Witold Więsław

Mathematics at Polish universities (Cracow and Vilnius) in the XVIII century

In: Eduard Fuchs (editor): Mathematics throughout the ages. Contributions from the summer school and seminars on the history of mathematics and from the 10th and 11th Novembertagung on the history and philosophy of mathematics, Holbaek, Denmark, October 28-31, 1999, and Brno, the Czech Republic, November 2-5, 2000. (English). Praha: Prometheus, 2001. pp. 101–121.

Persistent URL: http://dml.cz/dmlcz/401240

MATHEMATICS AT POLISH UNIVERSITIES (CRACOW AND VILNIUS) IN XVIII CENTURY

Witold Więsław

1 Introduction: Mathematical knowledge in Polish community in the XVIII century

Very small mathematical knowledge for the people at large was contained in the books of the type:
Oekonomika ziemianska by Jakub Kazimierz Haur from 1744 *(Agricultural Economy)*, or *Informacya matematyczna rozumnie ciekawego Polaka*, from 1743 by Wojciech Bystrzonowski *(Mathematical Information for Clever Interested Pole).* The first book contains practical information from the economy, while the second one contains only courses of exchange money in the country and abroad. Moreover, printed calendars were intended for everybody. King August II granted the charters for printing calendars either for different institutions such as Cracow Academy (13$^{\text{th}}$ October 1714), or for concrete mathematicians (the charter of August II for Paulus Pater from 28$^{\text{th}}$ January 1703).

2 General situation of universities in the XVIII century: Mathematics at universities

Teaching of mathematics before the reforms

In the times before Polish educational reforms, in the first half of the XVIII century, there was no scientific activity at Polish universities (Cracow and Vilnius Academies – *Akademia Krakowska* and *Akademia Wileńska*). Similar situation was in all of Europe. Scientific activity could be seen only at Academies and in correspondence of mathematicians in the XVIII century. For example, the number and the quality of scientific publications in *Acta Eruditorum* and then in *Nova Acta Eruditorum* was still decreasing. The only scientific publications were mainly theses for doctor's degree, having few pages, in which the most place was taken by the title page with names of many people at the university such as rector, dean of the faculty etc. For example, *Questio Geometrica* by Stanisław Mamczyński at Cracow Academy from the year 1754 contains only six pages with Euclid's proof of Pythagoras theorem and a few comments. Similar theses were presented at Vilnius Academy. Colonel Eugeniusz Corsonisch wrote in the 1780s many papers dealing with *Perfecta circuli quadratura*, putting there together everything that was known about approximations of π. University students had to read elementary textbooks written in Latin by Jesuits, e.g. *Compendiaria Matheseos Institutio* [...] P. Mako E S. I., Wratislaviae, 1766, and also free translations of foreign books, e.g. *Praelectiones Mathematicae ex Wolfianis elementis adornate*, Vilnae, Anno 1759, by Jakub Nakcjanowicz (see [11]). The above-mentioned and other similar textbooks contain very elementary mathematics.

A similar situation was with texbooks for high schools. Teaching of geometry was based on e.g. *Geometria practica* by A. Tacquet (1741), or on *Początki geometryi* by Marcin Poczobut (based on Clairaut) from 1772, to mention only two examples. Teaching of arithmetic was based on books written by Polish authors, e.g. *Alfha Matheseos Arithmetica Theorica et Practica,* Vilnae [1733], or *Arytmetyka czyli Nauka o Liczbie*, R. P. 1757, by Michał Kaczwiński. Some books contained all school mathematics, e.g. *Institutiones mathematicae* by Jonannis Kiesius, Varsaviae, Anno 1742, concisely and well written textbook using freely algebraical notation. This book was published by Piarists, well-known for good educational methods. Piarists were the ones who started the reform of education in Poland in the middle of the XVIII century.

3 The reform of educational system and the reform of universities

The situation before the reforms

There were almost one hundred universities in Poland in the XVIII century. Most of them were Jesuits schools (66), Piarists (19) and the rest Basilian, and the so-called academic schools, under the control of universities in Cracow and Vilnius.

