Lenka Čechová

The first Czech textbooks on the differential geometry

In: Eduard Fuchs (editor): Mathematics throughout the ages. Contributions from the summer school and seminars on the history of mathematics and from the 10th and 11th Novembertagung on the history and philosophy of mathematics, Holbaek, Denmark, October 28-31, 1999, and Brno, the Czech Republic, November 2-5, 2000. (English). Praha: Prometheus, 2001. pp. 296–305.

Persistent URL: http://dml.cz/dmlcz/401264

THE FIRST CZECH TEXTBOOKS ON THE DIFFERENTIAL GEOMETRY

Lenka Čechová

Abstract

The first Czech textbook on differential geometry was published by J. Sobotka as late as 1909. The situation in geometry was specific in the Czech lands at the beginning of the 20$^{\text{th}}$ century. The Czech geometers still concentrated mostly on the questions of constructive and projective geometry. Differential geometry was neglected at that time.

The aim of this contribution is to describe the circumstances of the rise of the first Czech textbook on differential geometry, and namely to compare this textbook with the another textbooks on differential geometry published in the Czech Lands until World War II.

1 Historical background

Thanks to the Industrial Revolution and a whole series of other factors (political, military, economic and social), the conditions for education and science in Central Europe (Bohemia and Moravia, Austria, Germany) changed significantly in the second half of the 19$^{\text{th}}$ century, as is well-known from historical studies. The technical universities, founded or reformed, devoted their attention mainly to geometry (useful in technical practice), but also to other mathematical disciplines.

The situation in the Czech lands was more specific and complicated. The Czech geometric school, which worked from the 1860s till the beginning of the 20$^{\text{th}}$ century, emerged in the Czech Lands. It had a great influence on the development of mathematics, especially geometry, in the Czech Lands.

Dr. Jaroslav Folta, in his historical analysis of the Czech geometric school [1, p. 83], says:

> *The Czech geometric school was to a certain extent created in contrast with the mathematical tradition established at the Bohemian Lands during the first half of the 19th century.*

> *The research of the Bohemian mathematicians of that time was concentrated more on the foundations of mathematical analysis (B. Bolzano), on the number theory (J. Ph. Kulik) or on theoretical and applied mechanics (F. J. Gerstner). A new tradition was established by the new social conditions connected with necessities of the continental (and especially Bohemian) development of the Industrial Revolution. The creation of the Prague polytechnical school on the model of the Mongeś programme for the Ecole Polytechnique in Paris, with an emphasis on the education of students in descriptive geometry founded the institutional and research base for the development of geometry. This process taking place under conditions of the growing Czech national movement and without any governmental support caused the intensive scientific activity in geometry and was formed into a territorial mathematical school.* [...]

Although the Czech geometric school contributed to the progress of Czech mathematics, namely constructive and projective geometry, the consequences of this phenomenon were not only positive. The Czech geometric school kept the traditions of these parts of geometry for very long and in the first half of the 20$^{\text{th}}$ century, these prevented wider use of new ideas. The concentration on the questions of constructive and projective geometry (both synthetic and algebraic) caused other branches of geometry (differential geometry, non-Euclidean geometry, etc.), which were paid increased attention in mathematics of the other countries at the time, to be neglected.

Except for the phenomenon of the Czech geometric school, there originated another problem at the end of the 19$^{\text{th}}$ century, because Czech mathematicians under growing Czech national movement (after the revolution in 1848) started to lecture[1] and publish in Czech.[2] It was necesary to create a whole mathematical terminology and textbooks in Czech language, to educate Czech teachers, and so on. This work took a lot of effort of Czech scientists.

[1] The first lectures in Czech language were given as lately as 1861–62 by RUDOLF SKUHERSKÝ (1828–1863) on Prague Polytechnic. Then, Prague Polytechnic was a German institution, which was divided into Czech and German in 1963–64, but this division was only formal. Finaly, they were separated in 1869 [1, p. 70].
After Skuherský there were two professors of descriptive geometry – FRANTIŠEK TILŠER (1825–1913) on the Czech and WILHELM FIEDLER (1832–1912) on the German one. Both of them influenced J. Sobotka; more in his biography bellow.

[2] The first volume of a regular Czech journal *Časopis pro pěstování matematiky a fysiky* was published in 1872 [2, p. 43].

2 Beginnings of differential geometry in the Czech lands

If we follow up the development of differential geometry, we can see that it had been neglected at the beginning of the 20^{th} century, too.

