Anzelm Iwanik

Extreme contractions on C(X) and $L^1(\mu)$

In: Zdeněk Frolík (ed.): Abstracta. 5th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1977. pp. 33--35.

Persistent URL: http://dml.cz/dmlcz/701085

FIFTH WINTER SCHOOL (1977)

EXTREME CONTRACTIONS ON $C(X)$ AND $L^1(\mu)$

A. IWANIK

Wrocław

Let X and Y be compact Hausdorff spaces and $C(X)$, $C(Y)$ the corresponding Banach lattices of real-valued continuous functions. We denote by U the unit ball in $\mathcal{L}(C(X), C(Y))$, by P the positive part of U, and by P_1 the subset of P consisting of all contractions taking 1 into 1.

An operator $T \in U$ is called nice if the adjoint T' takes extreme points of the unit ball in $C(Y)'$ into extreme points of the unit ball in $C(X)'$. It easy to see that every nice contraction is an extreme point of U. In the converse direction, M. Sharir [2] has shown that if Y is Stonian (i.e. extremally disconnected) then every extreme contraction is nice.

If Y is Stonian then $\mathcal{L}(C(X), C(Y))$ is a Banach lattice under its canonical ordering. Using Banach lattice techniques along with known results on positive operators (due to A. Ionescu Tulcea and C. Ionescu Tulcea, A. J. Ellis, and R. R. Phelps) we can restate Sharir's theorem in the following manner:

Theorem 1. Suppose X, Y are compact Hausdorff spaces and let Y be Stonian. Then for any $T \in \mathcal{L}(C(X), C(Y))$ the following conditions are equivalent:

(i) $T \in \operatorname{ex} U$,

(ii) $T^{\perp} \in \operatorname{ex} P$ and $|T1| = 1$,

(iii) $|T| \in \operatorname{ex} P_1$,

(iv) there exist a function $r \in C(Y)$ with $|r| = 1$ and a continuous map $\varphi: Y \longrightarrow X$ such that $Tf(y) = r(y)f(\varphi(y))$ for all $f \in C(X)$ and all $y \in Y$,

(v) T is nice.

Now suppose that X and Y are hyperstonian and denote by U_o the set of order continuous contractions in $\mathcal{L}(C(X), C(Y))$. The following result characterizes the set of extreme points of U_o

Proposition 1. $\operatorname{ex} U_o = U_o \cap \operatorname{ex} U$.

If (Q, Σ, μ) is a σ-finite measure space then there exists a hyperstonian space Y such that $L^{\infty}(\mu)$ is Banach lattice isomorphic with $C(Y)$. Also, if X is the corresponding hyperstonian space for a σ-finite measure ν, then we can show that the operator $T \in \mathcal{L}(C(X), C(Y))$ is order continuous if and only if it is the adjoint of an operator $S \in \mathcal{L}(L^1(\mu), L^1(\nu))$. Following this idea and using Theorem 1, Proposition 1, and results of [1] we obtain the following theorem.

Theorem 2. Let (Q, Σ, μ) be a σ-finite measure space and let ν be a σ-finite Borel measure on the real line R. Then $T \in \mathcal{L}(L^1(\mu), L^1(\nu))$ is an extreme contraction if and only if there exist $r \in L^{\infty}(\mu)$ with $|r| = 1$ and a nonsingular measurable transformation $\varphi: Q \longrightarrow R$ such that the adjoint T' of T is of the form

$$T'f(y) = r(y)f(\varphi(y)) \quad \text{a.e.}$$

for all $f \in L^{\infty}(\nu)$. (Here nonsingular means $\nu(B) = 0 \Rightarrow \mu(\varphi^{-1}(B)) = 0$).

References

[1] A. Iwanik, Pointwise induced operators on L_p-spaces, Proc. Amer. Math. Soc. 58 (1976), 173-178.

[2] M. Sharir, Characterizations and properties of extreme operators into C(Y), Israel J. Math. 12 (1972), 174-183.

INSTITUTE OF MATHEMATICS
TECHNICAL UNIVERSITY
WROCŁAW, POLAND