WSAA 4

Aleš Pultr

On categories over the closed categories of fuzzy sets

In: Zdeněk Frolík (ed.): Abstracta. 4th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1976. pp. 47--63.

Persistent URL: http://dml.cz/dmlcz/701043

FOURTH WINTER SCHOOL (1976)

ON CATEGORIES OVER THE CLOSED CATEGORIES OF FUZZY SETS

by

Aleš PULTR

Using the ordinary categorial language working with structured sets and mappings between them requires, roughly speaking, that

(i) it is just one type of mappings we are interested in, and

(ii) this type of mappings is closed under composition.

Fortunately enough, this is often the case. On the other hand, it is often not. One structure (e.g. topology) may give rise to various quite natural definitions of well-behaved mappings (in our example: continuous, open, closed, etc.). Worse still, one cannot sometimes solve the situation by dealing with distinct definitions of suitable mappings separately, because the condition (ii) may fail to be satisfied. Thus, one may be interested in metric spaces and mappings with small Lipschitz constants; but if one allows for mappings with a Lipschitz constant $K > 1$, one has to accept all the Lipschitz mappings to satisfy (ii). Similarly, one does not have a "category of graph homomorphisms up to small defects", a "category of non-constant mappings" etc. We want to show here that in many cases the language of

$\mathcal{V}$-categories (in particular, with $\mathcal{V}$ being a closed category of fuzzy sets) is appropriate.

§ 1. Closed categories.

1.1. The category of all sets and mappings will be denoted by Set. If $\mathcal{Q}$ is a category, the symbol $\mathcal{Q}$ is used also for the functor

$$\mathcal{Q}^{op} \times \mathcal{Q} \longrightarrow \text{Set}$$

given by $\mathcal{Q}(A,B) = \{\varphi \mid \varphi : A \to B \text{ in } \mathcal{Q}\}$, $\mathcal{Q}(\alpha,\beta)(\varphi) = \beta \circ \varphi \circ \alpha$.

1.2. A closed category $\mathcal{V}$ is a collection of data

$$\mathcal{V} = (\mathcal{V}_o, \otimes, H, E, k, a, b, c)$$

where $\mathcal{V}_o$ is a category,

$$\otimes : \mathcal{V}_o \times \mathcal{V}_o \longrightarrow \mathcal{V}_o,\ H: \mathcal{V}_o^{op} \times \mathcal{V}_o \longrightarrow \mathcal{V}_o$$

are functors, E is an object of $\mathcal{V}_o$ and

$k = k_{XYZ}: \mathcal{V}_o(X \otimes Y, Z) \cong \mathcal{V}_o(X, H(Y,Z))$,

$a = a_{XYZ}: (X \otimes Y) \otimes Z \cong X \otimes (Y \otimes Z)$,

$b = b_X \quad : X \otimes E \cong X$ and

$c = c_{XY} \quad : X \otimes Y \cong Y \otimes X$

are natural equivalences ($X \otimes Y$ stands for $\otimes(X,Y)$) satisfying, moreover certain rules (the coherence rules; it is not necessary to formulate them here).

The object E is called the unit, the functor $\otimes$ is usually called the tensor product (or tensor multiplication). If it coincides with the usual categorial product (more exactly, if there are natural transformations $p_{1XY}: X \otimes Y \to X$, $p_{2XY}: X \otimes Y \to Y$ such that $(p_{iXY})_{i=1,2}$ is always a product of X and Y - of course, in that case we write $X \times Y$ instead of $X \otimes Y$), $\mathcal{V}$ is said to be cartesian closed.

1.3. Remarks: 1. Originally, in [1] the expression closed category was used in a more general sense. The notion of "closed" from 1.2 coincides with "symmetric monoidal closed" from [1]. This shorter terminology is widely used ([5],[7]). On the other hand, it is necessary to stress that the more general enrichments of categories (just closed, just monoidal) are of importance, and that e.g. the notion of a $\mathcal{V}$-category we are going to discuss below can be easily given sense in the more general setting. The reader is therefore advised, if coming across the expression "closed category" in the literature, to check the precise meaning.

2. If there is no danger of confusion, one uses the same symbol for the whole collection of data constituting a closed category, and for its underlying category.

