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SURJECTIVITY AND BOUNDARY VALUE PROBLEMS

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In the paper we shall deal with an initial and a boundary problem for the functional differential equation with deviating argument $x'(t) = f[t, x_{\omega(t)}]$ in a Banach space whereby the functions of the state space are defined in the interval $(-\infty, 0]$ as well as with the generalized boundary value problem for a system of differential equations in R^n. The main tool for proving the existence of a solution to these problems will be some theorems on surjectivity of an operator.

1. Surjectivity of an operator.

Let $(E, |.|)$ be a real Banach space, $\phi \neq X \subset E$ and $S : X \to E$. We recall that S is *compact* if S is continuous and maps bounded sets into relatively compact sets. Similarly $T : X \to E$ is said to be a *condensing* map if T is continuous, bounded (i.e. maps bounded sets into bounded sets) and for every bounded set $A \subset X$ which is not relatively compact we have $\alpha(T(A)) < \alpha(A)$ where α is the Kuratowski measure of noncompactness. A simple example of a condensing map is one of the form $U + V$ where $U : X \to E$ is a strict contraction and $V : X \to E$ is a compact map.

Let $G \neq \phi$ be an open subset of E and denote by $\bar{G}$ the closure of G. Let $T : \bar{G} \to E$ be a condensing map, $a \in E$. If the set $\tilde{A} = \{x \in G : x - T(x) = a\}$ is compact (possibly empty), then the degree $\deg(I - T, G, a)$ is defined in the sense of Nussbaum [6] whereby I is the identity. Notice that $\tilde{A}$ will certainly be compact if G is bounded and T is such that $x - T(x) \neq a$ for all $x \in \partial G$ (boundary of G) ([6], p. 744). If T is compact, then the degree above agrees with the classical Leray-Schauder degree.

Denote B the real Banach space of all continuous functions $x : [0, \infty) \to E$ such that there exists $\lim x(t) = x(\infty)$ $(\in E)$ for $t \to \infty$. The norm in B is defined by $\| |x| \|_2 = \sup\{|x(t)| : 0 \le t < \infty\}$ for each $x \in B$. Let, further, $U(r) = \{x \in E : |x| < r\}$. Using the degree theory for condensing perturbations of identity, the topological principle in [8], p. 241, can be generalized as follows (for proof, see [9],[10]).

Theorem 1. Let $g : E \to B$ be a continuous mapping. Denote by $g(x,t)$

the value of $g(x) \in B$ at the point $t \in [0,\infty]$ ($g(x,\infty) = \lim g(x,t)$ for $t \to \infty$). Assume that

(i) $\nu(x) = \inf\{|g(x,t)| : 0 \le t \le \infty\} \to \infty$ for $|x| \to \infty$;

(ii) the mapping $I - g(.,t)$ is condensing for each $t \in [0,\infty]$;

(iii) for each $y \in E$ there is an $r_0 > 0$ such that

$$\deg(g(.,0) - y, U(r_0),0) \neq 0 ;$$

(iv) $g(x,.)$ is continuous in t, uniformly in $x \in \overline{U(r)}$ for each $r > 0$. Then for each $t \in [0,\infty]$

$$g(E,t) = E.$$

Proof. Let $y \in E$, $t_0 \in [0,\infty]$. By (i), there is an $r_0 > 0$, $|y| < r_0$, such that $y \notin g(\partial U(r_0),t)$ for each $t \in [0,\infty]$. Hence the mapping $G : \overline{U(r_0)} \times [0,\infty] \to E$ defined by $G(x,t) = x - g(x,t) + y$ is continuous and $G(x,t) \neq x$ for $x \in \partial U(r_0)$, $t \in [0,\infty]$. By (ii), $G(.,t)$ is a condensing map for $t \in [0,\infty]$ and (iv) implies that $G(x,.)$ is continuous in t, uniformly in $x \in \overline{U(r_0)}$. Hence, by Corollary 2 in [6], p.745, and (iii), for each t_0, $0 \le t_0 < \infty$,

$$\deg(I - G(.,t_0), U(r_0),0) = \deg(I - G(.,0), U(r_0),0) =$$
$$= \deg(g(.,0) - y, U(r_0),0) \neq 0.$$

As to the set $S = \{x \in U(r_0) : g(x,t_0) - y = 0\}$, either it is not compact or in case it is compact we can use Proposition 5 from [6], p. 744, and hence, in both cases it is nonempty.

