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# Quaternion quantum theory as a description of tachyons and the symmetry breaking.

J. Souček

The base of quaternionic quantum mechanics was set up in [1]. It was shown there that the right name for quaternionic quantum mechanics would be the space-like physics (the time evolution is substituted by the space evolution) and that it describes the theory of tachyons.

With more details, the Dirac equation for a quaternionic wave function $\psi(\vec{x},t) = \psi^0 + i_1 \psi_1 + i_2 \psi_2 + i_3 \psi_3 = \psi^0 + \vec{i}\,\vec{\psi}$ ($i_1$, $i_2$, $i_3$ are quaternionic units) have the form

$$\vec{i}\,\vec{\partial}\,\psi = (P\partial_0 + \sigma_3 m)\,\psi$$

where

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \quad P = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \partial_0 = \frac{\partial}{\partial t}, \quad \vec{\partial} = \frac{\partial}{\partial \vec{x}} .$$

The dispersion relation takes the form $p^2 = m^2 + E^2$ here, so it is really the Dirac equation for tachyons. It was stressed in [1] that the space variable $\vec{x}$ must be considered here as the evolution variable and that the operator $\vec{i}\,\vec{\partial}$ is the correct analogy of the operator $i\partial_0$ from the usual Schrödinger equation. The nonrelativistic approximation in the space-like physics is valid for $v >> c$ and corresponding Schrödinger equation has the form

$$\vec{i}\,\vec{\partial}\,\psi = \left(-\frac{1}{2m}\partial_0^2 + V(x_0)\right)\psi .$$

The phase invariance of the Dirac equation $\psi \mapsto \psi e^{\vec{i}\vec{\alpha}}$ leads us to the corresponding gauge field (in our case an Yang-Mills field) $A_\mu = \vec{i}\,\vec{A}_\mu$ with the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[\psi^+ \beta^{\mu}\psi(\partial_{\mu} - gA_{\mu}) - (\partial_{\mu} + gA_{\mu})\psi^+ \beta^{\mu}\psi\right] + \\ + \psi^+ \sigma_3 m\psi + \frac{1}{4}F^2_{\mu\nu}$$

where $\beta^0 = 1$, $\beta^k = -i_k$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$.

This form a nice dual counterpart to usual QED, where the dual gauge field is in fact an Yang-Mills field. It is worthwhile to notice that here the Yang-Mills field is coupled to only one (tachyonic) Dirac field. It was conjectured that the quaternionic "electrodynamics" could form a base for the description of quarks. The details of all this can be found in [1].

Let me join some more general comments on the space-like physics. The basic notion of it - the evolution of the system is the evolution in space variable - changes radically one of the (up to now) obvious and quite fundamental physical assumption. We want to argue that the obvious time-like character of our universe is in fact the consequence of the spontaneous symmetry breaking (in the Salam-Weinberg sense). Let us imagine for a moment an universe before symmetry breaking where all particle are massless. In this universe the character of evolution is far from obvious and can be interpreted equally well in terms of time-like or space-like physics. Of course, there is the difference between the dimensions of space and time, so the gauge field (describing phase invariance) in space-like physics is an Yang-Mills field with self-interaction, opposite to the elmg field in the time-like physics. This differenc could be the reason why the real symmetry breaking in the real universe goes towards the bradyonic side. But we can imagine another type of universe, where the symmetry breaking leads to tachyons and space-like physics, quite well.

Before the symmetry breaking the universe is invariant with respect to the space-time duality. So the space and time really gai the equal statute (in fact time and space in Einstein's relativity are mixed one to another, but they have not an equivalent position there - the full equivalence of them is not reached until the introduction of the time-space duality). When the symmetry breaking in S-W model occurs, the time-space duality simultaneously breaks down and then the time variable must be considered as the only possibility for the evolution variable.

So the basic physical axiom (the evolution in time) is not in fact an a priori given principle of all physics, but only the consequence of the symmetry breaking and the common fealing that the only possibility for evolution is the time variable is created by the fact that we are meeting all the time only bradyonic systems (in nonrelativistic approximation).

## Literature

J. Souček, Czech. J. Phys. B 29 (1979), 3/5.