3. The role of the here unspecified coherence rules is, roughly speaking, to make sure that a transit from one expression to another by means of the natural equivalences a, b, c does not depend on the way chosen. (Thus, e.g. that the composition $X \otimes Y \xrightarrow{c} Y \otimes X \xrightarrow{c} X \otimes Y$ is the identity, that $(X \otimes Y) \otimes E \xrightarrow{a} X \otimes (Y \otimes E) \xrightarrow{1 \otimes b} X \otimes Y$ coincides with $(X \otimes Y) \otimes E \xrightarrow{b} X \otimes Y$ etc.)

4. There is (up to obvious equivalence) at most one way

to make a category $\mathcal{A}$ to a cartesian closed one. The unit is then the singleton, the natural equivalences a, b, c are induced in the obvious way by the product properties. Thus obtained a, b, c are always coherent. Hence, the question whether a category $\mathcal{A}$ (with products) can be made to a cartesian closed category reduces to the question whether every

$$- \times X : \mathcal{A} \longrightarrow \mathcal{A}$$

is a left adjoint.

5. Partly, the notion of a closed category was motivated by the task to endow the set $\mathcal{V}(X,Y)$ with a suitable structure to make it an object of $\mathcal{V}$. This makes, of course, a sense only in a concrete category $(\mathcal{V},U)$, i.e. a category $\mathcal{V}$ together with a fixed forgetful functor $U: \mathcal{V} \longrightarrow$ Set, and results in the condition

$$U \circ H \cong \mathcal{V}(-,-).$$

We will show now that this is implicitly contained in the conditions of 1.2, more exactly, that it either follows or cannot hold at all. Really, we have $\mathcal{V}(X,Y) \cong \mathcal{V}(X \otimes E,Y) \cong$ $\cong \mathcal{V}(X,H(E,Y))$, so that $H(E,-) \cong 1_{\mathcal{V}}$. Consequently, if $U \circ H = \mathcal{V}$, we have $U \cong UH(E,-) \cong \mathcal{V}(E,-)$. On the other hand, if $U \cong \mathcal{V}(E,-)$, we have $UH(X,Y) \cong \mathcal{V}(E,H(X,Y)) \cong$ $\cong \mathcal{V}(E \otimes X,Y) \cong \mathcal{V}(X,Y)$.

1.4. Examples: In the following examples we will specify just the functors $\otimes$, H and, if necessary, the unit and the equivalence k. The other data will be obvious.

1. The category Set is cartesian closed, with H(X,Y) =

$= Y^X$ (k is given by $((k(\varphi))(x))(y) = \varphi(x,y)$.).

2. The category Ab of abelian groups and their homomorphisms, with $\otimes$ the usual tensor product and H = Hom.Ab cannot be made cartesian closed.

3. The category Top of all topological spaces and their continuous mappings, with $X \otimes Y$ being the cartesian product of the underlying sets of X, Y endowed by the coarsest topology such that $f: X \otimes Y \to Z$ is continuous iff all $f(x,-): Y \to Z$ and $f(-,y): X \to Z$ are, and H(X,Y) the set of all continuous mappings $X \to Y$ with the topology of pointwise convergence.

4. Top cannot be made cartesian closed but its full subcategory K-Top of compactly generated spaces can. The H(X,Y) is endowed here by a modification of the compact-open topology.

5. Consider a partially ordered set $(X, \leq)$ understood as a thin category (i.e., there is exactly one morphism $x \to y$ if $x \leq y$, none otherwise). The structure of a closed category on $(X, \leq)$ consists of an order preserving operation (let us denote it by $\cdot$) making X to a commutative monoid ($\equiv$ semigroup with unit), and a mapping $h: X \times X \to X$ antimonotone in the first and monotone in the second variable such that

$$x.y \leq z \quad \text{iff} \quad x \leq h(y,z).$$

Obviously, if $(X, \leq)$ is a complete lattice, a necessary and sufficient condition of the existence of such an h is that x.- preserves suprema.

6. In particular, $(X, \leq)$ is cartesian closed iff there is an h such that

$$x \wedge y \leq z \quad \text{iff} \quad x \leq h(y,z).$$

(I.e., if $(X, \leq)$ is a Heyting algebra; one writes often $y \Rightarrow z$ for $h(y,z)$. In the case of complete lattices one sees immediately that the necessary and sufficient condition from 5 translates to the complete distributivity, which is the well-known characteristics of Heyting algebras.)

7. Below, we will reproduce a Lawvere's example based on the following thin closed category $\mathcal{V} = (R^+, +, h, 0)$: R^+ is the set of all non-negative integers inversely ordered, $+$ is the usual addition, $h(x,y) = \max(0, y - x)$. (Obviously really $x + y \geq z$ iff $x \geq h(y,z)$.)

