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Summation of Polyparametrical Functional Series by the Method of Finite Hybrid Integral Transforms (Fourier, Bessel)

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Abstract. The design of physical and mechanical characteristics of thin isotropic nonhomogeneous (partly-homogeneous) plates of limited size according to the degree low lead to the construction of solution for the separate system of differential equations on the given initial conditions, boundary conditions and the conditions of thermomechanical contact in the connecting plains. Let one end of the plates is under the actions of the spasmodic heat regime, or the heat sources act on the plane part according to the spasmodic law. The stationary state of system is described by the functions depending on functional series consisting of the combination of the trigonometric and Bessel functions. Since we deal easier with functions than with series we encounter with the problem of function series summation. The article is devoted to the summation of just such series by the method of finite hybrid integral transforms Hankel 2-Hankel 2-Fourier, Hankel 1-Hankel 2-Fourier, Fourier-Hankel 2-Hankel 2, . . . [1].

By the Cauchy's method for the separate system of the ordinary differential equations we have constructed the solution of the corresponding boundary problem in the case of general assumption on the differential and connected operators. The condition of the nonlimited solving and the structure of the general solution for the boundary problem have been written in the explicit form. On the other hand solution of this problem has been constructed by the method of the finite hybrid integral transforms. Since this problem has one and only one solution we may compare the first solution with the second and, as a result, get the sums of functional series.

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This is the preliminary version of the paper.

1 Introduction

There are many engineering problems, which occur in design and calculate of stability for machine constructive elements, in the designing engineer structure and in the research of kinetic for physical and chemical processes. Since the constructive elements is under action of instantaneous heat-stroke and after it are work in stationary state, one would like to know the value of stationary heat strength. This problem is very impotent with respect to composite materials. The stationary state of system is described by the functions depending on functional series consisting of the combination trigonometric and Bessel functions. Since we deal easier with functions than with series we encounter with the problem of function series summation.

Let us describe our reasoning on the next heat problems.

Problem 1. Let heat characteristics of thin isotropic plate

$$\Pi = \{r : \ r \in (0, R), \ \ R < +\infty\}$$

are designed according to the continuous law. If one end $r = 0$ of the plate Π is under action of the spasmodic heat regime, other end $r = R$ is under zero temperature. The structure problem of non-stationary heat field in this plate lead to mathematical construction in region

$$D = \{(t, r) : \ t > 0, \ r \in \Pi\}$$

of finite solution for heat equation

$$\frac{\partial T}{\partial t} + \chi^2 T - \frac{\partial^2 T}{\partial r^2} = 0, \quad t > 0$$

on boundary conditions

$$T|_{r=0} = S_+(t), \quad T|_{r=R} = 0$$

and zero initial condition. Note $S_+(t) = \begin{cases} 1, & t > 0, \\ 0, & t \leq 0. \end{cases}$

Solution of this problem is

$$T(t, r) = \frac{2}{R} S_+(t) \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda_n^2 + \chi^2} \left(1 - 1^{-(\lambda_n^2 + \chi^2)t}\right) \sin \lambda_n r.$$

The stationary state is described by function

$$T_{ST.} = \lim_{t \to \infty} T(t, r) = \frac{2}{R} \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda_n^2 + \chi^2} \sin \lambda_n r \equiv \frac{\operatorname{sh} \chi (R - r)}{\operatorname{sh} \chi R} = S_1(r, \chi).$$

Notice that $S_1(r, \chi)$ is finite on $[0, R]$ solution of boundary problem

$$\left(\frac{d^2}{dr^2} - \chi^2\right) S_1 = 0, \quad S_1|_{r=0} = 1, \quad S_1|_{r=R} = 0.$$

Problem 2. Let now heat characteristics of thin isotropic plate are designed according to the linear law. The structure problem of non-stationary heat field in this plate lead to mathematical construction of finite solution for heat equation

$$\frac{\partial T}{\partial t} + \chi^2 T - \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right) T = 0, \quad \chi > 0, \ t > 0$$

on boundary conditions

$$\left. \frac{\partial T}{\partial r} \right|_{r=0}, \quad T|_{r=R} = S_+(t)$$

and zero initial condition. Solution of this problem is

$$T(t,r) = \frac{2}{R} S_+(t) \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda_n^2 + \chi^2} \frac{\left(1 - 1^{(\lambda_n^2 + \chi^2)t}\right) J_1(\lambda_n r)}{[J_0^2(\lambda_n R) + J_1^2(\lambda_n R)]}.$$

The stationary state is described by function

$$T_{CT.} = \frac{2}{R} \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda_n^2 + \chi^2} \frac{J_1(\lambda_n R) J_0(\lambda_n r)}{J_0^2(\lambda_n R) + J_1^2(\lambda_n R)} = \frac{I_0(\chi r)}{I_0(\chi R)} \equiv S_2(r, \chi),$$

where $I_\nu(x)$ – modify Bessel function of the first kind of order ν.

