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SOME RELATIONS BETWEEN TOPOLOGICAL AND ALGEBRAIC PROPERTIES OF TOPOLOGICAL GROUPS

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Several theorems are known about the mutual dependence between topological and algebraic properties of groups, especially of abelian topological groups. E. g. the equivalence of connexity and divisibility in compact abelian groups, or the algebraic characterization of the abelian groups which admit a compact topology (A. HULANICKI).

The following are some results of this kind which I obtained in the last years in cooperation with other authors:

I. The property of a topological group G to be generated by an arbitrary neighbourhood of the unit element e is, in the case of a locally compact group, not only necessary but also sufficient for G to be connected. Compact divisible groups have a stronger property, namely they are unions $\bigcup_{x \in U} [x]$ of cyclic subgroups, U being an arbitrary neighbourhood of e [1]. Since all connected compact groups (even non-abelian) are divisible [2], they may be represented in this manner.

II. A subgroup H of an abelian group G is called pure if the equation $nx = a$ $(a \in H)$ is soluble in H whenever it is soluble in G. Denote by K the class of abelian groups G such that either G itself or its character group $\hat{G}$ is generated by a compact neighbourhood of e. Then, for every $G \in K$, the closed subgroup $H \subset G$ is pure if and only if each (continuous) character of H of finite order can be extended to a character of the whole group G without raising its order. The assumption $G \in K$ cannot be dropped [3].

III. If $\aleph_{\alpha+1} = 2^{\aleph_\alpha}$ for every ordinal α, then it can be proved that:

(a) an infinite compact abelian[1]) group of cardinality $\leqq 2^{2^{\mathfrak{m}}}$ contains a dense subset of cardinality $\leqq \mathfrak{m}$,

(b) for an infinite locally compact abelian group of cardinality $\leqq 2^{2^{\mathfrak{m}}}$ the following conditions are equivalent:

(1) there are more than $\mathfrak{m}$ disjoint neighbourhoods in G,

(2) every dense subset of G has cardinality $> \mathfrak{m}$,

(3) G can be homomorphically and continuously mapped onto a discrete group of cardinality $> \mathfrak{m}$,

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[1]) From recent results communicated during the Symposion by Prof. P. S. ALEXANDROV it follows that the assumption of commutativity can be dropped.

(4) G can be continuously mapped onto an isolated space of cardinality $> \mathfrak{m}$.

These conditions are satisfied if G contains an isolated subgroup of cardinality $> \mathfrak{m}$. The converse is not true [4].

References

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[2] *Jan Mycielski*: Some Properties of Connected Compact Groups. Colloq. Mat. *5* (1958), 162–166.

[3] *S. Hartman* et *A. Hulanicki*: Les sous-groupes purs et leurs duals. Fundam. Math. *45* (1957), 71–77.

[4] *S. Hartman* et *A. Hulanicki*: Sur les ensembles denses de puissance minimum dans les groupes topologiques. Colloq. Mat. *6* (1958), 187–191.