§ 2. $\mathcal{V}$-categories.

2.1. Throughout this paragraph,

$$\mathcal{V} = (\mathcal{V}, \otimes, H, E, k, a, b, c)$$

is a fixed closed category. The notion of a $\mathcal{V}$-category (see e.g. [1],[9]) we are going to describe is a natural generalization of the notion of a category. Roughly speaking, it is based on the observation that in the definition of category one actually already uses morphisms and their composition - namely those of Set. The point is in replacing the category Set by a more general $\mathcal{V}$. To stress the point, we will give the definition of $\mathcal{V}$-category in confrontation with repeating the well-known definition of category. To avoid an unnecessary discussion, we will omit the (otherwise very important) condition on disjointness of the morphism sets.

2.2. A category $\mathcal{A}$	A $\mathcal{V}$-category $\mathcal{A}$

consists of the following data:
a class $|\mathcal{A}|$ (the elements of which are called the objects of $\mathcal{A}$), a correspondence M associating with every $(A,B) \in |\mathcal{A}| \times |\mathcal{A}|$

a set $M(A,B)$ (thus, an object $M(A,B)$ of Set),	an object $M(A,B)$ of $\mathcal{V}$,

a correspondence m associating with every $(A,B,C) \in |\mathcal{A}| \times |\mathcal{A}| \times |\mathcal{A}|$ the composition rule which is

a mapping (a morphism in Set) $m_{ABC}: M(B,C) \times M(A,B) \longrightarrow M(A,C)$,	a morphism in $\mathcal{V}$ $m_{ABC}: M(B,C) \otimes M(A,B) \longrightarrow M(A,C)$,

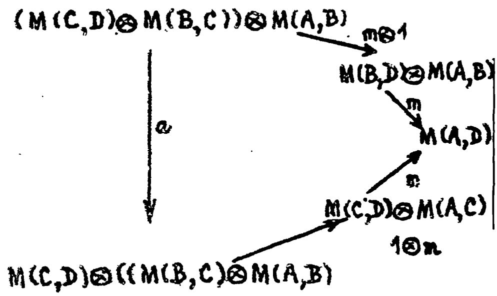
and a correspondence associating with every $A \in |\mathcal{A}|$

an element $1_A \in M(A,A)$ (which can be represented as a mapping $\varepsilon_A : E \longrightarrow M(A,A)$ where $E = \{0\}$, $\varepsilon_A(0) = 1_A$; recall that E is a unit corresponding to the product $\times$)	a morphism $\varepsilon_A : E \longrightarrow M(A,A)$ in $\mathcal{V}$,

such that

$m_{ABD}(m_{BCD}(\gamma,\beta),\alpha) =$
$= m_{ACD}(\gamma, m_{ABC}(\beta,\alpha))$
(the composition is associative)

the diagrams

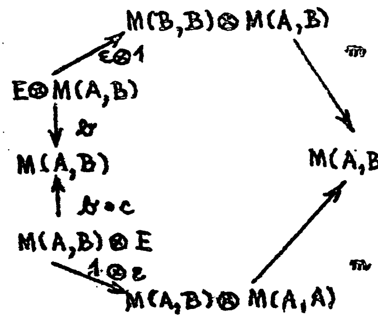


commute,

and

for $\alpha \in M(A,B)$
$m(1_B,\alpha) = m(\alpha,1_A) = \alpha$.

the diagrams



commute.

2.3. Examples: 1. Thus, the category in the usual sense is the Set-category.

2. Consider the category Ab = (Ab, ⊗ ,Hom, $\mathbb{Z}$,...) from 1.4.2. In an Ab-category we have, instead of sets of morphisms, abelian groups of morphisms. The composition rule is a homomorphism

$$M(B,C)\otimes M(A,B) \longrightarrow M(A,C).$$

But such homomorphisms are in a natural one-to-one correspondence (given by the unique extension) with the bilinear mappings

$$M(B,C)\times M(A,B) \longrightarrow M(A,C).$$

If we write this as a composition, we see that the bilinearity results in the distributivity laws

$$\alpha\circ(\beta+\gamma)=\alpha\circ\beta+\alpha\circ\gamma,\ (\beta+\gamma)\circ\alpha=\beta\circ\alpha+\gamma\circ\alpha\ .$$

Thus, the notion of an Ab-category coincides with the well-known notion of an additive category.