Although Monge's *Application de l' analyse à la geometrie* had been published in 1807 and Gauss' *Disquisitiones generales circa superficies curvas* in 1828 (these two books are nowadays recognized as the first books on differential geometry), these new ideas were coming to the publications of Czech geometers slowly. The ideas of Gauss were presented at the meeting of *Union of Czech Mathematicians* (in Czech: *Jednota českých matematiků*) in 1871 in a series of eight lectures under the title *On the Curvature of Surfaces* (in Czech *O křivosti ploch*) given by Č. Strouhal (viz [2, p. 44]). It is more than surprising but these new ideas couldn't find acceptance by Czech geometers (probably because they had been paying attention to the questions of constructive, projective and descriptive geometry) except for some geometers who studied abroad.

These were for example some articles of EMIL WEYR and particularly his younger brother EDUARD WEYR.[3] Eduard Weyr published 13 articles in which he considered differential geometry as well. These contributions, however, contain mostly only small results applied to the theory of curves or surfaces.[4]

3 The first book on the differential geometry

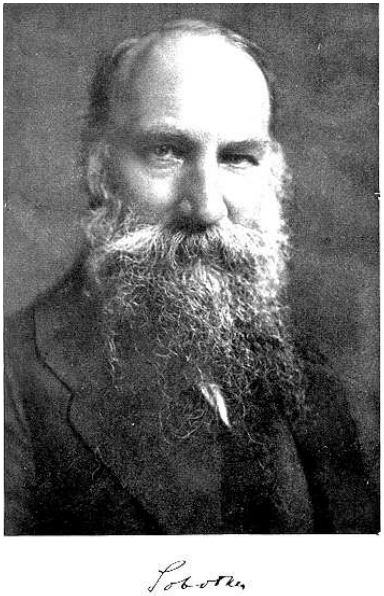
The first book on differential geometry in the Czech lands was published as late as in the years 1909–14. It was a textbook written as litography by JAN SOBOTKA – first full professor of descriptive geometry at Czech Technical Institute in Brno.

[3] EMIL WEYR (1848–1894) was influenced by LUIGI CREMONA (1830–1903) during his study in Milano, Italy, in 1870. He translated two of Cremona's books. Because of translating the second one – *Úvod do geometrické theorie křivek rovinných* (Introduction to the theory of plane curves) – he met Cremona again in 1873 (viz [3, p. 14]). EDUARD WEYR (1852–1903) studied in Götingen in 1872–3 and in Paris in 1873–4.

[4] More information about Eduard Weyr's differential geometry can be found in Prof. Nádeník's contribution to ([3, p. 67])

Jan Sobotka (1862–1931)

After finishing the German secondary technical school in Prague he continued his education at the Czech University and at the Czech Technical University in Praque in 1881–86. In 1886-91 he became assistant at Technical University, where he deputized F. Tilšer, who was then a parliamentary. In 1891 Sobotka studied with W. Fiedler at Technical University in Zurich. After this he continued to deputize F. Tilšer and in 1893 he left for scholarship to Wroclaw. As he couldn't find a job in the Czech Lands neither at a university, nor at a secondary school, he left for a secondary technical school in Vienna. Two years later he became assistant of descriptive geometry at Vienna Technical University. He was appointed extraordinary professor of descriptive geometry, projective geometry, and graphical calculus at Vienna Technical University in March 1897. In 1899 Sobotka became the first full professor of descriptive geometry at Czech Technical Institute in Brno, which had just founded there. In 1904 he left for Prague and became professor of mathematics at the Faculty of Arts of the University of Prague and after the founding of the Faculty of Natural Sciencies in 1920 he remained in this position for the rest of his life.