Corollary 2 as well as Proposition 5 from [6] can be applied to the case $t_0 = \infty$, too, since then $t = \operatorname{tg} \frac{\pi}{2} s$ maps $[0,1]$ continuously on $[0,\infty]$ and instead of the function $G(x,t)$ we consider $G_1(x,s) = G(x,\operatorname{tg} \frac{\pi}{2} s)$, $x \in \overline{U(r_0)}$, $s \in [0,1]$.

Remark. Clearly the assumption (iii) is satisfied if $g(x,0) = x$ for each $x \in E$.

On the basis of the Schauder theorem on domain invariance ([2], p. 72) the following result can be proved ([10]).

Theorem 2. Let $T : E \to E$ be such that

(a) $\lim\limits_{|x|\to\infty} |T(x)| = \infty$;

(b) $I - T$ is compact;

(c) T is locally one-to-one, i.e. for each point $x_0 \in E$ there is a neighbourhood N of this point such that $T|_N$ is one-to-one. Then $T(E)=E$.

Proof. The assumptions (b), (c) imply that T is an open mapping, i.e. it maps open sets onto open sets. Hence T(E) is an open subset of E. Let $\{y_n\} \subset T(E)$ be a convergent sequence and $y_0 = \lim_{n\to\infty} y_n$. Then we can find a sequence $\{x_n\}$ such that $T(x_n) = y_n$. Assumption (a) is equivalent to the statement that the inverse image of a bounded set at the mapping T is a bounded set. Hence the sequence $\{x_n\}$ is bounded together with the sequence $\{y_n\}$. By (b), there is a subsequence $\{x_m\}$ of $\{x_n\}$ and a point $x_0 \in E$ such that $x_m - y_m = x_m - T(x_m) \to x_0$ as $m \to \infty$. Then $\lim_{m\to\infty} x_m = y_0 + x_0$, and by continuity of T, $T(x_0 + y_0) = y_0$. Thus $y_0 \in T(E)$ and T(E) is closed. As E is connected, T(E) = E.

Corollary 1. Let $T : E \to E$ be such that

(a) $\lim_{|x|\to\infty} |T(x)| = \infty$;

(b) $I - T$ is compact;

(c) T is one-to-one.

Then T is a homeomorphism of E onto E and there is a compact mapping $T_1 : E \to E$ such that $T^{-1} = I - T_1$ where T^{-1} is the inverse mapping to T.

Proof. By Theorem 2 and its proof we have that T(E) = E and the mapping T^{-1} is continuous. Hence T is a homeomorphism. For T^{-1} we have the identity $I - T^{-1} = (T - I) \circ T^{-1}$. By (a), T^{-1} is a bounded mapping and thus, by (b), $I - T^{-1} = T_1$ is compact.

If $E = R^n$, then Theorem 1 is true without assuming assumptions (ii),(iv) and in Theorem 2 instead of the assumption (b) it suffices to assume the continuity of T. Choosing properly the mapping $g : R^n \to B$ (B now means the Banach space of all continuous functions $x : [0,1] \to R^n$ with the supnorm, $|.|$ is the euclidean norm in R^n and (.,.) the scalar product in this space) we get the following

Corollary 2. Let $T : R^n \to R^n$ be a continuous mapping such that

(i) $\lim_{|x|\to\infty} |T(x)| = \infty$;

(ii) either there is an $x_0 \in R^n$ such that
$T(x) - x_0 = k(x - x_0)$ implies $k \geq 0$ for each $x \in R^n$, $x \neq x_0$,
or
there is an $r_1 > 0$ such that $(x, T(x)) \geq 0$ for all $x \in R^n$, $|x| \geq r_1$

or

T is locally one-to-one.

Then

$$T(R^n) = R^n.$$

Proof. a. Consider the first case that there is an $x_0 \in R^n$ such that

(α) $T(x) - x_0 = k(x - x_0)$ implies $k \geq 0$ for each $x \in R^n$, $x \neq x_0$.