A cassation of Jesuits in 21st July 1773 brought up a question of the future of education in Poland. In the middle of the XVIII century Piarists started a kind of reform, but the problem of a unified reform in all schools was still open. On 24th October 1773 Seym, Polish Parliament, decided to constitute the Commission of National Education of Kingdom of Poland and Grand Duchy of Lithuania (abbr. KEN – in Polish: *Komisja Edukacji Narodowej*). KEN was the first ministry of education in the world. The main tasks of KEN were:

1) to take over and new organize post-Jesuit schools and their possessions;

2) to organize a new reformed system of education in Poland;

3) to prepare new teachers the school administration, and suitable school programmmes with modern textbooks, written in Polish.

Since KEN had many different tasks, it appeared to be necessary to call a special group to prepare details of the reform, mainly programmmes and textbooks. An informational campaign was organized in the meantime, informing about the reform and discussing different directions of possible changes in the education.

Society for Elementary Books

In 1775 KEN costituted a new institution – the Society for Elementary Books *(Towarzystwo do Ksiąg Elementarnych).* At first, the Society had to prepare new programmes and new textbooks in Polish. Later, the Society was preparing projects of new laws, the reform of universities, etc., even orders of educational equipment for schools. Priest GRZEGORZ PIRAMOWICZ was the Secretary of the Society; in fact, he was the heart of the Society. Soon, still in 1775, the Society decided to prepare new textbooks by an open competition. The information was published twice in Poland and twice abroad, in *Acta Eruditorum.* The first anonymous writer of a project of *the Arithmetic* was SIMON LHULLIER (1750–1840) from Geneve. In a similar way, in the following years, his projects of textbooks of geometry and algebra, won the competition.

LHUILLIER came to Poland, where he was a librarian of prince ADAM CZARTORYSKI, in 1777–1788. In the XVIII century it was a very important scientific position. Starting from 1795, LHUILLIER was professor at the university of Geneve, publishing many papers, mainly from geometry, in *Annales de Mathematiques pures et appliques*, at the beginning of the XIX century. For example, he extended the well-known formula of Leonhard Euler for polyhedrons.

Lhuillier's textbooks, originally written in French, were translated into Polish by the priest Andrzej Gawroński. His translations of Lhuillier's books are excellent. All thirty textbooks prepared by the Society were discussed every week at the meetings of the Society, together 631 meetings in the years 1775–1792. The Society took care also of the terminology – some old XVI–XVII century Polish terminology was reminded.

Although every textbook of Lhuillier is very good, they do not form a set of textbooks. His *Arithmetic* is very elementary, with many comments for teachers. The well-written *Geometry* (I and II) is based, of course, on Euclid, and is rather difficult. Finally, his *Algebra*, containing many, too many, examples, was difficult also for the teachers. Moreover, the Algebra was very expensive for students: it cost 6 times more than *Arithmetic*, and twice as much as *Geometry I*. *Arithmetic* had 8 editions up to 1830, *Geometry I* had 4 editions, *Geometry II* and *Algebra* – only one edition each. Lhuillier's *Algebra* was translated from Polish edition into German as *Anleitung zur Elementar Algebra*, Theil 1. Tübingen 1799, 298 S.; Theil 2.Tübingen 1801, 395 S.

Most textbooks of mathematics used before the reform were written in Latin and published often more than hundred years earlies.

Five years after the publication of the last textbook of mathematics (Lhuillier's *Algebra*, 1782), most of teachers used elementary books based on the new programme of mathematics.

The state of education in Lithuania was not so good as in the Kingdom of Poland. Jesuits of Polock were against the reforms. After 1772 Polock came to Russia. Jesuits remained there after the cassation, saved by Russia, and influenced the situation in education (and not only in education) in Grand Duchy of Lithuania.

Educational Laws

A Project of Educational Laws was prepared in 1781. The text, with rather small changes, was accepted by Seym *(Sejm)* in 1783. A system of education was planed in the following way: the Main Schools (*Szkoły Główne*) – universities (Cracow and Vilnius), High Schools (*Szkoły średnie*) under control of the Main Schools, and Elementary Schools (*Szkoły podstawowe*), often denominational schools. In order to open a school a necessary agreement of KEN was needed. Every school could be closed after a negative opinion of the General Inspector of KEN. Some of high schools, so called Faculty Schools, colaborated with Faculties of universities.

