Topological spaces

Introduction

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INTRODUCTION

Notions centering around the concepts of a continuous mapping and of a set endowed with continuous operations (compositions) play a fundamental role in general mathematical analysis. The main purpose of the present book is, broadly speaking, to investigate the concept of continuity; more specifically, topological spaces and related objects as well as their continuous mappings are examined from a general point of view with respect to their possible use, directly or indirectly, in the development of mathematical analysis. In this sense, the book is intended as an introduction to general analysis. The main stress is on the interrelations of various kinds of spaces and mappings, and on various kinds of "constructions", in a general sense, such as the projective and inductive generation of spaces. The examination of individual spaces or classes of spaces is given less attention; the numerous examples given serve rather to explain how or why a certain general device applies or does not apply in an individual case. Although the title of the book is, traditionally, "Topological Spaces", these spaces are scarcely given more attention than two other kinds of "continuous structures", namely uniformities and proximities; other types of structures have not been included but only because the size of the book would have been too great.

Thus, this book presents a certain part of general topology from a specific point of view, sometimes perhaps subjectively biased. It is by no means intended to give in any part of the book a complete or "best" treatment of the subject, particularly since, in the authors' opinion, fundamental mathematical ideas cannot be fully expressed in any fixed rigid conceptual system.

From a formal point of view, it can be said that the present book can be read without any knowledge of mathematics at all, with the exception that at some places such topics as the arithmetic of natural numbers, etc., are covered; all mathematical concepts, beginning with that of a class and a set, are introduced in an exact manner and no notions from mathematical logic are used. In fact, a reasonable familiarity with abstract mathematical thinking is necessary, and a certain acquaintance with some basic facts concerning metric and topological spaces may be quite useful. Although this is not a text-book, it can be reasonably supposed that a university student of mathematics will be able to read this book without excessive effort, but certainly not without some degree of concentration and study.

The book consists of seven chapters and an Appendix. Chapters I and II concern, roughly speaking, the theory of sets and classes as well as some topics from the theory of ordered sets and algebraic systems. Chapters III – VII and the Appendix concern general topology. They are followed by Exercises and by Notes containing bibliographical references and various remarks. Chapters I – VII are divided into sections numbered currently from 1 to 40. The final section, 41, is concerned with complete and compact spaces, and forms the Appendix just mentioned. These concepts are indispensable to modern topology. On the other hand, Chapters III – VII are concerned with very general situations, as a rule, and considering them along with the complete and compact case would require too much place. Therefore, it was felt appropriate to treat complete and compact spaces in a special section detached from the bulk of the book.

The number of exercises is relatively small; and they often contain concepts or results extending those of the main text. All the exercises are placed at the end of the book, ordered by the sections to which they pertain, and numbered separately for each section or group of sections; however, to some sections there are no corresponding exercises.

In the main text, exercises belonging to the same section are referred to simply as, e. g., ex. 3; references to exercises of other sections are as, e. g., 6 ex. 8 or 25 ex. 11 (the latter refers to the eleventh exercise in the group belonging to sections 23 – 25).

All sections (with one exception) are subdivided into subsections designated by letters A, B, ... Every subsection consists of items, e.g. 3 A.1, 3 A.2, etc., each of these containing, as a rule, a proposition and its proof or a definition as well as examples, remarks, etc.; however, some items may contain both a definition and a proposition or else an informal reasoning, notes of various kind and so on. As a rule, all propositions referred to in what follows are italicized, but only those which seem to be particularly important are labelled explicitly as "theorems"; expressions such as "proposition and definition", "corollary" are also used, but the label "proposition" does not appear. We distinguish definitions and conventions (there is no sharp dividing line, of course): in "conventions", as a rule, an abbreviation is introduced for a term or symbol already defined or it is stated that a term will be used in an extended sense and so on.

The terminology and notation deviates sometimes from current usage, and quite a number of new terms have been introduced, usually to designate concepts which are either new or rarely treated in the literature, or which are considered from a new point of view. In particular, some innovations are a consequence of considering sets along with "proper" classes, single-valued relations along with mappings, proximity and uniform spaces along with topological ones, etc. It has been by no means intended to replace the current terms by specially invented ones, but solely to use such terminology and notation which seemed to be best adapted to the purposes of this book without deflecting too much from current usage. The choice of terms and symbols, in the authors' opinion, is subject to several requirements. They should be exact, that is, they should specify unequivocally the mathematical objects concerned. On the other hand, they should be flexible and concise; this implies the use of terminology and notation in a manner unequivocal in a given context only. Finally, the intuitive connotation of terms and symbols is of considerable importance. It has been intended to satisfy these requirements in an optimal manner, but a partial success is the most that can be hoped for.

On the other hand, the notation can never play more than an auxiliary role. Thus, we freely use various abbreviations as well as expressions which are, strictly speaking, not correct (so-called *abus de langage*); for instance, a space and the set of its points are, as usual, often denoted by the same letter, and so on.

As for the literature, some references are given in the Notes at the end of the book. With one or two exceptions, the proper text contains no references or remarks of a historical character. This may be rightly considered as a shortcoming. It was felt, however, that this should not seriously affect the purpose of the book.

A survey of the contents of sections 1–41 is deferred to the introductory remarks and "orientations" at the beginning of every chapter. Here, only some special features of Chapters I and II will be mentioned. These chapters, devoted to the theory of sets and classes, to the elements of abstract algebra, to ordered sets and so on, contain considerably more material than is required by the subsequent topological chapters. It was felt, however, first, that as a consequence of the increasing importance of the theory of categories, it is necessary to include its elements in the book, although they seldom occur in the sequel; secondly, that the use of "proper" classes or some similar objects can no longer be avoided in a book of this type. This makes a Cantorian approach impossible, of course, and necessitates a kind of axiomatic exposition.

Thus, Chapters I and II contain: a non-formalized axiomatic presentation of the theory of classes and sets (as well as of natural numbers), on the basis of current logic and language; basic facts concerning relations, mappings and correspondences; elements of the theory of categories; basic facts, as well as some additional information, on algebra and the theory of order.

It is by no means necessary for the prospective reader to go through Chapter I and II (i.e. sections 1–13) step by step. It will be sufficient to read through sections 1–7 (proofs may be omitted) and then go directly to Chapter III, returning to the pertinent place in Chapter I and II whenever a term appears or a fact is used which needs an explanation.