3 (Lawvere). Take the $\mathcal{V}$ from 1.4.7 and see what happens: A $\mathcal{V}$-category is a class X together with a correspondence $M: X\times X \longrightarrow R^+$ and "morphisms"

$$M(y,z) + M(x,y) \geq M(x,z)$$
$$0 \geq M(x,x)$$

(and hence $M(x,x) = 0$).

Thus the notion of a $\mathcal{V}$-category with this particular $\mathcal{V}$ coincides with the notion of (in general, non-symmetric) quasi-metric space.

4. Every closed category $\mathcal{V}$ can be viewed in a natural way as a $\mathcal{V}$-category.

More examples of $\mathcal{V}$-categories will be given in § 4.

§ 3. The closed categories $(L,\sqcap)$-Fuzz.

3.1. Throughout this paragraph, L is a lattice with a least element o and a largest element e. Its ordering will be denoted by $\leq$.

3.2. An L-fuzzy set X (or just fuzzy set, if there is no danger of confusion) is a mapping

$$X:\ ?X \longrightarrow L$$

where ?X is a set. We write

$$x \in_a X \text{ for } X(x) \geq a.$$

Let X, Y be fuzzy sets. A morphism (cf. [4])

$$f:\ X \longrightarrow Y$$

is a mapping $f:\ ?X \longrightarrow ?Y$ such that for every $x \in ?X$, $Y(f(x)) \geq \geq X(x)$. Thus, in the convention above, $f:\ ?X \longrightarrow ?Y$ is a morphism $X \longrightarrow Y$ iff

for every $a \in L$, $x \in_a X$ implies $f(x) \in_a Y$.

The category formed by fuzzy sets and their morphisms will be denoted by

L-Fuzz.

Associating with a fuzzy set X the set ?X and with a morphism $X \longrightarrow Y$ the corresponding mapping $?X \longrightarrow ?Y$ we obtain a faithful functor

$$?\ :\ \text{L-Fuzz} \longrightarrow \text{Set}.$$

3.3. In [8] there was shown that the closedness structures ($\otimes$, H, ...) on L-Fuzz such that

$$?H(X,Y) = ?Y^{?X} \text{ and } H(X,Y)(f) = e \text{ for } f:\ X \longrightarrow Y$$

(i.e. such that all the mappings $?X \longrightarrow ?Y$ are in some extent members of H(X,Y), the morphisms having the strongest membership possible) are in a one-to-one correspondence with the tensor products on L (see 1.4.5) having e for the unit. This correspondence is given as follows: if $\square$ is the tensor product on L, for the corresponding $\otimes$, H holds

$?(X \otimes Y) = ?X \times ?Y, \quad (X \otimes Y)(x,y) = X(x) \,\square\, X(y),$

$f \in_a H(X,Y)$ iff for every $b \in L$, $x \in_b X$ implies $f(x) \in_{a \square b} Y$. The closed category with the closedness structure induced by $\square$ will be denoted by

$$(L, \square)\text{-Fuzz}.$$

3.4. Remark: The condition that the largest element e of L is the unit of $\square$ is equivalent with

$$x \square y \leq x \wedge y \ .$$

Really, if e is the unit, we have $x \square y \leq x \square e = x$ and similarly $x \square y \leq y$, so that $x \square y \leq x \wedge y$. On the other hand let $x \square y \leq x \wedge y$ and let j be the unit of $\square$. We have

$$e = e \square j \leq e \wedge j = j.$$

§ 4. $(L, \square)$-Fuzz-categories.

4.1. a) By the definitions above, we see that an $(L, \square)$-Fuzz-category $\mathcal{A}$ consists of a clas $|\mathcal{A}|$ of objects, fuzzy sets M(A,B) associated with couples A, B of objects, an associative composition (from now on, we will denote it by $\circ$)

$$\circ : ?M(B,C) \times ?M(A,B) \longrightarrow ?M(A,C)$$

such that

if $\beta \in_b M(B,C)$ and $\alpha \in_a M(A,B)$, $\beta \circ \alpha \in_{b \square a} M(A,C)$, and the units $1_A \in_e M(A,A)$ such that $1 \circ \alpha = \alpha$, $\alpha \circ 1 = \alpha$ whenever defined.