There was a unique plan of lectures in all high schools, the same day of a week and the same hours. Below I present the programme for mathematics.

Class I. Arithmetic 6 hours, by recommended elementary books

Class II. Arithmetic (continuation from Class I.)

Class III. A repetition of the Arithmetic (2). Geometry I (4)

Class IV. Geometry I (4) - finishing. Algebra (4).

Class V. (two years' class) Geometry II (4)

–. Algebra (2) - finishing. Elements of land-measuring (2)

Class VI. Logic (2).

Everything that was connected with the education was described in details in the Educational Laws, e.g. a school administration, students, their laws and rights, etc. A school teacher was called *professor*, the head of the school *rector*, a dean of the faculty was called *pro-rector*, at the universities as well as at high schools.

Reform of Nowodworski Schools

The reform of education was started in Cracow at Nowodworski Schools, which had some hundred years tradition. The whole reform was prepared, organized, and started by Hugo Kołłątaj in 1777. He was then 27 years old. Kołłątaj wrote new programmmes and found very good teachers, such as Jan Śniadecki, who spent the following years in Göttingen, in Holland and in France, cooperating with Simon Pierre Laplace in physical astronomy. Jan Śniadecki soon became professor of pure mathematics and astronomy at the Cracow University. (His brother Jędrzej was the professor of chemistry).

Hugo Kołłątaj reformed also Cracow Academy. However, he had some problems with bishop Sołtyk. Formally, the Cracow bishop was the principal of the university for the last three centuries. Bishop Sołtyk was against the reform of Cracow Academy. Consequently, Kołłątaj had to leave Cracow for some time, escaping before Bishop's Court. But finally the bishop, suspected for a insanity, was isolated, and Kołłątaj again was able to continue the reform of the university.

Not only elementary schools, high schools, but also the universities needed new books. A well-known book of ÉTIENNE BÉZOUT, *Cours de mathematiques a l'usage du corps royale de l'artillerie*, 1770. edition I, (vols. I–IV) was translated into Polish by JÓZEF JAKUBOWSKI. The translation was excellent. The book appeared in 1781–82. JAKUBOWSKI spent a few years in Western Europe, mainly in France, visiting European universities. We owe to him a great deal of contemporary Polish mathematical terminology.

JAN ŚNIADECKI wrote another book in Polish [15, 16]: *The Theory of Algebraical Calculus Applied to the Curve Lines (Rachunku algebraicznego Teorya Przystosowana do Geometryi Linii Krzywych)*, 1783 (II vols). Volume I contains mostly the algebra of polynomials of one variable, and some elementary calculus with applications to calculating square and cubic roots, discussion on the solvability of equations of degree 3 and 4. Moreover, some information about infinite series can be found there, for example, the general Newton expansion of $(1+x)^\alpha$ with a real exponent α. Volume II presents a systematic exposition of the plane analytical geometry as well as the elements of geometry of space. He studies there conic sections starting with an algebraic equation of degree 2 with two variables. Then, after suitable changes of the variables, he obtains the well-known objects and their description in the form of canonical equations. It seems to be new in the literature of his times. Unfortunately, the book was written in Polish, and, consequently it had no influence on mathematics in Europe. The book is very good, comparing it with books of other authors in the second half of the XVIII century. ŚNIADECKI published the book at his own expenses and during 42 years, he was not able to sell 1200 copies.

4 Lectures of mathematics at Cracow Academy

Cracow University was founded in 1364. In the first two centuries the state of mathematical sciences was good, but later, starting from the second half of the XVII century, it became worse and worse. However, I am not going to give the interesting history of this university here. It is very well described in different sources.

Relatively good information about mathematics at Cracow University is contained in publications such as:

Praelectiones Academicae quae in Principe Regni Schola Ima Octobris Anni 1786 ad diem ultimam Junii Anni 1787 [...], PROSPECTUS LECTIONUM ACADEMICARUM QUAE IN PRINCIPE REGNI

SCHOLA A die r. Octobris Anni 1788. ad diem ultimam Junii Anni 1789. publice tradentur.

