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CONTINUOUS MAPPINGS OF EXTENSIONS OF A TOPOLOGICAL SPACE

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1. An extension X' of a topological space X is said to be a true extension of X if the system $\{\bar{F}^{X'} \mid F \subset X\}$ is a basis of closed sets of the space X' and for any point $x \in X' \setminus X$ the set $\{x\}$ is a closed set of X'. Here we shall consider only true compact extensions of topological spaces. The statements will be formulated in terms of contiguity relations, the latter being a relation between elements of finite systems of closed sets of X with the following properties:

C 1: If every element of a system α contains a certain element of a system β and $\sigma(\beta)$ takes place (σ holds for the system β) then $\sigma(\alpha)$ takes place.

C 2: If $\sigma(\alpha \vee \beta)$ takes place then either $\sigma(\alpha)$ or $\sigma(\beta)$ or both take place ($\alpha \vee \beta = \{A \cup B \mid A \in \alpha, B \in \beta\}$).

C 3: If a system α contains the empty set then $\sigma(\alpha)$ does not take place ($\bar{\sigma}(\alpha)$ takes place).

C 4: If the intersection of all elements of a system α is not empty then $\sigma(\alpha)$ takes place.

If a contiguity relation σ holds for every finite subsystem of a system γ of closed sets of X, the system γ being finite or infinite, then the system γ is said to be a σ-contiguity system. One can consider maximal σ-contiguity systems and distinguish among them the disappearing systems, that is the systems with the empty intersection.

Let $\hat{X}$ be the set of all maximal disappearing σ-contiguity systems and we put $\hat{F} = \{\gamma \mid F \in \gamma, \gamma \in \hat{X}\}$ for every closed set F of X. Now denote $X \cup \hat{X}$ by σX and consider the topological structure on the set σX induced by the closed basis $\{F \cup \hat{F}\}$. The topological space σX is found to be a true compact extension of the space X and, moreover, every true compact extension of the space X is equivalent to an extension σX for a proper contiguity relation σ on X. That is why in the theory of true compact extensions one may consider only such extensions σX. At this point we should like to note [1] that for any finite system α of closed sets of X, $\sigma(\alpha)$ takes place iff $\bigcap_{F \in \alpha} \bar{F}^{\sigma X} \neq \emptyset$. This property defines σ-extensions up to the extension equivalence.

We shall say that a contiguity relation σ is weaker than a contiguity relation σ' and write $\sigma \leqq \sigma'$ if any σ'-contiguity system is a σ-contiguity system. If $f : \sigma' X \to \sigma X$ is a natural continuous mapping then $\sigma \leqq \sigma'$. Thus the condition $\sigma \leqq \sigma'$ is a necessary condition for some natural mapping $f : \sigma' X \to \sigma X$ to exist. Moreover, if σX

and $\sigma'X$ are Hausdorff extensions then this condition is sufficient but in general it is not so. The subject of our paper is to find conditions for the existence of such natural mappings.

2. It is possible to establish some sufficient conditions for the existence of a natural mapping $f: \sigma'X \to \sigma X$. But we need some additional notions for this purpose.

A system $H_1, H_2, \dots, H_q$ of closed sets of X is said to be a σ-majorant of a system $F_1, F_2, \dots, F_p$ if, given any system $F'_1, F'_2, \dots, F'_n$ of closed sets of X, $\sigma(F'_1, \dots, F'_n, F_1, \dots, F_p)$ implies $\sigma(F'_1, \dots, F'_n, H_1, \dots, H_q)$.

We shall say that a contiguity relation σ' satisfies the condition C_σ if $\sigma'(F_1, F_2, \dots, F_n)$ and $\bar{\sigma}(F, F_1, \dots, F_n)$ imply the existence of a σ'-majorant H of the system $F_1, F_2, \dots, F_n$ such that $\sigma(F, H)$ does not take place.

We shall say that a contiguity relation σ satisfies the condition $C^0_{\sigma'\sigma}$ if $\bar{\sigma}(F_1, F_2, \dots, F_n)$ implies the existence of a system $H_1, H_2, \dots, H_n$ such that $F_i \subset H_i$, $\bar{\sigma}'(H_1, H_2, \dots, H_n)$ and $\bar{\sigma}'(H_1, \dots, H_{j-1}, F, H_{j+1}, \dots, H_n)$ imply $\bar{\sigma}(F_1, \dots, F_{j-1}, F, F_{j+1}, \dots, F_n)$ for any closed set F.

