_\delta(A) \cap V'$, and $\varepsilon_0^\delta P_p^{su}(U, V, U', V', i; id_Q)$ holds. Select a homotopy $f_t:V \longrightarrow U$ with $f_0 = id$, $f_1(K) \subset U'$, and $d(f_t(x), x) < \varepsilon$ for every $x \in K_0$ and every $t \in I$. Since $f_1(V) \cup f_t(K_0) \subset U'$, we can assume without loss of generality that $f_1|K_0 = id$. By applying the technique of the proof of the Theorem (5.1) in [2], we can get a homotopy which connects f_0 and f_1 and is fixed on K_0. Hence, A is an SSDR of X.

Conversely, suppose that A is an SSDR of X, $X \subset Q$, and let a compact ANR neighborhood U of X in Q, a neighborhood U' of A in Q, and an $\varepsilon > 0$ be given. Pick a homotopy $D:X \times I \longrightarrow U$ such that $D_0 = id$, $D_1(X) \subset U'$, and $D(x, t) = x$ for all $x \in A$ and all $t \in I$. Since U is an ANR, there is a neighborhood V of X in U, a neighborhood $\tilde{V}'$ of A in U', and a homotopy $\tilde{D}:V \times I \longrightarrow U$ such that $\tilde{D}_0 = id$, $\tilde{D}|X \times I = D$, $D_1(V) \subset U'$, and $d(\tilde{D}(x, t), x) < \varepsilon/2$ for all $x \in \tilde{V}'$ and all $t \in I$. Choose a δ, $\varepsilon/2 > \delta > 0$, with $N_{3\delta}(A) \subset \tilde{V}'$ and put $V' = N_\delta(A)$. Our choices imply that $\varepsilon_0^\delta P_p^{su}(U, V, U', V', i; id_Q)$ is true. Hence, i is an e_0P_p-surjection.

(5.3) THEOREM. The composition $f \circ f'$ of e_0C_p-surjections $f':X'' \longrightarrow X'$ and $f:X' \longrightarrow X$ is an e_0C_p-surjection.

PROOF. Suppose X'', X', $X \subset Q$ and let $F':Q \longrightarrow Q$ and $F:Q \longrightarrow Q$ be extensions of f' and f, respectively. Consider a neighborhood U of X in Q, a neighborhood U'' of X'' in Q, and an $\varepsilon > 0$. Since f' is an e_0C_p-surjection, there is a neighborhood V' of X' in Q, a neighborhood V_1'' of X'' in Q, a δ^*, $\varepsilon > \delta^* > 0$, and a δ', $\varepsilon/2 > \delta' > 0$, such that $\delta^* \in \Lambda(F, \varepsilon/2)$ and $(\delta^*)_0^{\delta'}C_p^{su}(F^{-1}(U), V', U'', V_1'', f'; F')$ holds. Choose a neighborhood V of X in U, a neighborhood V_1' of X' in Q, and a $\delta > 0$ such that $(\delta')_0^{\delta}C_p^{su}(U, V, V', V_1', f; F)$ is true and put $V'' = V_1'' \cap (F')^{-1}(V_1')$. We leave to the reader the verification of $\varepsilon_0^{\delta}C_p^{su}(U, V, U'', V'', f \circ f', F \circ F')$.

(5.4) THEOREM. Let $f':X'' \longrightarrow X'$ and $f:X' \longrightarrow X$ be K-maps. If $f \circ f'$ is an e_0C_p-surjection and f' is an eC_p''-surjection [5], then f is an e_0C_p-surjection.

PROOF. Under the assumptions from the proof of the Theorem (5.3), consider a neighborhood U of X in Q, a neighborhood U' of X' in Q, and an $\varepsilon > 0$. Choose a neighborhood V of X in U, a neighborhood V'' of X'' in $U'' = (F')^{-1}(U')$, and a $\delta > 0$ such that $\varepsilon_0^{2\delta}C_p^{su}(U, V, U'', V'', f \circ f'; F \circ F')$ holds. Let $\eta \in \Lambda(F, \delta)$. Since f' is an eC_p''-surjection, there is a neighborhood V' of X' in U' such that $\eta(C_p'')^{su}(U', V'', V', f'; F')$ is true (i. e., for every $K_0 \in C_p''$ and a map $\varphi_0':K_0 \longrightarrow V'$ there is a map $\varphi_0'':K_0 \longrightarrow V''$ with $F \circ \varphi_0''$ η-close to φ_0'). One can check that $\varepsilon_0^{\delta}C_p^{su}(U, V, U', V', f; F)$ holds.

