Václav Zizler
Supereflexive Banach spaces

In: Zdeněk Frolík (ed.): Abstracta. 4th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1976. pp. 161--162.

Persistent URL: http://dml.cz/dmlcz/701069

FOURTH WINTER SCHOOL (1976).

SUPERREFLEXIVE BANACH SPACES

by

V. ZIZLER

A Banach space X mimics a Banach space Y if for each finite dimensional subspace $L \subset Y$ and $\varepsilon > 0$, there is a linear operator $T: L \longrightarrow X$ with $|T| \cdot |T^{-1}| \leq 1 + \varepsilon$

Examples: 1) $c_0(N)$ mimics any Banach space (this is easy to prove)

2) A. Dvoretzky: Any Banach space mimics Hilbert space

3) J. Lindenstrauss, H. Rosenthal: Any Banach space X mimics its bidual X^{**} - so called local reflexivity of any Banach space -

A norm of a Banach space X is uniformly rotund if for each $\varepsilon > 0$, $\inf\limits_{\substack{|x|=|y|=1 \\ |x-y| \geq \varepsilon}} (1 - |\frac{x+y}{2}|) > 0$

A set $\{x_1, \dots, x_{2n+1}\}$ of a Banach space X is an $(n - \varepsilon)$ tree in X if

$$x_j = 1/2 \cdot (x_{2j} + x_{2j+1}), \ |x_{2j} - x_{2j+1}| \geq \varepsilon \ , j=1,2,\dots,n.,$$

A proof of the following theorem of was discussed:

Theorem (R.C. James, P. Enflo). The following properties of a Banach space X are equivalent:

a) X mimics only reflexive Banach spaces (i.e. X is so called superreflexive)

b) X admits an equivalent uniformly rotund norm

c) X admits an equivalent uniformly Fréchet smooth norm

d) for each $\varepsilon > 0$, there is an integer n such that no $(n - \varepsilon)$ tree lies in the unit ball of X

References: The works of R.C. James and P. Enflo - see e.g. the last edition of Day's book on Normed spaces.