After the reforms, Cracow University had the following structure:

Physical Collegium containig: I. School of Mathematics; II. School of Physics; III. School of Medicine. School of Mathematics had 4 Chairs: Elementary Mathematics (FELIX RADWAŃSKI; next JAN KANTY KRUSIŃSKI); Mechanics (FELIX RADWAŃSKI); Higher Mathematics (JAN ŚNIADECKI from 1781), and Astronomy (also JAN ŚNIADECKI).

The second, Moral Collegium included: I. School of Theology (teaching only in Latin); II. School of the Law; III. School of the Literature.

In 1783, 61 students studied elementary mathematics and mechanics, and 27 astronomy and higher mathematics.

After the reforms, Polish became the language of eduaction at Polish universities instead of Latin, traditional during the last three centuries. It was one of the most important decisions of KEN.

The programme of mathematics did not change between 1785 and 1800, when the Austrian army occupied Cracow; then German became the language at the university and university programmes changed. Polish as the language of education at Cracow University came back in 1870, after the acceptation of the Austrian Governement.

In *ORDINATIO STUDIORUM* from 1774 we find:

Sub Facultate Philosophica Mathematicae Disciplinae:
In Ima Classe.
Arithmetica (Logistica.
(Decimalis.
Geometria Theoretica.
Trigonometria (Plana.
(Sphaerica.
Algebra.
In 2da Classe.
Mechanica, Statica, Hydrostatica,
Arometria, Hydraulica.
In 3tia Classe.
Optica, Perspectiva,
Catoptrica, Dioptrica,
Astronomia (Sphaerica,
(Theoretica.
In 4ta Classe.
Geographia, Hydrographia,
Chronologia, Gnomonica.

Architectura, (Militaris,
(Civilis.

I think that the translation is not necessary.

Some years later, in 1781, geometry was lectured by ADAM STEFAN JAGIELSKI (see manuscript [25]). In the manuscript, we find:

GEOMETRIAE THEORICO-PRACTICAE

PARS I.

CAPUT I. DE DEFINITIONIBUS, AXIOMATIS ET POSTULATIS.

CAPUT II. THEOREMATA TRIANGULORUM.

CAPUT III. DE THEOREMATIS TRIANGULORUM.

CAPUT IV. DE THEOREMATIS CIRCULI.

CAPUT V. DE PROBLEMATIS LINEARUM.

CAPUT VI. DE PROBLEMATIS LINEARUM INTER SE PROPORTIONALIUM.

CAPUT VII. DE PROBLEMATIS ANGULORUM ET CIRCULI.

CAPUT VIII. DE INSTRUMENTIS GEOMETRICIS.

CAPUT IX. DE MENSURIS AD CALCULO GEOMETRICO.

CAPUT X. DE PROBLEMATIS LONGOMETRIAE.

CAPUT XI. DE PROBLEMATIS ALTIMETRAE.

CAPUT XII. [No title]

PARS II.

CAPUT I. DE DEFINITIONIBUS ET DIVISIONIBUS.

CAPUT II. DE THEOREMATIS RECTANGULORUM ALIARUMQUE FIGURARUM RECTILIN.

CAPUT III. DE THEOREMATIS AREAE CIRCULI.

CAPUT IV. I. DE PROBLEMATIS FIGURARUM EFFORMANDARUM.
II. DESCRIPTIO POLYGONORUM.

CAPUT V . DE DIMENSIONIBUS SUPERFICIERUM SEU AREARUM.

CAPUT VI. DE DIVISIONE AREARUM.

CAPUT VII. DE TRANSMUTATIONE AC AUGMENTO AREARUM.

CAPUT VIII. DE TRANSMUTATIONE ET AUGMENTO CIRCULI.

CAPUT IX. DE PLANIMETRIA PRACTICA.

PARS III.

CAPUT I. DE DEFINITIONIBUS ET DIVISIONIBUS.

CAPUT II. DE THEOREMATIS SOLIDORUM.

CAPUT III. DE THEOREMATIS SPHAERAE.

CAPUT IV. DE PROBLEMATIBUS SOLIDORUM.