(5.5) THEOREM. If a class C_p e_0-dominates a class D_p and a K-map $f:X' \longrightarrow X$ is an e_0C_p-surjection, then f is also an e_0D_p-surjection.

PROOF. Suppose f is embedded into a Q-map F. Let a neighborhood U of X in Q, a neighborhood U' of X' in Q, and an $\varepsilon > 0$ be given. Choose neighborhoods V and V' and a $\delta > 0$ such that $(\varepsilon/2)_0^{\delta}C_p^{su}(U, V, U', V', f; F)$ holds. We claim that $\varepsilon_0^{\delta}D_p^{su}(U, V, U', V', f; F)$ is also true.

Indeed, let $(L, L_0) \in D_p$ and let $\varphi: L \longrightarrow V$ and $\varphi_0': L_0 \longrightarrow V'$ be maps with $F \circ \varphi_0'$ δ-close to $\varphi|L_0$. Let $\underline{V}$ be an open cover of V' with sets of diameter $< \varepsilon/2$. Let $\underline{U} = (\varphi_0')^{-1}(\underline{V})$. Since C_p e_0-dominates D_p, there is a $(K, K_0) \in C_p$ and maps $u:(L, L_0) \longrightarrow (K, K_0)$ and $d:(K, K_0) \longrightarrow (L, L_0)$ such that $d \circ u \simeq_0^{\underline{U}} id_{(L, L_0)}$. For maps $\psi = \varphi \circ d: K \longrightarrow V$ and $\psi_0' = \varphi_0' \circ (d|K_0)$ pick $\psi': K \longrightarrow U'$ such that $\psi'|K_0$ is $(\varepsilon/2)$-close to ψ_0' and $F \circ \psi' \simeq_0^{\varepsilon/2} \psi$. Clearly, $F \circ \varphi' \simeq_0^{\varepsilon} \varphi$ and $\varphi'|K_0$ is ε-close to φ_0', where $\varphi' = \psi' \circ u$.

For K-maps $f: X' \longrightarrow X$ and $g: Y' \longrightarrow Y$ we shall write $f \overset{\varepsilon}{\leq}_m g$ provided there are maps $u: X \longrightarrow Y$, $d: Y \longrightarrow X$, $u': X' \longrightarrow Y'$, and $d': Y' \longrightarrow X'$ such that the maps in the pairs $(u \circ f, g \circ u')$, $(f \circ d', d \circ g)$, $(d \circ u, id_X)$, and $(d' \circ u', id_{X'})$ are ε-close.

(5.6) THEOREM. Let $f: X' \longrightarrow X$ be a K-map. If for each $\varepsilon > 0$ there is an e_0C_p-surjection $g: Y' \longrightarrow Y$ with $f \overset{\varepsilon}{\leq}_m g$, then f is an e_0C_p-surjection.

PROOF. The proof (similar to the proof of the Theorem (3.4) in [8]) is left to the reader.

A compactum X is e_0C_p-<u>movable</u> ($e_0(C_p, D_p)$-<u>tame</u>) provided the identity map id_X is e_0C_p-movable ($e_0(C_p, D_p)$-tame). A K-map which is both an e_0C_p-surjection and an eC_p''-surjection is called an $e_0^*C_p$-<u>surjection</u>.

(5.7) THEOREM. If $f: X' \longrightarrow X$ is an $e_0^*C_p$-surjection and X' is e_0C_p-movable ($e_0(C_p, D_p)$-tame), then X is also e_0C_p-movable ($e_0(C_p, D_p)$-tame).

PROOF. We shall prove the statement about e_0C_p-movability and leave an analogous proof for $e_0(C_p, D_p)$-tameness to the reader.

