Grzegorz Karch

Scaling in nonlinear parabolic equations: locality versus globality

In: Zuzana Došlá and Jaromír Kuben and Jaromír Vosmanský (eds.): Proceedings of Equadiff 9, Conference on Differential Equations and Their Applications, Brno, August 25-29, 1997, [Part 3] Papers. Masaryk University, Brno, 1998. CD-ROM. pp. 137--138.

Persistent URL: http://dml.cz/dmlcz/700283

EQUADIFF 9 CD ROM, Brno 1997 PAPERS
Masaryk University pp. 137–138

Scaling in Nonlinear Parabolic Equations: Locality versus Globality

Grzegorz Karch

Instytut Matematyczny, Uniwersytet Wrocławski
pl. Grunwaldzki 2/4, 50-384 Wrocław, Poland
Email: karch@math.uni.wroc.pl
WWW: http://www.math.uni.wroc.pl/~karch

Abstract. The Cauchy problem for parabolic equations with quadratic nonlinearity is studied. We investigate the existence of global-in-time solutions and their large-time behavior assuming some scaling property of the equation as well as of the norm of the Banach space in which the solutions are constructed.

AMS Subject Classification. 35K55, 35K15

Keywords. the Cauchy problem, self-similar solutions

We study the Cauchy problem for the parabolic equation

$$u_t = \Delta u + \mathbf{B}(u, u)$$

supplemented by the initial condition

$$u(x, 0) = u_0(x).$$

Here $u = u(x, t)$, $x \in \mathbb{R}^n$, and $t \in [0, T)$ for some $T \in (0, \infty]$. We assume that the nonlinear term $\mathbf{B}(\cdot, \cdot)$ is defined by a bilinear form acting on $u(x, t)$ with respect to x only. This nonlinearity will also be assumed to satisfy a scaling property. To set it up, first given $f : \mathbb{R}^n \to \mathbb{R}$ we define the rescaled function $f_\lambda(x) = f(\lambda x)$ for each $\lambda > 0$. We extend this definition for all $f \in \S'$ in the standard way.

Definition 1. The bilinear form $\mathbf{B}(\cdot, \cdot)$ is said to have the scaling order equal to $b \in \mathbb{R}$ if

$$\mathbf{B}(f_\lambda, g_\lambda) = \lambda^b \Big(\mathbf{B}(f, g)\Big)_\lambda$$

for any $\lambda > 0$ and all $f, g \in \S'(\mathbb{R}^n)$, for which the both sides make sense.

Our main requirement is that *the bilinear form* $\mathbf{B}(\cdot, \cdot)$ *has the scaling order equal to* $b < 2$.

Now suppose we are able to construct local-in-time solutions in $C([0, T); E)$, where the Banach space E consists of tempered distributions. Assume, moreover, that the equation is invariant under some scaling transformations of the

The paper is the extended abstract of the paper [1].

independent and dependent variables. We show that these two assumptions combined with a scaling property of $\|\cdot\|_E$ allow us to obtain global-in-time solutions for suitably small initial data. To get such results we introduce a new Banach space of distributions which, roughly speaking, is a homogeneous Besov type space modeled on E. This approach allows us to get solutions for initial data less regular than those from E. In this abstract setting, we also study large-time behavior of constructed solutions. We find a simple condition (in terms of decay properties of the heat semigroup) which guarantees that solutions have the same asymptotic behavior as $t \to \infty$.

References

[1] Karch, G., *Scaling in nonlinear parabolic equations: locality versus globality*, Report of the Mathematical Institute, University of Wroclaw **92** (1997) 1–29, submitted.