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For considering extensions of the Bishop-Phelps theorem [1], Lindenstrauss [5] studied the following two possible properties for a real Banach space X .

PROPERTY A. For every Banach space Y the set

$$P(X,Y) = \{T \in L(X,Y) : \|T\| = \|Tx\| \text{ for some } x \text{ in } X \text{ with } \|x\| = 1\}$$

is norm dense in the space $L(X,Y)$ of all bounded linear operators on X to Y .

PROPERTY B. For every Banach space Y the set $P(Y,X)$ is norm dense in $L(Y,X)$.

For a recent study, see [4]. Here we proved

THEOREM [3]. If X fails to have the Radon-Nikodým property, then there exist equivalent norms $\|\cdot\|$ and $\|\|\cdot\|\|$ on X such that the identity operator is not in the closure of $P((X,\|\cdot\|), (X,\|\|\cdot\|\|))$. In particular, $(X,\|\cdot\|)$ does not have Property A and $(X,\|\|\cdot\|\|)$ does not have Property B.

The proof is obtained by modifying a proof in [2] where it is shown that if X has the RNP then it satisfies an apparently much stronger property than Property A.

An open question is: Is Property A an isomorphic property? (Equivalently: Is Property A equivalent to the RNP?)

References

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