Suppose f is embedded into a Q-map F, $\varepsilon > 0$, and U is a neighborhood of X in Q. Let $U^* = F^{-1}(U)$ and let $\delta \in \Lambda(F, \varepsilon/2)$. Since X' is e_0C_p-movable, there is a neighborhood U' of X' in U^* such that $\delta_0C_p^{mo}(U^*, U', X'; Q)$ holds (i. e., for each $(K, K_0) \in C_p$, a map $\varphi': K \longrightarrow U'$, and every neighborhood W' of X' in Q there is a map $\psi':K \longrightarrow W'$ with $\varphi' \simeq_0^{\varepsilon} \psi'$ in U^*). The assumptions about f imply the existence of neighborhoods V_1 and V of X in Q, a neighborhood V' of X' in Q, and an $\eta > 0$ such that $(\varepsilon/2)_0^{\eta}C_p^{su}(U, V_1, U', V', f; F)$ and $\eta(C_p'')^{su}(V_1, V', V, f; F)$ are true. Our choices guarantee that $\varepsilon_0 C_p^{mo}(U, V, X; Q)$ holds.

(5.8) THEOREM. Let $f':X'' \longrightarrow X'$ and $f:X' \longrightarrow X$ be K-maps. If f' is an $e_0^*C_p$-surjection and the composition $f \circ f'$ is $(*)e_0C_p$-movable ($(*)e_0(C_p, D_p)$-tame), then f is also $(*)e_0C_p$-movable ($(*)e_0(C_p, D_p)$-tame).

PROOF. Again we shall prove only the statement about e_0C_p-movability. Suppose f' and f are embedded into Q-maps F' and F, respectively, U is a neighborhood of X in Q, and $\varepsilon > 0$. Let $U' = F^{-1}(U)$. Choose neighborhoods U'' and V'' of X'' in Q, neighborhoods $\bar{V}'$ and V' of X' in Q, and numbers $\delta > 0$ and $\eta > 0$ such that $\eta \in \Lambda(F, \varepsilon/2)$ and $\eta_0^{\delta}C_p^{su}(U', \bar{V}', U'', V'', f'; F')$, $(\varepsilon/2)_0C_p^{mo}(U, U'', f \circ f'; F \circ F')$, and $\delta(C_p'')^{su}(\bar{V}', V'', V', f'; F')$ hold. One easily checks that $\varepsilon_0C_p^{mo}(U, V', f; F)$ is true.

(5.9) THEOREM. Let $f:X' \longrightarrow X$ be an e_0C_p-movable $e_0^*C_p$-surjection. Then

(a) f is $*e_0C_p$-movable, and

(b) X is e_0C_p-movable.

PROOF. Suppose f is embedded into a Q-map F, U is a neighborhood of X in Q, and $\varepsilon > 0$. Choose a neighborhood U' of X' in Q so that $(\varepsilon/2)_0C_p^{mo}(U, U', f; F)$ holds.

(a) We claim that $*\varepsilon_0C_p^{mo}(U, U', f; F)$ is true. Indeed, consider a neighborhood V' of X' in Q, a $(K, K_0) \in C_p$, and a map $\varphi':K \longrightarrow U'$. Pick neighborhoods $\bar{V}$ and V of X in Q, a neighborhood $\bar{V}'$ of X' in Q, and a $\delta > 0$ such that $(\varepsilon/2)_0^{\delta}C_p^{su}(U, \bar{V}, V', \bar{V}', f; F)$ and $\delta(C_p'')^{su}(\bar{V}', V, f; F)$ hold. Our choices imply the existence of maps $\psi:K \longrightarrow V$, $\psi_0':K_0 \longrightarrow \bar{V}'$, and $\psi':K \longrightarrow V'$ with $F \circ \varphi' \simeq_0^{\varepsilon/2} \psi$ in U, $F \circ \psi_0'$ is δ-close to $\psi | K_0$, and $F \circ \psi' \simeq_0^{\varepsilon/2} \psi$ in U. Clearly,

$F \circ \varphi' \simeq_0^{\varepsilon} F \circ \psi'$ in U.