CAPUT V. DE PROBLEMATIBUS SPHAERAE.

CAPUT VI. DE TRANSMUTATIONE SOLIDORUM. PROBLEMATA.

I. Datum Cylindrum in Parallelepipedum convertere.

II. Datum Conum in Pyramidem, vel Cylindrum in Conum convertere.

III. Prisma in Pyramidem, vel Cylindrum in Conum convertere.

IV. Dato Parallelepipedo, Cylindro, Cono et cae: aequalem cubum facere.

V. Datae Sphaerae equalem Cubum componere.

CAPUT VII. DE AUGMENTO ET DECREMENTO SOLIDORUM.

I. Cubum datum duplicare, triplicare et cae.

II. Datis duobus Cubis, unum aequalem efficere.

III. Ex duobus aut pluribus Sphaeris, unam efficere.

IV. Parallelepipedum augere, vel minuere in Proportione data.

V. Dato Cubo, Corpus aequale regulare construere.

CAPUT VIII. DE DIMENSIONIBUS AECONOMICIS.

PROOBLEMATA. VAS perfecte Cylindrum Metiri.

TRIGONOMETRAE THEORICO=PRACTICAE.
PARS I.

CAPUT I. DE DEFINITINIBUS ACPRAE NOTATIS HUC SPECTANTIBUS.

CAPUT II. DE THEOREMATIS AD RESOLUTIONEM TRIANGULORUM SPECTANTIBUS.

CAPUT III. DE NATURA AC PROPRIETATIBUS LOGARITHMORUM.

PARS II.

CAPUT I. DE PROBLEMATIS TRIANGULORUM RECTANGULORUM.

CAPUT II. DE PROBLEMATIS CIRCA RESOLUTIONEM TRIANGULORUM OBLIQUANGULORUM.
GEOGRAPHIA. HOROGRAPHIA. MECHANICAE THEORICO-PRACTICAE PARS I–II. ASTRONOMIA.

OPTICAE UNIVERSAE.

PARS I. SIVE VISIO DIRECTA.

PARS II. SIVE CATOPTRICA.

PARS III. SIVE DIOPTRICA.
HOROGRAPHIAE THEORICO=PRACTICAE.
ARCHITECTONICAE CIVILIS, THEORICO-PRACTICAE.
HYDROSTATICAE AC HYDRAULICAE.

PARS I. AEROMETRIAE.

Its beginning contains an elementary extract from Books I–VI of EUCLID's *Elements*, without proofs, only theorems and definitions are formulated. However, Axiom V of Euclid was omitted there. The next chapters are essentialy different from *Elements*. *Problemata* (Exercises) are solved geometrically. We present here selected solutions of *Problemata* from the manuscript.

Problem I (Caput VI). Squaring the circle (basis of cylinder) we obtain basis of parallelepipedum. Problem I (Caput VII). He states, without using algebraical symbols, that the problem leads to the construction of $\sqrt[3]{2}\ \sqrt[3]{3}$, etc., and Problem II (Caput VII) leads to the construction of $\sqrt[3]{x^3+y^3}$ for given x and y. For Problem III (Caput VII). JAGIELSKI writes: *change a sphere into a cube and apply the solution of Problem II.*

Problem V (Caput VII). He writes: *If a side of a cube is 1000, then equal [with equal volume] regular polyhedrons have the following sides: tetrahedron 2059, octahedron 1285, icosahedron 770, dodecahedron 507 and the sphere 1239.*

In a similar way as by JAGIELSKI in [25], geometry was lectured for two centuries before him.