Without loss of generality we may assume that $x_0 = 0$. Let the mapping $g : R^n \to B$ be defined by

$$g(x,t) = tT(0) \quad \text{for } x = 0,\ 0 \leq t \leq 1,$$

$$g(x,t) = [(1 - t)|x| + t|T(x)|].[|(1 - t)x + tT(x)|]^{-1}.$$
$$[(1 - t)x + tT(x)] \quad \text{for } x \neq 0,\ 0 \leq t < 1,$$

$$g(x,t) = T(x) \quad \text{for } x \neq 0,\ t = 1.$$

By (α) the mapping g is well defined. Further $g(x,.)$ is continuous in $[0,1]$ for each $x \in R^n$ and thus, g maps R^n into B. Clearly

(β) $g(x,0) = x, \quad g(x,1) = T(x) \quad$ for each $x \in R^n$.

Now we prove that g is continuous. Let $x \neq 0$ be an arbitrary but fixed point from R^n and y be a point sufficiently close to x. Then

$$|g(x,t) - g(y,t)| \leq \left|\frac{(1 - t)x + tT(x)}{|(1 - t)x + tT(x)|} - \frac{(1 - t)y + tT(y)}{|(1 - t)y + tT(y)|}\right| .$$
$$. [(1 - t)|x| + t|T(x)|] +$$
$$+ |(1 - t)(|x| - |y|) + t(|T(x)| - |T(y)|)|, \quad 0 \leq t \leq 1.$$

Clearly the second term on the right-hand side is less or equal to

(γ) $(1 - t)|x - y| + t|T(x) - T(y)|, \quad 0 \leq t \leq 1.$

As to the first term, there is a constant $k > 0$ such that this term is less or equal to

$$k|(1 - t)y + tT(y)|^{-1}.|[(1 - t)x + tT(x)] \bullet |(1 - t)y + tT(y)| -$$
$$- [(1 - t)y + tT(y)].|(1 - t)x + tT(x)|| \leq$$
$$\leq k|(1 - t)y + tT(y)|^{-1} . |[(1 - t)(x - y) + t(T(x) - T(y))] .$$
$$. |(1 - t)y + tT(y)| + [(1 - t)y + tT(y)]. [|(1 - t)y + tT(y)| -$$
$$- |(1 - t)x + tT(x)|]|.$$

Hence the first term is less or equal to

(δ) $2k[(1 - t)|x - y| + t|T(x) - T(y)|], \quad 0 \leq t \leq 1.$

The inequalities (γ) and (δ) give

$$|g(x,t) - g(y,t)| \le (2k+1)[(1-t)|x-y| + t|T(x) - T(y)|],$$
$$0 \le t \le 1,$$

which proves the continuity of g at $x \neq 0$. In a similar way it can be shown that g is continuous at 0.

Now we derive properties (i), (iii) of g from Theorem 1 and this will complete the proof of this part of Corollary 2. As $|g(x,t)| = (1-t)|x| + t|T(x)| \ge \min(|x|, |T(x)|)$, clearly (i) is satisfied. (iii) follows from (β).

b. Suppose that there is an $r_1 > 0$ such that

($\varkappa$) $\quad (x, T(x)) \ge 0 \quad$ for all $x \in R^n$, $|x| \ge r_1$.

Consider the mapping g which is defined for each $x \in R^n$, $0 \le t \le 1$, by

$$g(x,t) = (1-t)x + tT(x).$$

Clearly $g : R^n \to B$ and g is continuous. Further g satisfies (β). By ($\varkappa$), $|g(x,t)|^2 \ge (1-t)^2|x|^2 + t^2|T(x)|^2 \ge \frac{1}{2}[(1-t)|x| + t|T(x)|]^2$. Hence g satisfies assumption (i) as well as (iii) of Theorem 1. By this theorem the result follows.

c. The statement of Corollary 2 in case that T is locally one-to-one follows directly from Theorem 2.

2. Functional Differential Equations With Deviating Argument

First we formulate the initial-value problem for these equations which includes the problem from [12],[4] and is related to one in [1], [3]. For details and proofs, see [9]. We shall employ the notations:

$(E, |\cdot|)$ is a real Banach space.

