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BOX PRODUCTS OF BAIRE SPACES

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The author proposes that the study of box products of Baire spaces accompany the study of products of Baire spaces. Certain classical results carry over without change. For example:

Theorem 1. The box product of complete spaces is Baire.

The stationary set techniques of [FK] extend to box products. For example:

Theorem 2. For every regular cardinal κ there is a space X such that the box product of less than κ copies of X is Baire, while the box product of κ copies of X is not Baire.

Theorem 2 gives a family of spaces whose usual product is Baire but whose box product is not Baire.

Question 1. If the box product of a family of spaces is Baire, is the usual product Baire?

The techniques of Oxtoby [O] do not seem to extend to box products.

Question 2. Is the box product of second countable Baire spaces Baire?

In particular, let $\mathcal{T} = \{T_\alpha : \alpha < c\}$ be a family of disjoint sets with each T_α meeting every perfect subset of the Cantor set.

The question of whether the box product of $\mathcal{Y}$ is Baire is related to the following game. Two players, I and II, alternately choose a family $\{B^i_\alpha : \alpha < c\}$ of nonempty basic open sets of the Cantor set with $B^{i+1}_\alpha \subset B^i_\alpha$. Player II wins if for all distinct $\alpha, \beta < c$, $\bigcap\{B^i_\alpha : i \in \omega\} \cap \bigcap\{B^i_\beta : i \in \omega\} = \emptyset$.

<u>Question 3</u>. Does player II have a winning strategy? Is the game determined?

Assuming that every uncountable subset of the Cantor set has a perfect subset, player I has a winning strategy. But then the family $\mathcal{Y}$ does not exist. So Question 3 is most interesting when the Axiom of Choice is valid.

BIBLIOGRAPHY

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