In the next year, we read in the university program called *Prospectus Lectionum* (1782/83) that:

> *Felix Radwański* [...] *will lecture on algebra, one hour three times a week. The programme includes the theory of functions and equations, algebraical as well as transcendental, based on just published textbook of Jan Śniadecki. He will also lecture practical mechanics, one hour a week, on Sunday, for craftsmen.* [...]
>
> *Jan Śniadecki* [...] *will lecture on higher geometry, one and a half an hour three times a week. He will draw the theory of algebraical and transcendental curves from the nature of their equation. He will also discuss equations of surfaces. It will be useful for students who study mechanics. Then he will start to lecture beginnings of the calculus, based on a book of Cousin. Jan Śniadecki will also lecture astronomy, starting with spherical trigonometry. He is going to show how practical observations in astronomy lead to algebraical equations. It will help students to understand the connections of astronomy and geometry.*

In *Prospectus Lectionum* (1786/87) we read:

> *Felix Radwański* [...] *will lecture elementary mathematics, one hour three times a week. He will lecture elementary arithmetic (fractions, decimal fractions, continued fractions and their applications), as well as powers and roots. He will finish by showing in detail the theory of proportions, the*

Golden Rule, and others, as well as [arithmetical] progressions and logarithms. Then he will start with the beginnings of Euclid's Geometry, starting with fundamental information about measure (linear, plane and space). Next he is going to lecture space geometry also based on Euclid, including polyhedrons, regular and irregular. He will also lecture plane trigonometry with many practical applications. He will explain how to draw maps, economical and geographical. He will also lecture practical mechanics, one hour a week, for craftsmen. [...]

Jan Śniadecki [...] *will continue his lectures from the last academic year, based on the second part of his book. He is going to explain infinite series. He will show how to calculate inverse series [i.e. linear recurrent sequences], for example, how to expand rational functions into infinite series. It will imply the fundamental properties of continued fractions. Next he wants to lecture exponential functions and Euler's algorithms for expanding them into power series. It will imply the Theory of Logarithms, the methods of calculating them and making the tables of logarithms. Next he will discuss the methods for calculating sine, cosine etc., their expansions into infinite series and calculations of tables of trigonometric functions. After doing it he is going to present the conic sections and their applications in astronomy.* [...] *He will follow his own book.* [...] *The same will explain how to use mathematics in description of motion, fluids, from Mechanics, Hydromechanics, Meteorology, and mainly in applications of mathematics to the progress in the Astronomy.* [...] *After three months he will lecture practical Astronomy.*

In *Prospectus Lectionum* (1792/3) we read:

Jan Kanty Krusiński [...] *will continue the lectures based on Books 1-3 of Euclid's Elements. Next he is going to lecture properties of cones, spheres, cylinders and relations between them. After doing it he wants to presents results from Euclid's book called in Greek (in Latin Data), being continuation of his Elements. It will be useful in presentation Archimedes' results on surfaces of spheres and their sections. He plans also to lecture plane trigonometry with applications,*

> *and finally, beginnings of Algebra, based on Śniadecki's textobook.* [...]
>
> *Jan Śniadecki, Doctor of Philosophy, Public Professor of Higher Mathematics and Astronomy, Dean of the Physics Faculty, will continue his course of lectures in Higher Mathematics, he had begun in the last academic year. He will start with Calculus, which will be deduced from pure beginnings of geometry. He will explain everything in detail and with proofs and examples from Geometry of Lines and from Mechanics. Next he will explain Differential Calculus, not avoding anything that is necessary for understanding Differential Equations and the methods of their solution. He will lecture the methods of solving such equations, exact and approximate, by applying power series. If there is enough time, he also wants to present wide applications of this science in solution of many problems from Physics and Mechanics. The lectures are to be given one and a half hour three times a week.*
>
> *Jan Śniadecki will lecture Astronomy after Easter.*[...]
>
> *Felix Radwański will lecture Mechanics and Hydraulics, three times a week one hour.* [...] *He is going to discuss constructions of dams, flood gates and other water installations.*

From the examples above, we see that programmes of mathematical studies at Cracow University in the second half of XVIII century, during educational reforms, contain a good portion of mathematics, physics, astronomy and of many other branches of science, now attributed to technical universities, including a lot of practical knowledge. However, it is not quite clear, whether the programmes were indeed realised.

Traditionally, there was a so-called exhibit day in Polish schools at the end of the school year. It was a kind of public exam. Very often, questions and exercises from the exhibit day were published. Similar exhibit days were organized also at universities, but systematically, every week.

I give here some examples of exercises from Cracow Academy (see [26]).

Meeting V. 11 December 1784, at which Jozef Obminski, student of the second year, solves equations of degree 3 and 4. He explains also difficulties of Geometers in solving equations of higher degree. Examinators: students F. Szopowicz, S. Błaszczykiewicz, M. Kłosowicz, M. Kownacki.