Notice that $S_2(r,\chi)$ is finite on $[0,R]$ solution of the boundary problem

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \chi^2 \right) S_2 = 0, \quad \left. \frac{dS_2}{dr} \right|_{r=0}, \quad S_2|_{r=R} = 1.$$

Problem 3. Let us consider the finite thin plate

$$\mathit{\Pi}_1 = \{ r : \ r \in (0, R_1) \cup (R_1, R_2), \ \ R_2 < \infty \}$$

with continuous heat characteristics on the first part and linear heat characteristics on the second part. The structure problem of non-stationary heat field in this plate lead to mathematical construction in region

$$D_1 = \{ (t,r) : \ t > 0, \ r \in \mathit{\Pi}_1 \}$$

of finite solution for separate system of equations

$$\frac{1}{a_1^2}\frac{\partial T_1}{\partial t} + \chi_1^2 T_1 - \frac{\partial^2 T_1}{\partial r^2} = 0, \quad t > 0, \ r \in (0, R_1),$$

$$\frac{1}{a_2^2}\frac{\partial T_2}{\partial t} + \chi_2^2 T_2 - \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right) T_2 = 0, \quad \chi_2 > 0, \ t > 0, \ r \in (R_1, R_2)$$

on zero initial condition, boundary conditions

$$T_1|_{r=0} = S_+(t), \qquad \left. \frac{\partial T_2}{\partial r} \right|_{r=R_2} = 0$$

and conditions of non-ideal heat contact

$$\left[\left(R_0\frac{\partial}{\partial r}+1\right)T_1-T_2\right]\Big|_{r=R_1}=0,$$

$$\left(\frac{\partial T_1}{\partial r}-\gamma_1\frac{\partial T_2}{\partial r}\right)\Big|_{r=R_1}=0.$$

Solution of this problem is

$$T_1(t,r)=S_+(t)\sum_{n=1}^{\infty}\frac{\overline{\beta_{1n}\beta_{2n}}^2\omega_1^2(\lambda_n)\sin\overline{\beta_{1n}}r}{(\lambda_n^2+a_2^2\chi_2^2)\|V(r,\lambda_n)\|^2}\left(1-1^{-(\lambda_n^2+a_2^2\chi_2^2)t}\right),$$

$$T_2(t,r)=S_+(t)\frac{\gamma_1}{R_1}\sum_{n=1}^{\infty}\frac{\overline{\beta_{1n}\beta_{2n}}^2\omega_1(\lambda_n)}{(\lambda_n^2+a_2^2\chi_2^2)\|V(r,\lambda_n)\|^2}\left(1-1^{-(\lambda_n^2+a_2^2\chi_2^2)t}\right)\times$$
$$\times[\omega_3(\lambda_n)N_0(\overline{\beta_{2n}}r)-\omega_2(\lambda_n)J_0(\overline{\beta_{2n}}r)].$$

Notice that $\beta_{1n}=\sqrt{\lambda_n^2+a_2^2\chi_2^2-a_1^2\chi_1^2}$, $\beta_{2n}=\lambda_n$, $\overline{\beta_{jn}}=a_j^{-1}\beta_{jn}$, $j=\overline{1,2}$,

$$\omega_1(\lambda_n)=J_1(\overline{\beta_{2n}}R_2)N_0(\overline{\beta_{2n}}R_1)-N_1(\overline{\beta_{2n}}R_2)J_0(\overline{\beta_{2n}}R_1),$$
$$\omega_2(\lambda_n)=(R_0\overline{\beta_{1n}}\cos\overline{\beta_{1n}}R_1+\sin\overline{\beta_{1n}}R_1)N_1(\overline{\beta_{2n}}R_2),$$
$$\omega_3(\lambda_n)=(R_0\overline{\beta_{1n}}R_1+\sin\overline{\beta_{1n}}R_1)J_1(\overline{\beta_{2n}}R_2).$$

If $(a_1^2\chi_1^2-a_2^2\chi_2^2)\geq 0$ then $\beta_{1n}=\lambda_n$, $\beta_{2n}=\sqrt{\lambda_n^2+a_1^2\chi_1^2-a_2^2\chi_2^2}$. In this case

$$T_{1,CT.}(t,r)=\sum_{n=1}^{\infty}\frac{\overline{\beta_{1n}\beta_{2n}}^2\omega_1^2(\lambda_n)\sin\overline{\beta_{1n}}r}{(\lambda_n^2+a_2^2\chi_2^2)\|V(r,\lambda_n)\|^2}=\operatorname{ch}\overline{\chi_1}r+\frac{\varDelta_1(\overline{\chi}}{\varDelta(\overline{\chi})}\frac{\operatorname{sh}\overline{\chi_1 r}}{\chi_1}=S_3(r,\chi),$$

