

Toposym 1

L. A. Tumarkin

Concerning infinite-dimensional spaces

In: (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the symposium held in Prague in September 1961. Academia Publishing House of the Czechoslovak Academy of Sciences, Prague, 1962. pp. [352]--353.

Persistent URL: <http://dml.cz/dmlcz/700987>

Terms of use:

© Institute of Mathematics AS CR, 1962

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://project.dml.cz>

CONCERNING INFINITE-DIMENSIONAL SPACES

L. TUMARKIN

Moscow

The problem whether every infinite-dimensional compactum (= compact metric space) contains closed subsets of an arbitrary finite dimension, was formulated by myself some 35 years ago and it still remains open (even for closed one-dimensional subsets).

In this note some theorems concerning this problem are considered.

A metric space of infinite dimension is called countable-dimensional if it is a union of a countable number of 0-dimensional subsets. In the opposite case we call the space strongly infinite-dimensional.

The following definitions generalize a classical notion due to Urysohn.

An infinite-dimensional compactum is called a Cantorian manifold in the weak sense (or in the strong sense, respectively), if it cannot be decomposed by any finite-dimensional (or by any finite- or countable-dimensional, respectively) closed subset.¹⁾

The proof of the following theorem 1 is very easy:

Theorem 1. *An infinite-dimensional compactum X contains closed subset of an arbitrary finite dimension if and only if it contains some countable-dimensional closed set.*

Theorem 2 improves my older result [1].

Theorem 2. *Let X be an infinite-dimensional compactum. Then either*

a) *X contains a countable-dimensional closed set*

or

b) *X contains an infinite-dimensional Cantorian manifold in the strong sense.*

The two cases do not exclude each other.

However, the question whether every infinite-dimensional Cantorian manifold in the strong sense contains a countable-dimensional closed subset, still remains open.

Now we shall consider arbitrary separable metric spaces.

Theorem 3. *Under the assumption of the continuum hypothesis every strongly infinite-dimensional separable metric space X contains a set A with the following property:*

¹⁾ In the weak case we suppose moreover that the space can be decomposed by some countable-dimensional closed subset.

The intersection of A with every finite-dimensional or countable-dimensional subset of X is at most countable.

This theorem generalizes a result of W. HUREWICZ [2]. The proof makes use (as does the construction by Hurewicz) of the fact (proved by myself in the year 1925) that every n -dimensional subset of a separable metric space X (with a given metric) is contained in some G_δ -set of the same dimension, lying in the metric space X .

Concerning the countable dimensional spaces, I have proved in [3] the

Theorem 4. Every countable-dimensional separable metric space X is a union

$$X = \bigcup_{i=1}^{\infty} \mathfrak{M}_i$$

of 0-dimensional subsets \mathfrak{M}_i such that the sum of any finite number of them is still 0-dimensional:

$$\dim \bigcup_{i=1}^N \mathfrak{M}_i = 0 \text{ for any finite } N.$$

Let us finally point out that even the question whether in every countable dimensional separable metric X there is contained a subset of an arbitrary finite dimension, still remains open.

References

- [1] Л. А. Тумаркин: О бесконечномерных канторовых многообразиях. Докл. АН СССР, 115 (1957), 244—246.
- [2] W. Hurewicz: Une remarque sur l'hypothèse du continu. Fundam. Math., 19 (1932), 8—16.
- [3] Л. А. Тумаркин: О разбиении пространств на счетное число нульмерных множеств. Вестн. Моск. ун-та, серия I, математика, механика, № 1 (1960), 25—33.