Let us call the $\alpha \in_a M(A,B)$ the a-morphisms from A to B, and write $\alpha :_a A \to B$. The rule above says that a composition of an a-morphism with a b-morphism gives an $a \square b$-morphism.

b) Obviously, we can view an $(L, \sqcap)$-Fuzz-category as follows: A category $?\mathfrak{A}$ (this is the $(|\mathfrak{A}|, ?M, \circ, (1_A))$) together with mappings $\sigma_{AB}: ?M(A,B) \longrightarrow L$ (in the notation above, $\sigma(\alpha) = M(A,B)(\alpha)$) such that

$$\sigma(\alpha \circ \beta) \geq \sigma(\alpha) \sqcap \sigma(\beta) \text{ and } \sigma(1_A) = e.$$

4.2. In particular we are interested in those $(L, \sqcap)$-Fuzz-categories where the objects are sets endowed by structures of some common type, $?M(A,B)$ are some of (possibly all) the mappings between the underlying sets, and σ says how far the mapping in question preserves the structure (the mappings which "really preserve the structure" having $\sigma(f) = e$).

4.3. Let L be a lattice. Consider a system of refinements in the sense of [2]

$$\mathfrak{K}_a \xrightarrow{r_{ab}} \mathfrak{K}_b$$

for $b \leq a$ in L. By the definition of a refinement, obviously $r_{bc} \circ r_{ab} = r_{ac}$. We see immediately that the system can be described as an $(L, \wedge)$-Fuzz-category with $\alpha \in_a M(A,B)$ iff $\alpha \in \mathfrak{K}_a(A,B)$. (Thus, the basic situation of one refinement is governed by the smallest non-trivial boolean algebra $\mathbf{2}$.) On the other hand, every $(L, \wedge)$-Fuzz-category can be viewed as such a system of refinements. The case with a general tensoring $\sqcap$ is finer than that, being able to deal with estimates of well-behaving of mappings which are not categorial.

4.4. Remark: It is worth noting that $\wedge$ is the only tensor product with which the situation is reduced to a system of refinements. In fact, it is the only tensor product with unit e which is idempotent. Really, we know already that for such an $\sqcap$, $x \sqcap y \leq x \wedge y$. If $\sqcap$ is idempotent, we have, on the other hand,

$$x \wedge y = (x \wedge y) \sqcap (x \wedge y) \leq x \sqcap y,$$

so that $\sqcap = \wedge$.

4.5. Let us describe now two patterns for constructing $(L, \sqcap)$-Fuzz-categories associated with concretely described structures.

In a reasonable generality (see [6]), structures on sets can be described as follows: A functor

$$F: \mathrm{Set} \longrightarrow \mathrm{Set}$$

is given; an F-structure on a set X is a subset r of F(X) (in concrete cases the structures are often subjected, moreover, to special conditions, but we do not need to go into it here). A mapping $f: X \longrightarrow Y$ behaves well with respect to structures r, s on X, Y resp., if $F(f)(r) \subset s$ (or $F(f)(s) \subset r$, in the contravariant case). Thus, e.g. n-ary relations are Q_n-structures, where Q_n sends X to X^n, the well-behaved mappings are the relation preserving ones; topology is a (special) P^--structure, where P^- is the contravariant power-set functor, the well-behaving mappings coinciding with the continuous ones; etc.

I. Let us have given mappings $\nu : \exp F(X) \longrightarrow L$ such that $\nu(A \cup B) \geq \nu(A) \sqcap \nu(B)$, $\nu(\emptyset) = e$, and that for

$f: X \to Y$ $\nu(F(f)(A)) \geq \nu(A)$ (e.g., L may be the inversely ordered set of natural numbers plus ∞, $\square = +$, $\nu(A)$ the number of elements of A). For a mapping $f: X \to Y$ and F-structures r, s on X, Y put

$$D_{rs}(f) = F(f)(r) \setminus s.$$

We have

$D_{rt}(g \circ f) = F(g \circ f)(r) \setminus t = F(g)(F(f)(r)) \setminus t \subset F(g)(s \cup$
$\cup (F(f)(r) \setminus s)) \setminus t = (F(g)(s) \cup F(g)(D(f))) \setminus t \subset D_{st}(g) \cup$
$\cup F(g)(D_{rs}(f))$.

Thus, putting

$$\sigma(f) = \nu(D(f))$$

one obtains, using the point of view of 4.1 b), an $(L, \square)$-Fuzz-category in which the failure to preserve a structure is measured according to the extent of the damaged part in $f(X)$.

