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In: (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the second Prague topological symposium, 1966. Academia Publishing House of the Czechoslovak Academy of Sciences, Praha, 1967. pp. 361--362.

Persistent URL: http://dml.cz/dmlcz/700881

ALGEBRAIC GENERALIZATION OF THE TOPOLOGICAL THEOREMS OF BOLZANO AND WEIERSTRASS

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The present communication constitutes an exposition of the results published in [2] and [3]. They are based on the results of [1], in which we have introduced the notions of "*functional structure*" and of "*homomorphism*" between such structures, and have shown that, with appropriate definitions, a sort of "algebra of sets" is valid for these structures and their homomorphisms.

The couple "structure – morphism" studied by Bourbaki, as well as the couple "category – covariant functor" (of S. Eilenberg and S. Maclane) can be expressed in the form of the couple "functional structure – homomorphism".

These couples seem appropriate for the generalization and unification of apparently different classical results, e.g. for the following couples: "topological space – continuous function", "uniform space – uniformly continuous function", "ring – homomorphism", etc..

So, an abstract theorem for functional structures [2] contains as particular cases: 1) the well-known Bolzano's theorem on the preservation of connexity by continuous surjections, 2) an obvious result for uniform spaces and 3) the theorem on the one-to-one correspondence between the prime ideals of a ring B and the prime ideals of a ring A containing the kernel of a homomorphism of A onto B (A and B are commutative rings with unity).

The introduction of the notion of "*dimension of quasi-compactness*" in [3] allows to show that the well-known Weierstrass theorem on the preservation of quasi-compactness (i.e. of the Heine-Borel-Lebesgue property) by continuous surjections and the algebraic statement that linear mappings do not increase linear dimension (in vector spaces) are all particular cases of the same abstract theorem for functional structures, asserting that, *under certain rather general conditions, homomorphisms between functional structures do not increase the dimension of quasi-compactness.*

The author presumes that some parts of General Topology (concerning the interplay between topological maps and continuous functions) can be extended to functional structures.

In relation to the above considerations the author introduces an abstract notion of dimension [4], containing as particular cases many of the existing notions of dimension (including linear dimension, algebraic dimension, dimension of a prime ideal in a ring and Lebesgue topological dimension) as well as the classical Riemann and Lebesgue integrals. The use of functional structures as well as of a generalization of the notion of lower limit (or upper limit respectively) have been necessary for the definition of this abstract notion of dimension [4], [5].

References

[1] *S. Zervos:* Structures fonctionnelles et homomorphismes. Comptes Rendus de l'Académie des Sciences de Paris *260* (1965), 3809—3812.

[2] *S. Zervos:* Une généralisation du théorème de Bolzano pour la connexité. Ibid, *260* (1965), 5979—5982.

[3] *S. Zervos:* Une généralisation abstraite du théorème topologique de Weierstrass pour la préservation de la quasi-compacité; une notion de dimension de quasi-compacité. Ibid, *260* (1965), 6781—6784.

[4] *S. Zervos:* Une notion abstraite de dimension. Ibid, *261* (1965), 859—862.

[5] *S. Zervos:* Une définition générale de la dimension. Séminaire Pisot-Delange-Poitou, Institut H. Poincaré, Paris, 1965—66.