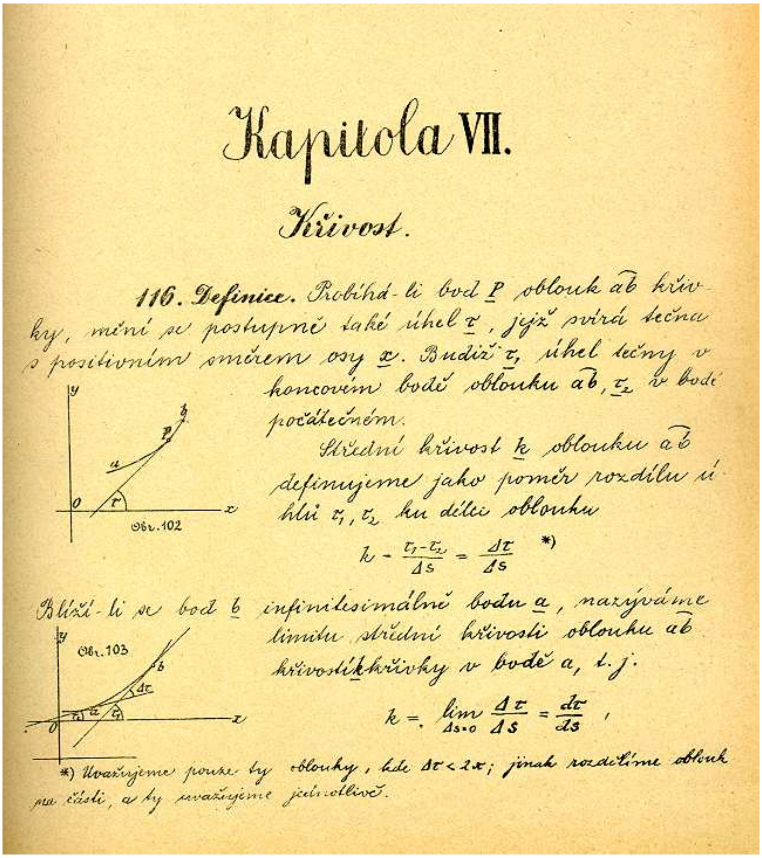


The main works of Jan Sobotka's are: Descriptive geometry of parallel projection (1906), Differential geometry (vollume I, II, III 1909-1914) and Geometrical affinities (1916). In addition, his works contain more than 150 treatises, mostly extensive treatises.

Sobotka J., *Differential geometry* [4]

Volume I: ***Plane curves*** (in Czech: *Křivky rovinné*), was published in 1909; **volume II:** ***Space curves; surfaces in rectangular coordinates.*** (in Czech: *Křivky prostorové; plochy v souřadnicích pravoúhlých.*) and **volume III:** ***Parametric expression of surfaces. Linear objects.*** (in Czech: *Parametrické vyjádření ploch. Útvary přímkové.* were published in 1914.

In the book, the consequences of the activities of the Czech geometric school can be observed. Jan Sobotka was, similarly as the majority of geometers of that time, active in the area of descriptive and projective geometry, which is reflected also in many aspects of this text. Constructive or projective methods of solution contrasted with differential methods are not an exception in Sobotka's book (e.g. next to the determination of asymptotes and osculation circles of the curves, the method of their construction is stated). This approach is seldom to be found in contemporary books devoted to differential geometry. The author was basically a synthetic geometer (as the majority of Czech geometers of his time).



Kapitola VII.

Křivost.

116. Definice. Probíhá-li bod p oblouk $\widehat{ab}$ křivky, mění se postupně také úhel τ, jejž svírá tečna s positivním směrem osy x. Budiž τ_1 úhel tečny v koncovém bodě oblouku $\widehat{ab}$, τ_2 v bodě počátečném.

Střední křivost k oblouku $\widehat{ab}$ definujeme jako poměr rozdílu úhlů τ_1, τ_2 ku délce oblouku

$$k = \frac{\tau_1 - \tau_2}{\Delta s} = \frac{\Delta \tau}{\Delta s} \quad {}^{*)}$$

Obr. 102

Blíží-li se bod b infinitesimálně bodu a, nazýváme limitu střední křivosti oblouku $\widehat{ab}$ křivostí křivky v bodě a, t. j.

$$k = \lim_{\Delta s \to 0} \frac{\Delta \tau}{\Delta s} = \frac{d\tau}{ds},$$

Obr. 103

*) Uvažujeme pouze ty oblouky, kde $\Delta\tau < 2\pi$; jinak rozdělíme oblouk na části, a ty uvažujeme jednotlivě.