The state space C is the Banach space of all continuous and bounded mappings $x : (-\infty, 0] \to E$ with the sup-norm $\|.\|$.

$\psi : [0,\infty) \to (0,\infty)$ is a nondecreasing continuous function.

The deviation $\omega : [0,\infty) \to R$ is a continuous mapping such that $\omega(0) = 0$.

$f : [0,\infty) \times C \to E$ is a continuous mapping.

$a^+ = \max(a,0)$ for each $a \in R$, $\operatorname{sgn} 0 = 0$, $\operatorname{sgn} a = 1$ for each $a > 0$.

Finally, if $x : (-\infty,\infty) \to E$ is a continuous mapping which is bounded in $(-\infty,0]$ and $u \in R$, then x_u is the function defined by

$$x_u(s) = x(u+s) \text{ for all } s, -\infty < s \le 0.$$

Clearly $x_u \in C$.

The initial-value problem in the case that $h \in C$ is uniformly continuous in $(-\infty, 0]$

(1) $\qquad x'(t) = f[t, x_{\omega(t)}]$

(2) $\qquad x_0 = h$

means the problem to find a function x which is continuous in $(-\infty,\infty)$, $x(t) = h(t)$ for all $t \in (\infty,0]$, x is differentiable in $[0,\infty)$ and it satisfies (1) at each point from $[0,\infty)$. Since ω, f are continuous and h is uniformly continuous, the problem (1),(2) is equivalent to the problem: To find a continuous solution of the integral equation

(3) $$x(t) = h(0) + \int_0^t f[s,x_{\omega(s)}]ds \quad (0 \le t < \infty)$$

which satisfies (2).

Consider the following assumptions:

(A1) The function $\int_0^t |f(s,0)|ds$ is ψ-bounded in $[0,\infty)$, i.e. $|\int_0^t |f(s,0)| ds|/\psi(t)$ $(0 \le t < \infty)$ is bounded.

(A2) There exists a nonnegative, locally integrable in $[0,\infty)$ real function n such that
$$|f(t,z_1) - f(t,z_2)| \le n(t) \|z_1 - z_2\|$$
for every $z_1, z_2 \in C$ and $t \in [0,\infty)$.

(A3) The function $\int_0^t n(s)ds$ is ψ-bounded in $[0,\infty)$.

(A4) There exists a q, $0 \le q < 1$, such that
$$\int_0^t n(s)\ \mathrm{sgn}\ \omega^+(s)\psi[\omega^+(s)]ds \le q\psi(t) \quad (0 \le t < \infty).$$

(A5) There is a $K > 0$ such that $\int_0^t |f(s,0)|ds \le K$ for all t, $0 \le t < \infty$.

(A6) There is a q, $0 \le q < 1$, such that $\int_0^t n(s)ds \le q$, $0 \le t < \infty$.

The existence of a unique ψ-bounded solution to (1),(2) is guaranteed by

Lemma 1. If the assumptions (A1)-(A4) are satisfied, then there exists a unique ψ-bounded in $[0,\infty)$ solution x(t) of (1),(2), i.e. $|x(t)|/\psi(t)$ is bounded in $[0,\infty)$.

Proof. Let D be the vector space of all continuous mappings $x : (-\infty,\infty) \to E$ which are bounded in $(-\infty,0]$ and ψ-bounded in $[0,\infty)$, $D_h = \{x \in D : x(t) = h(t), -\infty < t \le 0\}$. Let F be the Banach space of all continuous and ψ-bounded mappings $x : [0,\infty) \to E$ with the norm $\|x\|_1 = \sup_{0 \le t < \infty} |x(t)|/\psi(t)$. Then in view of the assumptions of the lemma the mapping T defined by
$$T(x)(t) = h(t), \quad -\infty < t \le 0,$$
$$T(x)(t) = h(0) + \int_0^t f[s,x_{\omega(s)}]ds, \quad 0 \le t < \infty,$$
maps D_h into D_h or considering only the restriction of functions from D_h to $[0,\infty)$, $T : G \to G$ where $G = \{x \in F : x(0) = h(0)\}$ is a closed

subset of F. By (A2) and (A4) $|T(x)(t) - T(y)(t)|/\psi(t) \le$ $\le \int_0^t n(s)\|x_{\omega(s)} - y_{\omega(s)}\|ds/\psi(t) \le \|x - y\|_1 \int_0^t n(s)\ \mathrm{sgn}\ \omega^+(s).$ $\psi[\omega^+(s)]ds/\psi(t) \le q\|x - y\|_1$. The Banach fixed point theorem gives the result.

