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EMBEDDING WEAKLY COMPACT SETS

by

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There is the well known result of Amir and Lindenstrauss that every weakly compact set in a Banach space can be embedded (even affinely) into a space $c_0(\Gamma)$ for some set Γ (of course, on the space $c_0(\Gamma)$ the weak topology is considered)

In the paper of Y. Benyamini and T. Starbird: "Embeddings Weakly Compact Sets into Hilbert Space" (to appear in Israel J. of Math.) the possibility of a strengthening of the result above is investigated.

The two following theorems are obtained:

Theorem 1: There exists a weakly compact set in a Banach space which does not embed into a Hilbert space.

Theorem 2: For a weakly compact set K in a Banach space the following conditions are equivalent:

(i) K embeds into a Hilbert space

(ii) K embeds into a superreflexive space

(iii) K embeds into a space $c_0(\Gamma)$ for some set Γ in such a way that the following condition is satisfied (we identify K with its image in $c_0(\Gamma)$):

for all $\varepsilon > 0$ there exists a natural number $N(\varepsilon)$ such that for all $k \in K$ the cardinality

$$\text{card}\{\gamma \in \Gamma;\ |k(\gamma)| > \varepsilon\} < N(\varepsilon)$$

The other bibliography concerning the embeddings weakly compact sets:

D. Amir, J. Lindenstrauss: The structure of weakly compact sets in Banach spaces, Annals of Math. 88(1968), pp. 35-46

W.J. Davis, T. Figiel, W.B. Johnson, A. Pelcynski: Factoring weakly compact operators, J. Funct. Anal. 17(1974), pp. 311-327.