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ON AN INEQUALITY CONCERNING CARTESIAN MULTIPLICATION

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1. For a family F of sets let DF be the supremum of the cardinal numbers of disjointed subfamilies of F. Let F^{I2} be the set of all the cartesian products $X \times Y$ with $X, Y \in F$. Analogously, for any ordinal number r let Ir be the interval of the ordinals $< r$ and let F^{Ir} be the system of the cartesian products of all r-sequences of members of F.

2. For a space S let GS be the system of all the open sets of G; we put $DS = D(GS)$; $DS^{Ir} = D(G(S^{Ir}))$; the number DS is called the cellularity or disjunction degree of the space S.

The question arises to find the relations between the numbers DF^{Ir} $(r = 1, 2, \ldots)$ for any set family F and particularly for $F = GS$, S being any given topological space.

3. Let (G, ϱ) be a binary graph i. e. G is a set and ϱ is a binary reflexive and symmetrical relation in G. Let I be a non void set and for every $i \in I$ let (G_i, ϱ_i) be a binary graph; we define the product of the graphs (G_i, ϱ_i) as (G, ϱ), where $G = \prod G_i$ and where for $x, y \in G$ the relation $x\varrho y$ means $\bigwedge_i x_i \varrho_i y_i$, i. e. for every $i \in I$ one has $x_i \varrho_i y_i$ (let us remind that $x \in \prod G_i$ means that x is a mapping of I such that $x_i \in G_i$ for every $i \in I$). Let $k_c(G, \varrho)$ (resp. $k_{\bar{c}}(G, \varrho)$ or $k_a(G, \varrho)$) be the supremum of the cardinal numbers of chains (resp. antichains) of (G, ϱ).

The problem arises to find the connections between the numbers $k_a G^{Ir}$ and $k_a G$.

4. Theorem. *For any set system F with infinite DF one has: $(DF)^n \leqq DF^{In} \leqq 2^{DF}$ for any natural number n.* (II) *For any ordinal α there is a system F_α of sets such that $DF_\alpha = \aleph_\alpha$, $DF^{I2} = 2^{\aleph_\alpha}$, and consequently $DF_\alpha < D(F_\alpha^{I2})$.*

5. Theorem. *For any binary graph (G, ϱ) one has $(k_a G)^n \leqq k_a G^{I2} \leqq 2^{k_a G}$; if $k_a G \geqq \aleph_0$, then $k_a G^{In} \leqq 2^{k_a G}$ for every natural number n.*

6. Theorem. *For any metrical infinite space S and any positive integer n one has $k_a S = k_a S^{In}$.*

7. Theorem. *For totally ordered sets O the relation (1) $k_a O = k_a O^{I2}$ is equivalent to the following reduction principle: Every infinite ramified set R of regular cardinality kR contains a degenerated subset D of cardinality kR (any ordered set O*

in which every principal ideal $O(.,x) = \{y; y < x; \ y \in O\}$ *is a chain is said to be ramified; O is degenerated if both: principal ideals and dual principal ideals of O are chains). The relation* (1) *is connected to the well-known Suslin problem.*

8. Problem. As yet one does not know any topological infinite space S satisfying $DS < DS^{I2}$; the problem is to exhibit such a space.