A page from Sobotka's *Differential geometry*

In the book, the consequences of the activities of the Czech geometric school can be observed. Jan Sobotka was, similarly as the majority of geometers of that time, active in the area of descriptive and projective geometry, which is reflected also in many aspects of this text. Constructive or projective methods of solution contrasted with differential methods are not an exception in Sobotka's book (e.g. next to the determination of asymptotes and osculation circles of the curves, the method of their construction is stated). This approach is seldom to be found in contemporary books devoted to differential geometry. The author was basically a synthetic geometer (as the majority of Czech geometers of his time). He also solved the problems of differential geometry mostly using synthetic methods and the final goal of most question was the construction for him, which is untypical for pure differential geometry. In contrast to other pure synthetic geometers of his time, however, he did not avoid using analytic methods in his work, although he used them mostly for the constructive goal. This connection makes his sense for detail visible.

The reason for including other methods belonging to constructive, projective etc. geometry was probably the aim to cover the relations between the individual areas of geometry. The author's wide range of knowledge as well as his own results in other branches of geometry gave him all the prerequisites for this approach. The reader really has the feeling, at times, that up to then, he only knew the pieces and now he hears all of the opera. Chapter V. [4, vol. I], for example, is very well compiled in this sense. On the other hand, we can find passages, where different views on the same problem render the book seem confused and hardly legible (for example chapters devoted to singular points [4, vol. I]).

As soon as we begin to read Sobotka's Differential Geometry more closely and compare it with contemporary works published in Germany (e.g. [8],[7]), we cannot overlook that some passages are to a certain extent archaic, for example, the proposition on page 4 is very questionable [4, p. 4]:

> *The notion of a curve is of the empiric origin, as all the geometric notions in general; therefore if we are looking for the answer to the question given, we have to start from the visual appearance.*

Sobotka's textbook, however, was in its time one of the first steps from the relatively rigid tradition towards modern geometry. Thus although this book never stood up to the world's top-level geometry, its deep meaning for Czech geometry cannot be overlooked (see the Conclusion).

4 The following textbooks on differential geometry (until World War II)

Another textbook of differential was published as early as 1915 (only a year after the last volume of Sobotka!):

Hostinský B., *Differential geometry of curves and surfaces* [5]

It was written by BOHUSLAV HOSTINSKÝ (1884–1951) – professor of theoretical Physics at the Czech University in Brno. At first sight, this book is completely diferent in comparison with Sobotka's books.

The lengthy synthetic methods are not used here to such extent as in Sobotka's books. Descriptions of the constructions are left out completely. Last, but not least, Hostinský's book is not a litograph (like Sobotka's), which contributes to better legibility and more modern appearance.

In the first part, the theory of plane and space curves and the theory of surfaces are presented. In the second part, similarly as in Darboux's book [7], the usage of ordinary and partial differential equations is explained. This should not be surprising from the author who was a physicist. In the introduction, he himself says:

> *First, I put the emphasis on the usage of differential equations in geometry.* [...] *Secondly, I devoted special attention to the kinematic method, on which the classic works of Darboux are based. In most books and treatises on differential geometry, however, this method is not used.* [...]

The book was favoured by the readers, which can be confirmed by the fact that it was published in 1942 for the second time and in 1950 for the third.

The second edition from 1942 contains an interesting appendix *On further problems and methods in differential geometry* (in Czech: *O dalších problémech a metodách diferenciální geometrie*). Here the author does not present the geometrical theorems with proofs systematically, but aims only to draw the reader's attention to some interesting problems in differential geometry.

As far as the terminology is concerned, the author himself states in the introduction that he "accepted a number of terms from Sobotka's litographed lectures on differential geometry" and that "[he] did not introduce special novelties in the terminology." [5]

The whole book, especially its last part, is full of citations. More detailed description of the book exceeds the framework of this text.

Development proceeded very intensively and another textbook, which was published in 1937, was improved to the world standar. This book was written by VÁCLAV HLAVATÝ (1894 – 1969):

Hlavatý V., *Differential geometry curves and surfaces and tensor calculus* [6]

It is not uninteresting that the author of this book should have been J. Sobotka again, as V. Hlavatý mentions in the Introduction [6]:

> *In the introduction to his book Differential Calculus (Reports of the Union of Czechoslovak Mathematicians and Physicists, Prague 1923 [Sborník jednoty československých matematiků a fyziků], Prof. Petr says that the second volume of the differential calculus, which should be devoted to the geometric applications, should be written by Prof. Sobotka. After the death of Prof. Sobotka, this task was assigned to me.*[...]

In the book, the theory of curves and surfaces using Gauss' method and the method of tensor calculus is introduced.

