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AN EXAMPLE OF J.W. ROBERTS OF A CONVEX COMPACT SUBSET IN A LINEAR METRIC SPACE WITH NO EXTREME POINTS

by

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Very recently J.W. Roberts has constructed a convex compact subset K of a linear metric space (X,d) (obviously, non-locally convex) with no extreme points. This answers a well known problem of an existence of such a set (cf. for example the book of R.R. Phelps "Lectures on Choquet theory"). The construction of the author can be summarized in the following way:

Let E be the linear space of all real-valued step functions defined on the unit interval of the form $f = \sum a_i \chi_{[\alpha_i, \beta_i)}$ where α_i, β_i are binary rational numbers. In the space E consider the set

$$C = \{f \in E: f \geq 0, \int f dt \leq 1\} .$$

Using some delicate finite dimensional arguments, one can prove (the proof is relatively complicated) that there exists a linear metric d on E such that if X and $\tilde{C}$ are the completions of E and C (respectively) in the metric d then $\tilde{C}$ is a convex compact cone in X with only one extreme point (namely - the origin).

To obtain the desired example it suffices to define

$$K = \tilde{C} - \tilde{C} .$$