By considering the bounded solutions of the problem (1), (2) we can prove

Lemma 2. If the assumptions (A1)-(A4) are satisfied and ψ is bounded, then for the unique bounded solution x(t) of (1),(2) there exists $\lim_{t\to\infty} x(t) = c\ (\in E)$.

Proof. By (3) and (A2), for $0 \le t_1 < t_2 < \infty$ we have $|x(t_2) - x(t_1)| \le \int_{t_1}^{t_2} |f(s,0)|ds + \int_{t_1}^{t_2} n(s)\|x_{\omega(s)}\|ds$. In view of (A1), (A3) and the boundedness of ψ, by the Cauchy-Bolzano criterion the result follows.

Denote this unique bounded solution of (1),(2) as x(t,h). Then the continuity of the bounded solution of (1),(2) in h is proved in

Lemma 3. Suppose that (A2),(A5) and (A6) are satisfied. Then for any $h_1, h_2 \in C$, h_1, h_2 are uniformly continuous in $(-\infty,0]$ and $h_1(0) = h_2(0) = 0$

$$\|x_t(.,h_2) - x_t(.,h_1)\| \le \|h_2 - h_1\| v(t), \qquad 0 \le t < \infty ,$$

where v(t) is the unique real bounded continuous solution of

$$(4) \qquad v(t) = 1 + \int_0^t n(s)\ v[\omega^+(s)]ds, \qquad 0 \le t < \infty .$$

Proof. Denote $u(t) = \|x_t(.,h_2) - x_t(.,h_1)\|$, $0 \le t < \infty$. By (3) and (A2) it follows that

$$|x(t,h_2) - x(t,h_1)| \le |h_2(0) - h_1(0)| + \int_0^t n(s)u[\omega^+(s)]ds, \qquad 0 \le t < \infty,$$

and hence $u(t) \le \|h_2 - h_1\| + \int_0^t n(s)u[\omega^+(s)]ds$, $0 \le t < \infty$. Since u is bounded and continuous, by the generalized Gronwall lemma the result follows.

Lemma 4. Assume that (A2),(A5) and (A6) are satisfied. Let $h \in C$, $h(0) = 0$ and let h be uniformly continuous in $(-\infty,0]$. Let $\{z_k\}$, $z_k \in E$, $k = 1,2,\dots$, be a sequence with $\lim_{k\to\infty} |z_k| = \infty$. Denote $m_k = \inf\{|x(t,h + z_k)| : 0 \le t < \infty\}$. Then

$\lim_{k\to\infty} m_k = \infty$.

Proof. It is similar to that of Lemma 3 in [8], p. 240.

By using Theorem 1 where $g(x_0,t) = x(t,h + x_0)$ the following *boundary value problem* for (1) can be solved. An arbitrary point $x_1 \in E$ and an initial function $h \in C$, h is uniformly continuous in $(-\infty,0]$, $h(0) = 0$, are given. To find a point $x_0 \in E$ such that

(5) $$\lim_{t\to\infty} x(t,h + x_0) = x_1 .$$

Theorem 3. Assume that (A2'),(A5),(A6) as well as the assumption:

(A7) There exists a q_1, $0 \le q_1 < 1$, such that for the bounded continuous solution $v(t)$ of the equation (4) the inequality
$v(t) \le 1 + q_1, \quad 0 \le t < \infty$,
is true,

are satisfied. Let $x_1 \in E$ and let $h \in C$, h be uniformly continuous in $(-\infty,0]$, $h(0) = 0$. Then there exists exactly one $x_0 \in E$ such that (5) is true.

