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NON-CYCLIC TRANSFORMATIONS AND UNIFORM CONVERGENCE OF PICARD SEQUENCES

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Definition. A continuous transformation of a topological space S into itself is said to be *non-cyclic* iff $f(x) \neq x$ implies $f^2(x) \neq x$.

In a recent paper, extending a theorem due to S. C. Chu and R. D. Moyer, the second author proved that, if f is a continuous transformation of a compact and connected space S into itself, whose topology is deduced from a total ordering, then all the Picard sequences converge iff f is non-cyclic (and this last property is proved to be equivalent to five other properties).

In this paper the authors characterize (under the same hypotheses on S) the non-cyclic transformations for which the Picard sequences converge uniformly with respect to $x \in S$. Besides, some partial answers to the same problem in a more general setting are given.

Notations: $F(f)$ indicates the set of all fixed points of f. $F^*(f)$ indicates the set $\bigcap_N f^n(S)$.

Obviously we have $F^*(f) \supset F(f)$.

Theorem. *If S is totally ordered, connected and compact in the order topology, and f a non-cyclic transformation of S into itself, then the following properties are equivalent:*

(a) *$F(f)$ is connected;*
(b) *$F^*(f) = F(f)$;*
(c) *the convergence of the Picard sequences is uniform.*

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