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A CLASS OF CONNECTED SPACES WITH MANY RAMIFICATIONS

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In this note we study the following class RAM of spaces, which emerged as counterexamples in the characterization of linearly orderable and weakly linearly orderable, connected spaces.

Definitions. $X \in$ RAM if X is a connected topological space which has a point x_0 such that every connected subset of X containing x_0 is closed.

Following the terminology of [1] we say that a space X is weakly linearly orderable iff there exists a linear order on X for which the order topology is weaker (coarser) than the given topology. If both topologies coincide X is said to be linearly orderable. Moreover we say that X has property (H) iff every connected subset has at most two non-cutpoints.

In [1] it was proved that for a connected T_2-space X, weak linear orderability is equivalent to (H) + each point separates in at most two components (X is linearly orderable iff moreover X is locally connected). It was asked whether property (H) suffices. Now it is easily seen from properties 2, 7 and 8 below that each $X \in$ RAM has property (H) but can never be weakly linearly orderable.

For $X \in$ RAM we have the following properties:

1. First notice that every connected space contains at least one nonclosed connected subset (except spaces with $\leqq 1$ point).

2. The x_0 from the definition of $X \in$ RAM is unique. For every $x \in X \setminus \{x_0\}$ the subspace $X \setminus \{x\}$ has infinitely many components.

3. X is not separable metrizable (nor is any open subset).

4. For each $x \in X$ there exists an open subset $O \subset X$ such that $x \in O \in$ RAM and x is the "x_0 of O".

5. X is nowhere locally compact.

6. X is nowhere locally connected.

7. RAM $\neq \emptyset$, even $\exists Y \in$ RAM Y is T_2 and Y has countably many points.

8. Each connected subset of X has at most one non-cutpoint.

We do not even know whether no $X \in$ RAM is metrizable, although we conjecture that for each $X \in$ RAM each continuous real-valued function is constant.

Properties 1 and 2 are easily verified, while 3 follows from the following theorem which can essentially be found in [2], p. 75, K 4.

Theorem. *If X is an m-separable connected T_1-space then the number of points $x \in X$ for which $X \setminus \{x\}$ has at least three components does not exceed m.*

The properties 4–8 are proved by exploring the relation R on X defined by: xRy iff $x = x_0$ or x separates x_0 from y. This turns out to be a partial ordering, which is related to the topology in a rather unusual way.

References

[1] *H. Kok:* On conditions equivalent to the orderability of a connected space. Nieuw Arch. Wisk. *18* (1970), 250–270.

[2] *L. F. McAuley:* On decomposition of continua into aposyndetic continua. Trans. Amer. Math. Soc. *81* (1956), 74–91.