Theorem 1. *Let a contiguity relation σ' on X satisfy the conditions $C_{\sigma'}$ and C_σ and a contiguity relation σ satisfy the condition $C^0_{\sigma'\sigma}$. Then there exists a natural continuous mapping $f: \sigma'X \to \sigma X$.*

To prove this theorem, we show, at first, some auxiliary statements assuming that the conditions of Theorem 1 are fulfilled.

Lemma 1.1. *Let γ' be a maximal σ'-contiguity system and $F \bar{\in} \gamma'$. Then there exists $H \in \gamma'$ such that $\sigma'(H, F)$ does not take place.*

Proof. Let $F \bar{\in} \gamma'$. Then there exist $F_1, F_2, \dots, F_n$ belonging to γ' such that $\sigma'(F, F_1, \dots, F_n)$ does not take place and therefore according to $C_{\sigma'}$ there exists a σ'-majorant H of the system $F_1, F_2, \dots, F_n$ such that $\sigma'(H, F)$ does not take place. Since $H \in \gamma'$, Lemma 1.1 is proved.

Lemma 1.2. *Let γ' be a maximal σ'-contiguity system, $\gamma = \{F \mid \sigma(F, F'_1, \dots, F'_m)$ for any $F'_1, F'_2, \dots, F'_m$ belonging to $\gamma'\}$. Then γ is the only maximal σ-contiguity system containing γ'.*

Proof. It is enough to show that γ is a σ-contiguity system since any σ-contiguity system containing the system γ' is within γ. For this it is necessary to show that for any $F'_1, F'_2, \dots, F'_m$ belonging to γ', $\sigma(F_i, F'_1, \dots, F'_m)$, $i = 1, 2, \dots, n$, implies $\sigma(F_1, \dots, F_n, F'_1, \dots, F'_m)$.

For $n = 1$ this statement is trivial. Assuming the statement to be true for n let us prove it for $n + 1$.

Let $F_1, F_2, \dots, F_{n+1}$ be closed sets such that $\sigma(F_i, F'_1, \dots, F'_m)$ takes place for any $F'_1, F'_2, \dots, F'_m$ belonging to γ', $i = 1, 2, \dots, n + 1$, and $\sigma(F_1, \dots, F_{n+1}, F'_1, \dots, F'_m)$ does not take place for some $F'_1, F'_2, \dots, F'_m$. Then there exists $H_1, \dots, H_{n+1}, H'_1, \dots$

..., H'_m with the properties ensured by the condition $C^0_{\sigma'\sigma}$. In particular, $\sigma'(H_1, \ldots$ $\ldots, H_{n+1}, H'_1, \ldots, H'_m)$ does not take place. Then one of the sets $H_1, H_2, \ldots, H_{n+1}$ does not belong to γ'. Let H_1 be such a set. By Lemma 1.1 there exists $F'_0 \in \gamma'$ such that $\sigma'(H_1, F'_0)$ does not take place and therefore $\sigma'(H_1, \ldots, H_n, F'_0, H'_1, \ldots, H'_m)$ does not take place and due to $C^0_{\sigma'\sigma}$, $\sigma(F_1, \ldots, F_n, F'_0, F'_1, \ldots, F'_m)$ does not take place. The contradiction proves Lemma 1.2.

Lemma 1.3. *Let γ' be a maximal σ'-contiguity system, $\gamma = \{F \mid \sigma(F, F')$ for any $F' \in \gamma'\}$. Then γ is the only maximal σ-contiguity system containing γ'.*

Proof. Using Lemma 1.2 it is enough to show that if $\sigma(F, F')$ takes place for any $F' \in \gamma'$ and a set F then $\sigma(F, F'_1, \ldots, F'_m)$ takes place for F and any $F'_1, F'_2, \ldots, F'_m$ belonging to γ'. Suppose that it is not the case, that is, $\sigma(F, F'_1, \ldots, F'_m)$ does not take place for some $F'_1, F'_2, \ldots, F'_m$ belonging to γ'. Then due to C_σ there exists a σ'-majorant H of the system $F'_1, F'_2, \ldots, F'_m$ such that $\sigma(F, H)$ does not take place. Since $H \in \gamma'$, the contradiction proves Lemma 1.3.

