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Definition. A locally convex space is called submetrizable if it admits a coarser metrizable locally convex topology.

Lemma 1. If X is a Hausdorff locally convex space and $Y \subset X$ a linear subspace such that $\dim(X/Y)$ is at most countable and such that Y is submetrizable, then also X is submetrizable.

Lemma 2. Let X be a locally convex space. Then X is separable if and only if the weak dual $(X, \sigma(X, X'))$ is submetrizable.

Proposition. Let $\omega := \mathbb{C}^{\mathbb{N}}$ be provided with its product topology \mathcal{P} and let \mathcal{F} be any separable locally convex topology on ω . Then the supremum $\mathcal{P} \vee \mathcal{F}$ is also separable.

Example. By an example of I. Amemiya - Y. Kōmura (Math. Ann. 177 (1968)) and R. Knowles - T. Cook (Proc. Camb. Phil. Soc. 74(1973)) there exists a locally convex topology \mathcal{F} on ω such that (ω, \mathcal{F}) is separable and such that every bounded set in (ω, \mathcal{F}) has finite dimensional linear span. Now the supremum $\mathcal{F} \vee \mathcal{P}$ is separable, and every bounded set in $(\omega, \mathcal{F} \vee \mathcal{P})$ has finite dimensional linear span. Thus $\mathcal{F} \vee \mathcal{P}$ is quasicomplete and strictly stronger than \mathcal{F} .

This example answers a question by E. Thomas.