A. Zarelua

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A.ZARELUA

Tbilisi

Integral dependence, integral closure, integrally closed ring - all these are well-known notions of commutative algebra. On the other hand there exists a functor from the Banach commutative algebras category to the compact spaces category due to space of maximal ideals. In this line of thought to familiar theorems of commutative algebra must correspond some theorems on compact spaces. Here is one of them which is the easiest to formulate:

Theorem. A continuous mapping $f:X \longrightarrow Y$ of compact spaces is zero-dimensional (resp. monotone) iff the integral closure of the algebra $f^*C_K(Y)$ is dense in the algebra $C_K(X)$ (resp. the algebra $f^*C_K(Y)$ is integrally closed in the algebra $C_K(X)$), $K = R, C$.

This theorem may be considered as a strict form of Katetov's zero-dimensional (monotone) mapping characterization theorem and actually has many specifications and versions for mappings, which are near to the ones spoken about.

Applications are the following:

1) approximation theorem for zero-dimensional mappings by finite-to-one mappings,

2) algebraic characterization of mappings, which may be extended on some compact extension to zero-dimensional ones,

3) acyclicity theorems for inverse image functor f^* in sheave theory for some kind of zero-dimensional mappings,

4) universal spaces for spaces which have zero-dimensional mapping in the fixed space,

5) simple algebraic characterization of monotone mappings.

Using the latter one and the well-known description of perfect compactifications as monotone images of Čech compactification, one may obtain an easy verifiable criterion for a compactification to be perfect, which is applied to prove that the Wiener compactification of harmonic space in the sense of Constantinescu-Cornea-Meghea is perfect.