Now we prove Theorem 1. Let $f : \sigma'X \to \sigma X$ with $f(x) = x$ for $x \in X$, $f(\gamma') = \gamma$ for $\gamma' \subset \gamma$ and $\gamma' \in \sigma'X \setminus X$. By Lemma 1.2 the mapping f is defined uniquely and it is enough to show f to be continuous, that is, to show that for any closed set F_0 of X the set $f^{-1}(\bar{F}_0^{\sigma X})$ is a closed set of $\sigma'X$.

Let $\gamma'_0 \bar{\in} f^{-1}(\bar{F}_0^{\sigma X})$, that is, $\gamma'_0 \subset \gamma_0 \bar{\in} \bar{F}_0^{\sigma X}$. Then $F_0 \bar{\in} \gamma_0$ and by Lemma 1.3 there exists $F_1 \in \gamma'_0$ such that $\sigma(F_0, F_1)$ does not take place. Thus due to $C^0_{\sigma'\sigma}$, there exist H_0, H_1 with the properties ensured by this condition. In particular, $F_0 \subset H_0$, $F_1 \subset H_1$, $\sigma'(H_0, H_1)$ does not take place. Since $F_1 \in \gamma'_0$, $F_1 \subset H_1$ so $H_1 \in \gamma'_0$ and therefore $H_0 \bar{\in} \gamma'_0$, that is, $\gamma'_0 \bar{\in} \bar{H}_0^{\sigma'X}$. On the other hand, if a maximal σ'-contiguity system γ' does not belong to $\bar{H}_0^{\sigma'X}$ then $H_0 \bar{\in} \gamma'$ and by Lemma 1.1. there exists $F \in \gamma'$ such that $\sigma'(H_0, F)$ does not take place. Thus, taking into account the choice of H_0, we see that $\sigma(H_0, F)$ does not take place, therefore $\sigma(F_0, F)$ does not take place. It means that for any $\gamma'_0 \bar{\in} f^{-1}(\bar{F}_0^{\sigma X})$ there exists a closed set $\bar{H}_0^{\sigma'X}$ of $\sigma'X$ containing $f^{-1}(\bar{F}_0^{\sigma X})$ but not containing γ'_0, which again means $f^{-1}(\bar{F}^{\sigma X})$ is closed and Theorem 1 is proved.

3. Consider now another system of conditions for contiguity relations σ', σ ensuring not only the existence of a natural continuous mapping $f : \sigma'X \to \sigma X$ but also some properties of such a mapping.

Denote the set of all subsets of a set X by PX ($PX = \{A \mid A \subset X\}$), according to which we have $P^2X = \{A \mid A \subset PX\}$. Consider a mapping $\mathscr{A} : PX \to P^2X$ associating with every set $A \subset X$ some system $\mathscr{A}(A)$ of subsets of X. We call the mapping $\mathscr{A} : PX \to P^2X$ majorizing if the following conditions are fulfilled

A_1: If $F' \in \mathscr{A}(F)$ then $F \subset F'$.

A_2: If $F' \subset F''$, $F' \in \mathscr{A}(F)$ then $F'' \in \mathscr{A}(F)$.

A_3: $\bigcap_{F' \in \mathscr{A}(F)} F' = F$.

A majorizing $\mathscr{A}$ is a $\sigma'\sigma$-majorizing if $\sigma(F_1, F_2, \ldots, F_n)$ takes place iff for any $\mathscr{A}$-majorants $F'_1, F'_2, \ldots, F'_n$ of sets $F_1, F_2, \ldots, F_n$, $\sigma'(F'_1, F'_2, \ldots, F'_n)$ takes place.

We shall say that a $\sigma'\sigma$-majorizing $\mathscr{A}$ satisfies the condition $C\mathscr{A}$ if for any $\mathscr{A}$-majorant F' of a set F and any H, $\bar{\sigma}'(H, F')$ implies $\bar{\sigma}(H, F)$.

Theorem 2. *Let a contiguity relation σ' on X satisfy the condition $C_{\sigma'}$ and let there exist a $\sigma'\sigma$-majorizing $\mathscr{A}$ satisfying the condition $C\mathscr{A}$. Then there exists a natural continuous mapping $f: \sigma'X \to \sigma X$ (onto σX).*

To prove this theorem we show at first some auxiliary statements assuming that the conditions of Theorem 2 are fulfilled.