Proof. Define a mapping $g : E \to B$ in this way. Given an $x_0 \in E$, let $g(x_0,t) = x(t,h + x_0)$ for $0 \le t < \infty$ and let $g(x_0,\infty) = \lim_{t\to\infty} x(t,h + x_0)$. Lemma 1 and 2 guarantee that g is well defined. By Lemma 3 g is continuous and Lemma 4 implies that the condition (i) in Theorem 1 is satisfied. Clearly (iii) in that theorem holds. Let $r > 0$, $t_1 < t$, $|x_2| \le r$. Then $|g(x_2,t) - g(x_2,t_1)| \le \int_{t_1}^{t} |f(s,0)|ds + \int_{t_1}^{t} n(s)\|x_{\omega^+(s)}(.,h + x_2)\|ds \le \int_{t_1}^{t} |f(s,0)|ds + \int_{t_1}^{t} n(s)\,[M_1 + (\|h\| + r)K_1]ds$, where $M_1 = \sup_{0\le s<\infty} \|x_{\omega^+(s)}(.,0)\|$, $K_1 = \sup_{0\le s<\infty} v(\omega^+(s))$. If $t < t_1$, we get a similar inequality. This implies that (iv) is satisfied.

Consider the mapping $U = I - g(.,t)$ for a fixed $t \in [0,\infty]$. Then by Lemma 3 and (A7) $|U(x_0) - U(y_0)| \le \int_0^t n(s)\|x_{\omega^+(s)} - y_{\omega^+(s)}\|ds \le |x_0 - y_0|(v(t) - 1) \le q_1|x_0 - y_0|$. Hence U is a strict contraction and thus a condensing mapping. By Theorem 1, $g(E,t) = E$ for each $t \in [0,\infty]$. Since U is a strict contraction, $|g(x_0,t) - g(y_0,t)| \ge (1 - q_1)|x_0 - y_0|$ which implies that $g(.,t)$ is a homeomorphic mapping of E onto itself.

Remarks. 1. In case $E = R^n$, Theorem 3 is valid without assuming (A7). Of course uniqueness of x_0 need not be true.

2. Theorem 3 extends the main result from [8], p. 239,

and in the case $\omega(t) = t$, $0 \le t < \infty$, is stronger than Theorem I in [11], p.3.

3. Generalized Boundary Value Problem for Differential Systems

The generalized boundary value problem for a differential system

(6) $\quad x' = f(t,x), \quad t \in i, \quad x \in R^n$,

and a given continuous mapping T (not necessarily linear) of the space $C(i,R^n)$ of all continuous n-dimensional vector functions defined in i into R^n can be defined as a problem of finding a solution x(t) of the system (6) on the interval i for which T(x) is a given vector r in R^n, i.e.

(7) $\quad T(x) = r$.

The topology in $C(i,R^n)$ is given in two different cases. If $i = [a,b]$ is a compact interval, then we consider the topology of uniform convergence, while in case i is a noncompact interval, e.g. $i = (a,\infty)$, then we use the topology of locally uniform convergence.

Theorem 4. Let $f = f(t,x)$ be a continuous function on $i \times R^n$ and let the equation (6) have the following properties:

(a) There is a point $t_0 \in i$ such that for each vector $x_0 \in R^n$ there exists a unique solution x(t) on i to the initial-value problem (6),

(8) $\quad x(t_0) = x_0$

and either:

(b) For each solution x of (6),(8) the following implication is true:

If $T(x) = kx(t_0)$, $x(t_0) \neq 0$, then $k \ge 0$,

or:

(c) The problem (6),(7) has at most one solution for each $r \in R^n$.

Then in the case (a),(b) a sufficient condition and in the case (a),(c) a necessary and sufficient condition that there exist at least one solution of the problem (6),(7) for each $r \in R^n$ is that the following *compactness condition* be satisfied:

(d) If $\{x_k\}$ is a sequence of solutions of (6) on the interval i such that $\{T(x_k)\}$ is bounded, then there is a subsequence $\{x_{k(l)}\}$ such that $\{x_{k(l)}\}$ is converging in $C(i,R^n)$.

The proof is based on Corollary 2 and the Kamke convergence lemma.

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