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Supereflexive Banach spaces

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FOURTH WINTER SCHOOL (1976).

## SUPERREFLEXIVE BANACH SPACES

by

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A Banach space  $X$  mimics a Banach space  $Y$  if for each finite dimensional subspace  $L \subset Y$  and  $\varepsilon > 0$ , there is a linear operator  $T: L \rightarrow X$  with  $\|T\| \cdot \|T^{-1}\| \leq 1 + \varepsilon$

Examples: 1)  $c_0(N)$  mimics any Banach space (this is easy to prove)

2) A. Dvoretzky: Any Banach space mimics Hilbert space

3) J. Lindenstrauss, H. Rosenthal: Any Banach space  $X$  mimics its bidual  $X^{**}$  - so called local reflexivity of any Banach space -

A norm of a Banach space  $X$  is uniformly rotund if for each

$$\varepsilon > 0, \inf_{\substack{\|x\|=\|y\|=1 \\ \|x-y\| \geq \varepsilon}} \left(1 - \left\| \frac{x+y}{2} \right\| \right) > 0$$

A set  $\{x_1, \dots, x_{2n+1}\}$  of a Banach space  $X$  is an  $(n - \varepsilon)$  tree in  $X$  if

$$x_j = \frac{1}{2} \cdot (x_{2j} + x_{2j+1}), \|x_{2j} - x_{2j+1}\| \geq \varepsilon, j=1, 2, \dots, n.$$

A proof of the following theorem of  
was discussed:

Theorem (R.C. James, P. Enflo). The following properties of a Banach space  $X$  are equivalent:

a)  $X$  mimics only reflexive Banach spaces (i.e.  $X$  is so called superreflexive)

- b)  $X$  admits an equivalent uniformly rotund norm
- c)  $X$  admits an equivalent uniformly Fréchet smooth norm
- d) for each  $\epsilon > 0$ , there is an integer  $n$  such that no  $(n - \epsilon)$  tree lies in the unit ball of  $X$

References: The works of R.C. James and P. Enflo - see e.g. the last edition of Day's book on Normed spaces.