Lemma 2.1. *Let γ' be a maximal σ'-contiguity system. If $F \bar{\in} \gamma'$ then there exists $F' \in \gamma'$ such that $\sigma'(F', F)$ does not take place.*

Proof. The proof is quite analogous to that of Lemma 1.1.

Lemma 2.2. *Let γ' be a maximal σ'-contiguity system and $\gamma = \{F \mid \sigma(F, F')$ for any $F' \in \gamma'\}$. Then γ is the only maximal σ-contiguity system containing γ'.*

Proof. It is sufficient to show that γ is a σ-contiguity system. If $\sigma(F_1, F_2, \ldots, F_n)$ does not take place then $\sigma'(H_1, H_2, \ldots, H_n)$ does not take place for some $\mathscr{A}$-majorants of the sets $F_1, F_2, \ldots, F_n$. Then at least one of $H_1, H_2, \ldots, H_n$ does not belong to γ', for example H_1. Now note that by Lemma 2.1 there exists $F' \in \gamma'$ such that $\sigma'(H_1, F')$ does not take place. Therefore $\sigma(F_1, F')$ does not take place and $F_1 \bar{\in} \gamma$. Thus $F_i \in \gamma$, $i = 1, 2, \ldots, n$, implies $\sigma(F_1, F_2, \ldots, F_n)$. This proves Lemma 2.2.

Lemma 2.3. *Let γ be a maximal disappearing σ-contiguity system. Then there exists a maximal disappearing σ'-contiguity system γ' such that $\gamma' \subset \gamma$.*

Proof. Consider the system $\mathscr{A}(\gamma) = \{F' \mid F' \in \mathscr{A}(F), F \in \gamma\}$ of closed sets of X. If $F'_1, F'_2, \ldots, F'_n$ belong to $\mathscr{A}(\gamma)$ then these sets are the $\mathscr{A}$-majorants of some $F_1, F_2, \ldots, F_n$ belonging to γ and as $\sigma(F_1, F_2, \ldots, F_n)$ takes place, $\sigma'(F'_1, F'_2, \ldots, F'_n)$ takes place, too. Thus the system $\mathscr{A}(\gamma)$ is the σ'-contiguity system, $\mathscr{A}(\gamma)$ being a disappearing system due to A_3. Let γ' be a maximal σ'-contiguity system containing $\mathscr{A}(\gamma)$, $F' \in \gamma'$, $F_1, F_2, \ldots, F_n$ belong to γ and $H', H_1, \ldots, H_n$ be $\mathscr{A}$-majorants of sets $F', F_1, \ldots, F_n$, respectively. Since $H_1, H_2, \ldots, H_n$ belong to $\mathscr{A}(\gamma)$, $\sigma'(F', H_1, \ldots, H_n)$ takes place and all the more $\sigma'(H', H_1, \ldots, H_n)$ takes place. It implies $\sigma(F', F_1, \ldots, F_n)$ by arbitrary choice of $\mathscr{A}$-majorants. This shows that $F' \in \gamma'$ implies $F' \in \gamma$, which proves Lemma 2.3.

Now we prove Theorem 2. The mapping $f: \sigma'X \to \sigma X$ is defined in the usual way, f being the mapping of $\sigma'X$ onto σX by Lemma 2.3. The continuity of this mapping is proved in the same way as in Theorem 1, the only difference being in that $C\mathscr{A}$ is used instead of $C^0_{\sigma'\sigma}$.

4. If $f: \sigma'X \to \sigma X$ is a natural continuous mapping, F is a closed set of X then in a general case $f(\bar{F}^{\sigma'X}) \neq \bar{F}^{\sigma X}$ even if the conditions of Theorem 2 hold. The following example shows this:

Let $X = A \cup B$, $A = \{(0, y) \mid 1 < y \leqq 2\}$, $B = \{(x, 0) \mid 1 < x \leqq 2\}$. The sets A, B are both open and closed subsets of X, the topological structure on B is standard, the topological structure on A is induced by the basis of closed sets consisting of finite unions of sets of the form $A_a = \{(0, y) \mid 1 < y \leqq a\}$ and finite sets. Define the contiguity relation σ on X setting $\sigma(F_1, F_2, \ldots, F_n)$ to take place iff at least one of the following conditions is fulfilled

1. $\bigcap_{i=1}^{n} F_i \neq \emptyset$.