II. Let us have given for every F-structure r on X we are interested in (thus, not necessarily for every $r \subset F(X)$) an L-fuzzy set $\tilde{r}$ such that

$$?\tilde{r} = F(X) \text{ and } \tilde{r}^{-1}(e) = r$$

and such that, moreover, if $u \in r$ implies $F(f)(u) \in_a \tilde{s}$, then $u \in_b \tilde{r}$ implies $F(f)(u) \in_{a \square b} \tilde{s}$. Then put

$$\sigma(f) = H(\tilde{r}, \tilde{s})(F(f))$$

where H is associated with $\square$ (see 3.3).

Here, roughly speaking, one measures not how large the damaged part is, but how large is the damage (everything may be damaged a bit, nothing too badly; in such a case, in I the mapping would be valuated as hopelessly bad, here only moderately so). See 4.6.1 .

4.6. Examples: In the following examples, if the pattern from 4.5.I is used, L is the inversely ordered set of natural numbers plus ∞, $\square = +$, $\nu(A)$ is the number of elements of A.

1) One-to-one mappings and in what extent a mapping is not such: Consider the contravariant power-set functor P^- and put $r_X = \{A \subset X \mid \text{card } A \leq 1\}$. Obviously, $f: X \longrightarrow Y$ is one-to-one iff $P^-(f)(r_Y) \subset r_X$.

a) The procedure I assigns to a mapping f the number of elements of Y into which more than one element of X is mapped.

b) Let L be the set of all positive natural numbers plus ∞ , again inversely ordered. Put

$\tilde{r}_X(A) = 1$ if card $A \leq 1$, $\tilde{r}_X(A)$ = card A otherwise. Obviously, we can use for $\square$ the usual multiplication of numbers.

Compare the valuation of mappings f, g: $N \times 2 \longrightarrow N$ defined by $f(n,i) = n$ and $g(n,0) = 0$, $g(n,1) = n$. In a), g is preferred to f, in b), f to g.

2) Using I for the structure of binary relations one obtains a description of the system of graphs and what is called their homomorphisms with defects. It would not be much use to try to describe it as a category in the ordinary sense (in particular in the finite case, one gets any mapping after sufficiently many compositions of homomorph-

isms with small defect).

3) Take, again, the structure of binary relations ($r \subset X \times X$), L like in 4.6.1 b). Put $(x,y) \in_n r$ iff $(x,y) \in r \cup r \circ r \cup \ldots \cup \underbrace{r \circ \ldots \circ r}_{n \text{ times}}$. Again, we see that in the procedure II we can use the ordinary multiplication.

4) Similarly, as in 2), if we use I for homomorphisms of algebras, say, with one binary operation, the defect of a mapping is obtained expressed as the number of the instances of the inequality $f(x).f(y) \neq f(x.y)$.

5) Consider the category of metric spaces. The proper choice of morphisms are the contractions. Considering the Lipschitz mappings we obtain again a category, but a lot is lost: e.g. an isomorphism is not necessarily an isometry any more. We can, however, consider the system as an $(L, \square)$-Fuzz-category where L is the inversely ordered set of real numbers ≥ 1, and $f:_a X \to Y$ iff its Lipschitz constant is less or equal to a. This fits into the pattern II above and the reader is invited to show how (Hint: the contractions preserve a system of binary relations.).

References:

[1] S. Eilenberg, G.M. Kelly: Closed Categories, in: Proc. Conf. on Categorial Algebra, La Jolla 1965(Springer-Verlag 1966), 421-562

[2] Z. Frolík: Three technical tools in uniform spaces, Seminar Uniform Spaces 1973-1974, Matem. ústav ČSAV, 3-26

[3] J.A. Goguen: L-fuzzy sets, J. Math. Anal. Appl. 18(1967), 145-174

[4] J.A. Goguen: Categories of V-sets, Bull. AMS 75(1969), 622-624

[5] G.M. Kelly, S.MacLane: Coherence in closed categories, J. of Pure and Appl. Algebra, Vol.1(1971), 97-140

[6] L. Kučera, A. Pultr: On a mechanism of defining morphisms in concrete categories, Cahiers de Topologie et Géométrie Différentielle XIII-4, 397-409

[7] F.W. Lawvere: Metric spaces, generalized logic, and closed categories, Rendiconti del Seminario Matematico e Fizico di Milano, Vol. XLIII(1973), 135-166

[8] A. Pultr: Closed categories of fuzzy sets, to appear in Proceedings of the Conference "Automaten- und Algorithmentheorie", Weissig, 1975

[9] J.W. de Schipper: Symmetric Closed Categories, Mathematical Centre Tracts, Mathematisch Centrum Amsterdam 1975