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METRIC SPACES IN WHICH PROHOROV'S THEOREM IS NOT VALID

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A well-known Prohorov's theorem says that for every topologically complete metric space X, for every compact set M of measures on X with mass 1 (with the weak topology) and for every $\varepsilon > 0$ there exists a compact set $A \subset X$ such that $\mu(X - A) < \varepsilon$ for each $\mu \in M$. It has been a problem whether this theorem holds in every separable metric space which is a Borel subset of its completion. This problem can be solved by the help of the following theorem.

Theorem 1. *A coanalytic separable metric space is topologically complete if and only if it contains no countable dense-in-itself G_δ subspace.*

If Prohorov's theorem holds in some metric space X then it is easy to prove that it holds also in every G_δ subspace of X. With respect to Theorem 1 and to Sierpiński's result according to which every two countable dense-in-itself metric spaces are homeomorphic it is clear that the negative answer for some countable dense-in-itself metric space implies the negative answer for every separable coanalytic metric space which is not topologically complete. Such countable metric space can be constructed (namely the space of rational numbers), therefore the following theorem holds.

Theorem 2. *If X is a separable coanalytic metric space then Prohorov's theorem holds in X if and only if X is topologically complete.*

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