More detailed comparison of the work of B. Hostinský and V. Hlavatý is very interesting, but exceeds the framework of this contribution.[5] Let us compare the mode of introducing the definition of a basic notion – the notion of a *curve*:

B. Hostinský (similarly to Sobotka) limits himself on the so-called *analytic curves* in the definition of curve, to the detriment of generality:

If x and y are perpendicular (Descartes) coordinates of the point A in plane and if y is a function of the variable x, i.e.:

$$y = f(x), \tag{1}$$

the point A creates a curve; (1) is its equation. We assume that $f(x)$ is analytic function of the variable x, i.e. that it can be developed in Taylor series

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \ldots + \frac{h^n}{n!}f^{(n)}(x) + \ldots \tag{2}$$

that is convergent, if $|h|$ does not exceed a certain value; $f^{(n)}(x)$ denotes n-th derivation of $f(x)$. (1) is then called analytic curve. [5, p. 1].

V. Hlavatý introduces the definition of a curve with the help of a real parameter:

Let us assume that three functions of a single parameter t

$$x = x(t), \; y = y(t), \; z = z(t), \tag{2,1}$$

[5] The author is preparing this detailed analysis as a part of her dissertation.

with the following properties be given:

a) They are defined uniquely and are continuous for all the values of the argument t from the interval $*\Omega$.

b) Such an interval Ω form $*\Omega$ exists that for all the values of the argument t from Ω, the functions (2,1) *have continuous derivation according to this argument up to the degree at least $r \geq 1$*

c) Single-line matrix

$$\mu = (x', y', z')$$

does not have the rank $h = 0$ for all the values t from Ω.

Def (2,1). *The set of points whose perpendicular coordinates x, y, z are given by the equations* (2,1)*, while the functions $x(t)$, $y(t)$, $z(t)$ comply with the conditions a), b), c) is called a curve (defined parametrically on the interval $*\Omega$). The argument t is called the parameter of the curve.*

By comparing both definitions, we can see that the definition given by Hlavatý is more general, i.e. that there are curves in the sense of Hlavatý's definition that are not curves in the sense of Hostinský's definition:

- Example 1: Unit circle – a set of points x whose rectangular coordinates x, y conform to the equation $x^2 + y^2 = 1$.
- Example 2: If we consider the function of one real variable

$$f(x) = \begin{cases} \mathrm{e}^{-\frac{1}{|x|}} & \text{pro } x \neq 0; \\ 0 & \text{pro } x = 0, \end{cases}$$

  then this function defines the curve in the sense of Hlavatý, but does NOT define a curve in the sense of Sobotka.

Conclusion

Nowadays, one can say that each of the above-mentioned textbooks accomplished unrecoverable part:

J. Sobotka [4] published the first extensive and concise text on differential geometry and created the terminology for differential geometry, which is being used today, up to small changes. He prepared good soil for the creation of further works.

B. Hostinský [5] deprived interpretation of lengthy syntetic methods and concentrated attention on application of differential geometrz to physic. (On the other hand some passages are mathematically not so precise.)

V. Hlavatý [6] created a modern textbook including tensor calculus and leading Czech geometry to the world-class level.

References

[1] Folta J., *Česká geometrická škola. Historická analýza (The Czech geometric school. Historical analysis. In Czech, English abstract.)*, Studie CSAV No. 9, Praha 1982.

[2] Veselý F., *100 let JČMF*, Praha 1962.

[3] Bečvář J. a kol., *Eduard Weyr (1852–1903)*, Praha 1995

[4] Sobotka J., *Diferenciální geometrie*, Volumes I to III, Praha 1909–14.

[5] Hostinský B., *Diferenciální geometrie křivek a ploch*, Praha 1915, 2nd ed. 1942, 3rd ed. 1950.

[6] Hlavatý V., *Diferenciální geometrie křivek a ploch a tensorový počet*, Praha 1937.

[7] Darboux G., *Lecons sur le théorie générale des surfaces*, Volumes I to IV, Paris 1887–1896, Reprint 3rd ed. New York 1972.

[8] Bianchi, Luigi. *Lezioni di geometria differenziale.* Volumes I to III, 2nd edition 1902–1909. (German translation: *Vorlesungen über Differentialgeometrie.* Leipzig und Berlin, 1910.)

Lenka Čechová
Department of Mathematics
Masaryk University
Janáčkovo nám. 2a
662 95 BRNO
Czech Republic
e-mail: mlc@math.muni.cz

All quotations were translated from Czech by Helena Durnová.