

Jan K. Páchl

Compactness and other questions in spaces of uniform measures

In: Zdeněk Frolík (ed.): Abstracta. 4th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1976. pp. 129--130.

Persistent URL: <http://dml.cz/dmlcz/701057>

Terms of use:

© Institute of Mathematics of the Academy of Sciences of the Czech Republic, 1976

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://project.dml.cz>

FOURTH WINTER SCHOOL (1976)

COMPACTNESS AND OTHER QUESTIONS IN SPACES OF UNIFORM
MEASURES

by

Jan PACHL

I have tried to discuss one particular aspect of so far unexplored duality between locally convex spaces and uniform spaces; viz. questions connected with "free" functors from Unif to LocConv and related questions of weak integrals of vector-valued functions. Uniform and free uniform measures arising in this way generalize several long-studied topics in measure theory (such as σ -additive and separable measures on top. spaces, cylindrical measures, σ -additive measures or σ -algebras). In 1975's Winter School I posed the following two problems:

1) In $\mathcal{M}_U(X)$ for an arbitrary uniform space X , is it true that any weakly compact set is compact ?

2) Is there a "nice" class \mathcal{C} of uniform spaces such that $\mathcal{M}_F(X) =$ the Radon measures with a compact support in \hat{X} , for any $X \in \mathcal{C}$?

Answers: ad 1) Yes. Much more is true:

a) the same holds for sequential compactness

b) the space \mathcal{M}_U is always weakly sequentially complete

c) the same holds for vector-valued measures

d) the same holds for (vector-valued) free uniform measures

ad 2) Yes. The class of sub-inversion-closed uniform spaces is coreflective and the equality above holds for any sub-inversion-closed space.

Several references:

- Berezanskij I.A.: Measures on uniform spaces and molecular measures, *Trudy Moskov. mat. obšč.* 19(1968),3-40 (Russian, English translation: *Trans. Moscow Math. Soc.* 19 (1968) 1-40)
- Frolík Z.: Mesures uniformes, *C.R. Acad. Sci. Paris* 277(1973), A 105-108
- " : Représentation de Riesz des mesures uniformes, *ibid.*, A 163-166
- Rajkov D.A.: Free locally convex spaces of uniform spaces (Russian) *Math. Sb.* 63(105)(1964), 582-590
- Le Cam L.: Note on a certain class of measures (unpublished)
- Pachl J.: Free uniform measures, *Comment. Math. Univ. Carolinae* 15(1974), 541-553
- " : Free uniform measures on subinversion-closed spaces, submitted to *Comment. Math. Univ. Carolinae*
- " : Compactness in spaces of uniform measures, mimeographed notes of Zdeněk Frolík Seminar (Abstract Analysis), Prague, 1974/75 [extended version is submitted to *Cz. Math. J.*]