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T. Soundararajan

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TOTALLY DENSE SUBGROUPS OF TOPOLOGICAL GROUPS

T. SOUNDARARAJAN

Madurai

A subgroup H of a topological group G is called totally dense if for each closed subgroup S of G, $S = \mathrm{cl}\,(S \cap H)$. A totally dense subgroup is dense, but the converse need not be true. Some of the results on this subject are:

1. *A subgroup H of the topological group G is totally dense if and only if for each $x \in G$, $\mathrm{cl}\,[x] = \mathrm{cl}\,(\mathrm{cl}\,[x] \cap H)$.*

2. If G is a totally dense subgroup so does any quotient group of G.

3. If G is a compact group, H a closed normal subgroup and G/H has a proper totally dense subgroup then G has a proper totally dense subgroup. This result is not true for locally compact groups.

4. A locally compact group G has a totally dense cyclic group if and only if it is either discrete cyclic or is the compact group of all p-adic integers for some prime p.

5. Let G be a compact Abelian group. Then every dense subgroup of G is totally dense if and only if either G is finite or G is the compact group of all p-adic integers for some prime p.

6. Let G be a compact totally disconnected Abelian group. Then the torsion part of G is totally dense if and only if $G = \prod_{p \text{ prime}} G_p$ where G_p is a compact torsion p-primary Abelian group.

7. **Definition.** *A subgroup H of a topological group G is called a G-S subgroup if H is totally dense and further distinct subgroups of H have distinct closures in G.*

8. Any compact group G has at most one G-S subgroup.

9. A compact Abelian group G has a G-S subgroup if and only if $G = \prod_{p \text{ prime}} F_p$ where each F_p is a finite p-primary Abelian group with the discrete topology.

10. If a compact group G has a G-S subgroup then G must be totally disconnected.

Open Questions

1. If $G = \prod_n C(p^n)$ with the discrete topology for $C(p^n)$ does G have a totally dense subgroup?

2. Give an explicit structure for compact totally disconnected groups having a $G - S$ subgroup.

MADURAI UNIVERSITY, MADURAI, INDIA