

Toposym 2

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BAIRE SETS WHICH ARE BORELIAN SUBSPACES

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J. Knowles and C. Rogers [2] recognized the good behaviour of Borelian subspaces which are Baire sets; these sets are called BB-sets here (while the term descriptive Baire sets was used in [2]). They proved the characterization (2) of BB's in Theorem B below, and used that characterization in developing of properties of BB's following the patterns used in [1] or [3] for Borelian spaces; the resulting proofs were rather complicated.

Here we want to indicate a smooth development of properties of BB's. Theorems B(2), C and D were proved in [2]. The complete proofs (with Theorems E and D in a more general setting) will appear in Proc. Roy. Soc. A (1967) under a similar title.

By a space we always mean a uniformizable (=completely regular) topological space, and Σ will stand for the space of irrationals. A disjoint ($\{fx \mid x \in P\}$ is disjoint) upper semicontinuous compact (the sets fx are compact) multivalued mapping of P into Q will be called a DUCC mapping. By [1, Theorem 7] a space P is Borelian iff there exists a DUCC mapping of Σ on P (Borelian spaces were introduced and studied in [1]).

Theorem A. *Each of the following conditions is necessary and sufficient for a Borelian subspace X of a space P to be a Baire set in P :*

(a) *There exists a countable collection \mathcal{F} of continuous functions on P which distinguishes the points of X from the points of $P - X$ (i.e., if $x \in X$, $y \in P - X$ then $fx \neq fy$ for some f in \mathcal{F}).*

(b) *There exists a continuous mapping f of P into a separable metrizable space such that $f[X] \cap f[P - X] = \emptyset$.*

(c) *If k is DUCC mapping of Σ onto X then there exists an f as in (b) such that $f \circ k$ is disjoint (and so a DUCC mapping).*

(d) *There exists a DUCC mapping of Σ onto X and an f as in (b) such that $f \circ k$ is disjoint.*

(e) *Condition (b) with $f[X]$ a Baire set, in the range space of f .*

Theorem B. *Each of the following two conditions is necessary and sufficient for a subset X of P to be a BB-set in P :*

(1) X is Borelian, and if k is a DUCC mapping of Σ onto X then there exists a continuous mapping F of Σ into $C^*(P)$ such that kx is the zero set of Fx for each x in Σ .

(2) There exist k and F with properties stated in (1).

Theorem C. Every Baire set in a BB-set in P is a BB-set in P . The BB's of P form a σ -ring.

Theorem D. Every Borelian subspace of P is a Baire set provided that every closed set is a G_δ . In particular, in a perfectly normal space every Borelian subspace is a Baire set.

The main step is the following Separation Theorem.

Theorem E. If X is an analytic subspace of a space P and if Y is an F_σ in P disjoint to X , then there exists a Baire set Z in P with $X \subset Z$, $Z \cap Y = \emptyset$.

References

- [1] Z. Frolík: A contribution to the descriptive theory of sets and spaces. General Topology and its Relations to Modern Analysis and Algebra. (Proceedings of the Symposium held in Prague in September, 1961), 157—173.
- [2] J. Knowles and C. Rogers: Descriptive sets. Proc. Roy. Soc. A 291 (1966), 353—367.
- [3] C. Rogers: Descriptive Borel sets. Proc. Roy. Soc. A 286 (1965), 455—478.

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