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L_p-estimates for solutions of Dirichlet and Neumann problems to heat equation in the wedge with edge of arbitrary codimension ⋆

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Abstract. Coercive estimates in anisotropic weighted L_p-spaces are obtained for solutions of the Dirichlet and Neumann problems to the heat equation in the wedge with arbitrary codimensional edge (in particular, in the cone).

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Denote $x = (x', x'')$ the point in $\mathbb{R}^n$, $x' \in \mathbb{R}^m$, $x'' \in \mathbb{R}^{n-m}$ $(2 \le m \le n)$.

Let $K_m(\omega) = \{x' \ : \ x'/|x'| \in \omega\}$ be a cone in $\mathbb{R}^m$, cutting a domain $\omega \subset S_1$ with a smooth boundary.

In the case $m < n$ we denote by $\mathcal{K}_m(\omega) = K_m(\omega) \times \mathbb{R}^{n-m}$ the wedge in $\mathbb{R}^n$ with $(n-m)$-dimensional edge (if $m = n$ we set $\mathcal{K}_m(\omega) = K_m(\omega)$).

We introduce the weighted spaces $L_{p,(\mu)}(\mathcal{K}_m)$ with the norm

$$\|u\|_{p,(\mu),\mathcal{K}_m} = \|u \cdot |x'|^{\mu}\|_{p,\mathcal{K}_m}, \quad \mu \in \mathbb{R},$$

($\|\cdot\|_p$ stands for the standard norm in L_p).

We also introduce two scales of anisotropic weighted spaces:

$$L_{p,q,(\mu)}(\mathcal{K}_m \times [0,T]) = L_q\big([0,T] \longrightarrow L_{p,(\mu)}(\mathcal{K}_m)\big)$$

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A note on results published in [2].

with the norm

$$\|u\|_{p,q,(\mu)} = \Big\| \|u(\cdot,t)\|_{p,(\mu),\mathcal{K}_m} \Big\|_{q,[0,T]};$$

$$\widetilde{L}_{p,q,(\mu)}(\mathcal{K}_m \times [0,T]) = L_{p,(\mu)}\big(\mathcal{K}_m \longrightarrow L_q([0,T])\big)$$

with the norm

$$|\!|\!| u |\!|\!|_{p,q,(\mu)} = \Big\| \|u(x,\cdot)\|_{q,[0,T]} \Big\|_{p,(\mu),\mathcal{K}_m}.$$

Let us consider the Dirichlet and Neumann initial-boundary value problems for the heat equation in $\mathcal{K}_m(\omega)$:

$$(\mathcal{D}) \quad \begin{cases} u_t - \triangle u = f(x,t), & x \in \mathcal{K}_m(\omega), \quad t > 0 \\ u\big|_{x\in\partial\mathcal{K}_m(\omega)} = 0, & u\big|_{t=0} = 0; \end{cases} \tag{1}$$

$$(\mathcal{N}) \quad \begin{cases} u_t - \triangle u = f(x,t), & x \in \mathcal{K}_m(\omega), \quad t > 0 \\ \frac{\partial u}{\partial \mathbf{n}}\big|_{x\in\partial\mathcal{K}_m(\omega)} = 0, & u\big|_{t=0} = 0 \end{cases} \tag{1'}$$

($\mathbf{n}$ stands for the unit outward normal).

Theorem 1. *Let $\Lambda_{\mathcal{D}}$ be the first eigenvalue of the Dirichlet problem to the Beltrami-Laplacian in ω:*

$$-\triangle' \,\mathcal{U} = \Lambda_{\mathcal{D}}\, \mathcal{U} \quad \text{in } \omega, \quad \mathcal{U}\big|_{\partial\omega} = 0.$$

Let $\lambda_{\mathcal{D}}$ be the positive root of the equation

$$\lambda^2 + (m-2)\cdot\lambda - \Lambda_{\mathcal{D}} = 0.$$

Let $p, q \in \,]1,+\infty[$, and

$$2 - \frac{m}{p} - \lambda_{\mathcal{D}} < \mu < m - \frac{m}{p} + \lambda_{\mathcal{D}}.$$

Then a solution of (1) satisfies the inequalities

$$\|u_t\|_{p,q,(\mu)} + \|D(Du)\|_{p,q,(\mu)} + \|u\cdot|x'|^{-2}\|_{p,q,(\mu)} \le C\ \|f\|_{p,q,(\mu)}, \tag{2}$$

$$|\!|\!| u_t |\!|\!|_{p,q,(\mu)} + |\!|\!| D(Du) |\!|\!|_{p,q,(\mu)} + |\!|\!| u\cdot|x'|^{-2} |\!|\!|_{p,q,(\mu)} \le C\ |\!|\!| f |\!|\!|_{p,q,(\mu)}. \tag{3}$$

Theorem 2. *Let $\Lambda_{\mathcal{N}}$ be the first* **nonzero** *eigenvalue of the Neumann problem to the Beltrami-Laplacian in ω:*

$$-\triangle' \,\mathcal{U} = \Lambda_{\mathcal{N}}\, \mathcal{U} \quad \text{in } \omega, \quad \frac{\partial \mathcal{U}}{\partial \mathbf{n}}\Big|_{\partial\omega} = 0.$$

Let $\lambda_{\mathcal{N}}$ be the positive root of the equation

$$\lambda^2 + (m-2)\cdot\lambda - \varLambda_{\mathcal{N}} = 0.$$

Let $p, q \in \,]1, +\infty[$, and

$$2 - \frac{m}{p} - \min\{\lambda_{\mathcal{N}}, 2\} < \mu < m - \frac{m}{p}.$$

Then a solution of (1') satisfies the inequalities

$$\|u_t\|_{p,q,(\mu)} + \|D(Du)\|_{p,q,(\mu)} \le C\ \|f\|_{p,q,(\mu)}, \tag{2'}$$

$$\|\!|u_t|\!\|_{p,q,(\mu)} + \|\!|D(Du)|\!\|_{p,q,(\mu)} \le C\ \|\!|f|\!\|_{p,q,(\mu)}, \tag{3'}$$

Remark 1. In (2), (2'), (3), (3') C does not depend on T.

Remark 2. For $m = 2$, $p = q$ the results of Theorems 1 and 2 were established in [1].

All the details and closed results are contained in [2].

References

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