(b) Pick neighborhoods $\bar{V}$ and V of X in Q, a neighborhood V' of X' in Q, and a $\delta > 0$ such that $(\varepsilon/2)_0^{\delta} C_p^{su}(U, \bar{V}, U', V', f; F)$ and $\delta(C_p'')^{su}(V', V, f; F)$ hold. Clearly, $\varepsilon_0 C_p^{mo}(U, V, X; Q)$ is true.

We close with the characterization of e_0C_p-surjections in terms of level maps of associated inverse ANR-sequences. A level map $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ is an e_0C_p-surjection provided for every $\varepsilon > 0$ and each index i there is a $j \geq i$ and a $\delta > 0$ such that the following condition holds.

$\varepsilon_0^{\delta} C_p^{su}(i, j; \underline{f})$: For every pair $(K, K_0) \in C_p$ and maps $\varphi:K \longrightarrow X_j$ and $\varphi_0':K_0 \longrightarrow X_j'$ with $f_j \circ \varphi_0'$ δ-close to $\varphi|K_0$, there is a map $\varphi':K \longrightarrow X_i'$ such that $p_{ij}' \circ \varphi_0'$ is ε-close to $\varphi'|K_0$ and $f_i \circ \varphi' \simeq_0^{\varepsilon} p_{ij} \circ \varphi$.

(5.10) THEOREM. A K-map $f:X' \longrightarrow X$ is an e_0C_p-surjection iff every level map $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ of ANR-sequences which induces f is an e_0C_p-surjection.

PROOF. It suffices to prove the following: If level maps $\underline{f}:\underline{X}' \longrightarrow \underline{X}$ and $\underline{g}:\underline{Y}' \longrightarrow \underline{Y}$ of ANR-sequnces induce the same map $f:X' \longrightarrow X$ and $\underline{f}$ is an e_0C_p-surjection, then $\underline{g}$ is also an e_0C_p-surjection.

Let an index i and an $\varepsilon > 0$ be given. Let $\eta \in \Gamma(X_i, \varepsilon/3)$ and $\bar{\varepsilon} \in \Lambda(g_i, \eta/2)$. By the Lemma 1(i) in [17], one can find an $i^* \geq i$, and maps $c:X_{i^*}' \longrightarrow Y_i'$ and $d:X_{i^*} \longrightarrow Y_i$ such that

$$\rho(c \circ p_{i^*}', q_i') < \bar{\varepsilon},$$

and

$$\rho(d \circ p_{i^*}, q_i) < \eta/2.$$

Furthermore, by the Lemma 1(ii) in [17], for any sufficiently large $k^* \geq i^*$, we have

($) $$\rho(g_i \circ c \circ p_{i^*k^*}', d \circ p_{i^*k^*} \circ f_{k^*}) < \eta.$$

Now, select a $j^* \geq k^*$ and a $\bar{\delta} > 0$ such that $(\varepsilon^*)_0^{\bar{\delta}} C_p^{su}(k^*, j^*; \underline{f})$ holds, where $\varepsilon^* \in \Lambda(c \circ p_{i^*k^*}', \varepsilon/2) \cap \Lambda(d \circ p_{i^*k^*}, \eta)$. Repeating the argument by which we got ($), we conclude that there is a $k \geq$

i, a $j \geq k$, and maps $a:Y_k \longrightarrow X_{j*}$ and $b:Y'_k \longrightarrow X'_{j*}$ such that

$$\varrho(f_{j*}\circ b\circ q'_{kj},\ a\circ q_{kj}\circ g_j) < \bar{\delta}/2,$$
$$\varrho(c\circ p'_{i*k*}\circ b\circ q'_{kj},\ q'_{ij}) < \varepsilon/2, \text{ and}$$
$$\varrho(d\circ p_{i*k*}\circ a\circ q_{kj},\ q_{ij}) < \eta.$$

Let $\delta \in \Lambda(a\circ q_{kj},\ \bar{\delta}/2)$. One can routinely check that $\varepsilon_0^{\delta}C_p^{su}(i,\ j;\ \underline{g})$ is true. Hence, $\underline{g}$ is an e_0C_p-surjection.

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