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Let $\mathscr{U}$ be a fundamental system of absolutely convex closed neighbourhoods of zero of a locally convex linear Hausdorff space $E[\tau]$. Let V be a $\tau_s(E)$-dense (other notations: weak* dense, $\sigma(E', E)$-dense) subspace of E'. Let $\beta(V, U)$ stand for the number $\max\{\varrho \geqq 0: \mathrm{Cl}\,(V \cap U^0) \supset \varrho U^0\}$ where U^0 denotes the polar of U and Cl denotes closure in E' in the weak topology $\tau_s(E, E')$. Then the number $\inf\{\beta(V, U) : : U \in \mathscr{U}\}$ is called the β-characteristic of the subspace V. This reduces to the conventional Dixmier characteristic of a subspace in a conjugate Banach space, if E is specialized to a normed linear space (cf. [1], [3]).

Theorem. *A locally convex Hausdorff space is semireflexive iff every $U \in \mathscr{U}$ (and also every bounded closed convex set in E) is closed for every locally convex Hausdorff topology τ' comparable with τ.*

This criterion for semireflexivity can then be weakened for a large class of locally convex spaces. This weakened criterion says that it is enough to take Mackey neighbourhoods of $E[\tau]$ and those locally convex Hausdorff topologies on E which are comparable with $\tau_k(E', E)$ and which occur as an initial topology (= projective limit topology) defined by maps

$$E \to E_W = E/N_W\,, \quad W \in \mathscr{W}$$

where $\mathscr{W}$ is any family of absolutely convex absorbing sets in E such that $(\mathscr{U}, \tau_k)$ is a refinement of $\mathscr{W}$ and where N_W denotes the null space of the seminorm specified by W. En route to the proof of this Theorem we get a proposition for separable locally convex Hausdorff spaces which is of some intrinsic interest.

Proposition. *Let $E[\tau]$ be a separable locally convex Hausdorff space such that $E'[\tau_b(E)]$ is quasicomplete. Let V be a $\tau_s(E)$-dense and $\tau_b(E)$-closed subspace of E'. Then for every $U \in \mathscr{U}$ the quotient topology defined by $\tau_k(V, E)$ on $E_{\bar{U}} = E/N_{\bar{U}}$ is finer than some norm topology on $E_{\bar{U}}$ where $\bar{U} = \tau_s(V, E)$-closure of U in E.*

Specialising this Proposition to Banach spaces we get the following result (cf. [4]) of Petunin: *Let E be a separable Banach space and V be a $\tau_s(E)$-dense norm-closed subspace of E'. Then $\tau_k(V, E)$ on E is finer than some norm topology on E.*

The notations that are not explained are as in [2]. The proofs will appear elsewhere.

References

[1] *Dixmier:* Duke Math. J. *15* (1948), 1057—1071.
[2] *Köthe:* Topologische Lineare Räume, 1960.
[3] *Krishnamurthy:* Trans. Amer. Soc. *130* (1968), 525—531.
[4] *Petunin:* Soviet Maths. (Doklady) *2* (1961), 1160—1162.

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