2. $F_i \supset A$, $i = 1, 2, \ldots, m$, and with $i > m$ the point $(1, 0)$ is a limit point of $\bigcap F_i$ in the usual topology.

The majorizing $\mathscr{A}$ is defined as follows:

$\mathscr{A}(F) = \{F' \mid F \subset F'\}$ for $F \subset A$, $F \neq A$;
$\mathscr{A}(F) = \{F' \mid F' \cap A \neq \emptyset\}$ for $F \subset B$, $(1, 0)$ is the limit point of F;
$\mathscr{A}(A) = \{A \cup B_a\}$, $B_a \supset \{(x, 0) \mid 1 < x \leqq a\}$;
$\mathscr{A}(F) = \{F' \mid F \subset F'\}$ for $F \subset B$, $(1, 0)$ is not the limit point of F.

It is easy to see that the majorizing $\mathscr{A}$ is an $\omega\sigma$-majorizing ($C\mathscr{A}$ being satisfied) but $f(\bar{A}^{\omega X}) \neq \bar{A}^{\sigma X}$.

In connection with the above discussion consider a system of conditions for contiguity relations σ', σ (stronger than in 3) namely instead of the condition $C\mathscr{A}$ consider the following condition

$C\mathscr{A}^*$: For any $\mathscr{A}$-majorants $F'_1, F'_2, \ldots, F'_n$ of sets $F_1, F_2, \ldots, F_n$ and any set F, $\bar{\sigma}'(F, F'_1, \ldots, F'_n)$ implies $\bar{\sigma}(F, F_1, \ldots, F_n)$.

Theorem 3. *Let a contiguity relation σ' on X satisfy the condition $C_{\sigma'}$ and let there exist a $\sigma'\sigma$-majorizing $\mathscr{A}$ which satisfies the condition $C\mathscr{A}^*$. Then there exists a natural continuous mapping $f : \sigma'X \to \sigma X$ for which $f(\bar{F}^{\sigma'X}) = \bar{F}^{\sigma X}$ for any $F \subset X$.*

Proof. The existence and continuity of the natural mapping $f : \sigma'X \to \sigma X$ results from Theorem 2. Thus, the only thing we need is to prove the equality $f(\bar{F}^{\sigma'X}) = \bar{F}^{\sigma X}$ for any $F \subset X$. Let $\gamma \in \bar{F}^{\sigma X}$, $F \in \gamma$, $\mathscr{A}(\gamma)$ be the system consisting of $\mathscr{A}$-majorants of the sets belonging to γ. Then the system $\{F\} \cup \mathscr{A}(\gamma)$ is the σ'-contiguity system due to $C\mathscr{A}^*$ and is a part of a maximal σ'-contiguity system γ'. But it was noticed in the proof of Lemma 2.3 that $\gamma' \subset \gamma$, $F \in \gamma'$. Therefore $\gamma' \in \bar{F}^{\sigma'X}$, $\gamma = f(\gamma')$, $\gamma \in f(\bar{F}^{\sigma'X})$. Hence $\bar{F}^{\sigma X} \subset f(\bar{F}^{\sigma'X})$. Since the inverse inclusion follows from the continuity of f, Theorem 3 is proved.

We shall say that a contiguity relation σ satisfies the condition C_σ^* if $\sigma'(F_1, F_2, \ldots, F_n)$ and $\bar{\sigma}(F'_1, \ldots, F'_m, F_1, \ldots, F_n)$ imply the existence of a σ'-majorant H of the system $F_1, F_2, \ldots, F_n$ such that $\sigma(F'_1, \ldots, F'_m, H)$ does not take place.

Theorem 4. *Let a contiguity relation σ on X satisfy the condition $C_{\sigma'}^*$ and let there exist a natural continuous mapping $f : \sigma'X \to \sigma X$ such that $f(\bar{F}^{\sigma'X}) = \bar{F}^{\sigma X}$*

for any $F \subset X$. Then there exists a $\sigma'\sigma$-majorizing $\mathscr{A}$ which satisfies the condition $C\mathscr{A}^*$.