$$T_{2,CT.}(t,r)=\frac{\gamma_1}{R_1}\sum_{n=1}^{\infty}\frac{\overline{\beta_{1n}\beta_{2n}}^2\omega_1(\lambda_n)[\omega_3(\lambda_n)N_0(\overline{\beta_{2n}}r)-\omega_2(\lambda_n)J_0(\overline{\beta_{2n}}r)]}{(\lambda_n^2+a_2^2\chi_2^2)|V(r,\lambda_n)\|^2}=$$
$$=-\frac{\overline{\chi_2}}{\varDelta(\overline{\chi})}[K_1(\overline{\chi_2}R_2)I_0(\overline{\chi_2}r)+I_1(\overline{\chi_2}R_2)K_0(\overline{\chi_2}r)]\equiv S_4(r,\chi),$$
$$\overline{\chi}=\{\overline{\chi_1},\overline{\chi_2}\},\quad \overline{\chi_j}=a_j\chi_j,$$

$$\varDelta(\overline{\chi})=\gamma_1\overline{\chi_2^2}\left(R_0\operatorname{ch}\overline{\chi_1}R_1+\frac{\operatorname{sh}\overline{\chi_1}R_1}{\overline{\chi_1}}\right)(I_1(\overline{\chi_2R_1})K_1(\overline{\chi_2}R_2)-I_1(\overline{\chi_2R_2})K_1(\overline{\chi_2}R_1)-$$
$$-\overline{\chi_2}\operatorname{ch}\overline{\chi_1}R_1\left(I_0(\overline{\chi_2}R_1)K_1(\overline{\chi_2}R_2)+I_1(\overline{\chi_2}R_2)K_0(\overline{\chi_2}R_1)\right),$$

$$\varDelta_1(\overline{\chi})=-\nu_1\overline{\chi_2^2}(R_0\overline{\chi_1}\operatorname{sh}\overline{\chi_1}R_1+\operatorname{ch}\overline{\chi_1}R_1)(I_1(\overline{\chi_2}R_1)K_1(\overline{\chi_2}R_2)-I_1(\overline{\chi_2}R_2)\times$$
$$\times K_1(\overline{\chi_1}R_1))+\overline{\chi_1\chi_2}\operatorname{ch}\overline{\chi_1}R_1(I_0(\overline{\chi_2}R_1)K_1(\overline{\chi_2}R_2)-I_1(\overline{\chi_2}R_2)K_0(\chi_2R_1)),$$

$$\|V(r,\lambda_n)\|^2=\frac{\omega_1^2(\lambda_n)}{2}(R_1^2-\frac{\sin 2\overline{\beta_{1n}}R_1}{\overline{\beta_{1n}}}+\frac{2}{(\pi\overline{\beta_{1n}})^2}-\frac{R_1^2}{2}\omega_4(\lambda_n),$$

$$\omega_4(\lambda_n)=\omega_2^2(\lambda_n)(J_0(\overline{\beta_{2n}}R_1)+J_1(\overline{\beta_{2n}}R_1))-2\omega_2(\lambda_n)\omega_3(\lambda_n)[J_0(\overline{\beta_{2n}}R_2)\times$$
$$\times N_0(\overline{\beta_{2n}}R_1)+J_1(\overline{\beta_{2n}}R_1)N_1(\overline{\beta_{2n}}R_1)]+\omega_3^2(\lambda_n)[N_0^2(\overline{\beta_{2n}}R_1)+N_1^2(\overline{\beta_{2n}}R_1)].$$

Notice that $S_3(r,\overline{\chi})$ and $S_4(r,\overline{\chi})$ is finite on Π_1 solution of the boundary problem

$$\left(\frac{d^2}{dr^2} - \overline{\chi_1}^2\right) S_3(r,\overline{\chi}) = 0, \quad r \in (0, R_1),$$
$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \overline{\chi_2}^2\right) S_4(r,\overline{\chi}) = 0, \quad \overline{\chi_2} > 0, \; r \in (R_1, R_2)$$

on boundary conditions

$$S_3|_{r=0} = 1, \qquad \left.\frac{dS_4}{dr}\right|_{r=R_2} = 0$$

and contact conditions

$$\left.\left[\left(R_0\frac{d}{dr} + 1\right) S_3 - S_4\right]\right|_{r=R_1} = 0,$$
$$\left.\left(\frac{dS_3}{dr} - \gamma_1\frac{dS_4}{dr}\right)\right|_{r=R_1} = 0.$$

My research is devoted to the summation of just such series by the method of finite hybrid integral transforms.

By the Cauchy's method for the separate systems of the ordinary differential equations we have constructed the solution of the corresponding boundary problem in the case of general assumption on the differential and connected operators. The condition of the non-limited solving and the structure of the general solution for the boundary problem have been written in the explicit form. On the other hand solution of this problems has been constructed by the method of the finite hybrid integral transforms. Since this problems has one and only one solution we may compare the first solution with the second and, as a result, get the sums of functional series.

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