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ON THE FIXED POINT PROPERTY FOR SET-VALUED MAPPINGS OF HEREDITARILY DECOMPOSABLE CONTINUA

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Let X and Y be two topological spaces. We say that $F: X \to Y$ is a closed set-valued mapping from X into Y if $F(x)$ is a non-empty closed subset of Y. A closed set-valued mapping $F: X \to Y$ is said to be upper (lower) semi-continuous if $\{x \in X : F(x) \cap A \neq \emptyset\}$ is closed (open) in X whenever A is closed (open) in Y. F is said to be continuous if it is both upper and lower semi-continuous. If $F(x)$ is connected for each $x \in X$, then F is called continuum-valued.

Let $\mathfrak{C}$ be a class of closed set-valued mappings of a topological space X into itself. We say that X has the fixed point property for $\mathfrak{C}$ (the F.p.p. for $\mathfrak{C}$) if, for each $F \in \mathfrak{C}$, there exists $x \in X$ such that $x \in F(x)$.

Three conditions for metric continua X are considered in the paper:

(I) X has the F.p.p. for upper semi-continuous, continuum-valued mappings;

(II) X is hereditarily unicoherent;

(III) X has the F.p.p. for continuous, closed set-valued mappings.

Main problems:

Problem 1. Characterize all continua X with property (I);

Problem 2. Characterize all continua X with property (III);

have only some partial solutions. It follows from results of A. D. Wallace ([5], Theorem A, p. 757), R. L. Plunkett ([4], Theorems 1 and 2, p. 161 and 162) and L. E. Ward, Jr. ([7], Lemma 4, p. 162 and Theorem 3, p. 164) that

Theorem 1. *If a continuum X is locally connected, then*

$$(\mathrm{I}) \Leftrightarrow (\mathrm{II}) \Leftrightarrow (\mathrm{III}).$$

L. E. Ward, Jr. proved ([7], Corollary, p. 163, and [6], Theorem 2, p. 926) the following

Theorem 2. *If a continuum X is arcwise connected, then*

$$(\mathrm{I}) \Leftrightarrow (\mathrm{II}) \Rightarrow (\mathrm{III}).$$

The problem if (III) implies (II) for arcwise connected continua X was posed for the first time in [7], p. 160. There is a conjecture suggesting that the answer is affirmative ([8], p. 92).

The aim of the paper is to prove

Theorem 3. *If a continuum X is hereditarily decomposable, then*

$$(I) \Rightarrow (II) \Rightarrow (III).$$

The proof of the first implication is patterned after Ward's proof of the same implication in Theorem 2, using a result of H. C. Miller ([3], Theorem 2.6, p. 187). The second implication follows from results of H. Cook ([1], Theorem 1, p. 20), S. Mardešić and J. Segal ([2], Theorem 1*, p. 148) and P. O. Wheatley ([9], p. 546).

It is natural to ask if both the inverse implications to those in Theorem 3 hold for hereditarily decomposable continua X. I conjecture that the answer is yes.

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