Proof. Put $\mathscr{A}(F) = \{F' \mid F \subset F', f^{-1}(\bar{F}^{\sigma X}) \subset \bar{F}'^{\sigma' X}\}$ for any $F \subset X$. It is easy to show $F \to \mathscr{A}(F)$ to be majorizing. Let $F'_1, F'_2, \ldots, F'_n$ be $\mathscr{A}$-majorants of sets $F_1, F_2, \ldots, F_n$. If $\sigma(F_1, F_2, \ldots, F_n)$ takes place, that is $\bigcap_{i=1}^{n} \bar{F}_i^{\sigma X} \neq \emptyset$, then $\bigcap_{i=1}^{n} \bar{F}_i'^{\sigma' X} \supset \bigcap_{i=1}^{n} f^{-1}(\bar{F}_i^{\sigma X}) \neq \emptyset$. Thus $\sigma'(F'_1, F'_2, \ldots, F'_n)$ takes place. Now let $\sigma'(F'_1, F'_2, \ldots, F'_n)$ take place for any $\mathscr{A}$-majorants of sets $F_1, F_2, \ldots, F_n$, correspondingly. Consider a system $\mathscr{A}(F_1, F_2, \ldots, F_n)$ of $\mathscr{A}$-majorants of sets $F_1, F_2, \ldots, F_n$. It is easy to show by $C^*_{\sigma'}$ that $\mathscr{A}(F_1, F_2, \ldots, F_n)$ is a σ'-contiguity system, thus the intersection of closures of its elements in $\sigma'X$ is not empty. This intersection is equal to $\bigcap_{i=1}^{n} f^{-1}(\bar{F}_i^{\sigma X})$, therefore $\bigcap_{i=1}^{n} \bar{F}_i^{\sigma X} \neq \emptyset$, that is, $\sigma(F_1, F_2, \ldots, F_n)$ takes place. Thus, $\mathscr{A}$ is a $\sigma'\sigma$-majorizing.

Let $F_1, F_2, \ldots, F_n$ be closed sets of X, $H_1, H_2, \ldots, H_n$ their $\mathscr{A}$-majorants, $\sigma'(F, H_1, \ldots, H_n)$ not taking place, that is $\bar{F}^{\sigma' X} \cap (\bigcap_{i=1}^{n} \bar{H}_i^{\sigma' X}) = \emptyset$. Then $\bar{F}^{\sigma' X} \cap f^{-1}(\bigcap_{i=1}^{n} \bar{F}_i^{\sigma X}) = \emptyset$ and, finally, as $f(\bar{F}^{\sigma' X}) = \bar{F}^{\sigma X}$, we obtain $\bar{F}^{\sigma X} \cap (\bigcap_{i=1}^{n} \bar{F}_i^{\sigma X}) = \emptyset$, that is, $\sigma(F, F_1, \ldots, F_n)$ does not take place. This proves $C\mathscr{A}^*$ and Theorem 4, as well.

Corollary 1. *Let a contiguity relation σ' on X satisfy the condition $C^*_{\sigma'}$. Then there exists a natural continuous mapping $f : \sigma'X \to \sigma X$ with $f(\bar{F}^{\sigma' X}) = \bar{F}^{\sigma X}$ for every $F \subset X$ iff there exists a $\sigma'\sigma$-majorizing $\mathscr{A}$ satisfying the condition $C\mathscr{A}^*$.*

This statement is simplified in the case of $\sigma' = \omega$, that is the contiguity relation under intersection ($\omega(F_1, F_2, \ldots, F_n)$ takes place iff $\bigcap_{i=1}^{n} F_i \neq \emptyset$).

Corollary 2. *There exists a natural continuous mapping $f : \omega X \to \sigma X$ with $f(\bar{F}^{\omega X}) = \bar{F}^{\sigma X}$ for every $F \subset X$ iff there exists an $\omega\sigma$-majorizing $\mathscr{A}$ satisfying the condition $C\mathscr{A}^*$.*

This statement gives us a description of the class, say $\mathscr{A}^*$, of natural continuous images of Wallman extension ωX which contains the so-called $\omega\alpha$-extensions [2]. Note that for any $\omega\alpha$-extension X' of a space X there exists a natural both closed and continuous mapping $f : \omega X \to X'$, while the corresponding natural mapping for any extension of the class $\mathscr{A}^*$ satisfies a weaker condition.

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