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The Fréchet space ω admits a strictly stronger separable and quasicomplete locally convex topology

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Definition. A locally convex space is called submetrizable if it admits a coarser metrizable locally convex topology.

Lemma 1. If X is a Hausdorff locally convex space and $Y \subset X$ a linear subspace such that $\dim(X/Y)$ is at most countable and such that Y is submetrizable, then also X is submetrizable.

Lemma 2. Let X be a locally convex space. Then X is separable if and only if the weak dual $(X, \sigma(X,X'))$ is submetrizable.

Proposition. Let $\omega := \mathbb{C}^{\mathbb{N}}$ be provided with its product topology $\mathcal{P}$ and let $\mathcal{T}$ be any separable locally convex topology on ω. Then the supremum $\mathcal{P} \vee \mathcal{T}$ is also separable.

Example. By an example of I. Amemyia - Y. Kōmura (Math.Ann. 177 (1968)) and R. Knowles - T. Cook (Proc.Camb.Phil.Soc. 74(1973)) there exists a locally convex topology $\mathcal{T}$ on ω such that $(\omega, \mathcal{T})$ is separable and such that every bounded set in $(\omega, \mathcal{T})$ has finite dimensional linear span. Now the supremum $\mathcal{T} \vee \mathcal{P}$ is separable, and every bounded set in $(\omega, \mathcal{T} \vee \mathcal{P})$ has finite dimensional linear span. Thus $\mathcal{T} \vee \mathcal{P}$ is quasicomplete and strictly stronger than $\mathcal{T}$.

This example answers a question by E. Thomas.