Note here that it was 15 years before the final solution of the problem of algebraic equations by Paolo Ruffini (*Teoria Generale delle Equazioni*, 1799) who proved that the general equation of degree greater than 4 cannot be solved by radicals. Next the result was rediscovered by Niels Henrik Abel in 1826 and published in *Crelle's Journal* (vol. 1).

Meeting XV. 12 March 1785, at which Michał Dymidowicz, student of the fourth year, describes all properties of conic sections.

Meeting XIV. 17. March 1786, at which Adam Kopijewski, student of the fourth year, describes methods of solutions of some differential equations.

5 Lectures of mathematics at Vilnius Academy

Vilnius Academy was founded in 1579. In the first half of the XVIII century situation of Vilnius University was extremely bad. During the North War, which lasted approximately 30 years, Vilnius was occupied many times either by Russian or by Swedish army. Plagues decimated the community in this region of Poland. For this reasons the condition of the university was not too good. However, as we will see below, it was not so bad as it could be. Publications like *Universitas et Academia Vilnensis aliam a Valeriano Protasewicz Vilnensium Antistite Condita: A Gregorio XIII. P. M. A Stephano Bathoreo* [. . .] *Anno Domini 1781. VILNAE Typis Sacrae Regiae Majestatis Penes Academiam;* or *Prospectus Lectionum in Alma Academia et Universitate VILNENSI NUPER SCHOLAE PRINCIPIS M. D. LIT. Nomine Insignita ad Instaurationem Studiorum Ex Anno 1783. in Annum 1784* give us the possibility to reconstruct the programmes of mathematics.

The programme of mathematics at Vilnius Academy in the academic year 1707/1708 (see manuscript [8]) contains some elementary geometry, namely:

Elementorum Geometriae

PARS Una. *Euthymetria. Seu DE Dimensione Linearum.*

PARS 2.da. *Epipedometria Seu DE Dimensione Superficierum.*

PARS 3tia. *Stereometria Seu. DE. Dimensione Solidorum.*

TRIGONOMETRIA Plana. Vilnae 1707. in. 1708.

Caput 1Mum. *De Definitionibus Trigonometriae Et Constructione Canonis Sinuum. Tangentium et Logaryt[h]morum.*

Caput 2dum. *De Triangulorum Rectangulorum resolutione.*

Caput 3tium. *De Resolutione Triangulorum obliquangulorum. Institutiones Analyticae*

Pars 1ma. *De Arithmetica speciosa.*

Caput 1mum. *Prologomena Analyticos universae.*

Caput 2dum. *De calculo numerorum integrorum seu rationalium.*

Caput 3tium. *De Calculo Fractorum et exponentialium.*

Caput 4tum. *De Calculo radicalium seu irrationalium.*

Sectio 2da. *De Algebra in Particulari.*

Calut 1mum. *Prologomena Algebraica.*

Caput 2dum. *De Usu Algebrae in Geometria et Trigonometria plana.*

SECTIO 3tia. *De Geometria Sublimiori.*

Caput 1mum. *De Sectionibus Conicis.*

AP[P]ENDIX AD Geometriam practicam *In quo Deffinitio mensurae apponit* [...]

OPTICA [...] ***COSMOGRAPHIA*** [...] ***ARCHITECTURA MILITARIS*** [...]

This programme, which contains measuring the length, surface, and volume, with applications of plane trigonometry and using logarithms, was then standard at universities. The arithmetic and algebraic part of the programme contains arithmetic of integers and rationals, operations on irrational numbers (radical expressions), solving linear equations, systems of linear equations (2×2) and quadratic equations. Moreover, the text includes exercises leading to Diophantine equations of degree one. The solutions are given in parametric forms. Finally, the anonymous author formulates fundamental theorems of plane trigonometry in the form of algebraic equations and he solves standard exercises from analytical geometry. Thus the level of mathematical lectures at Vilnius University in year 1707/1708 was comparable with other European universties.

Arithmetic [1] for the students of Vilnius Academy is a typical textbook, containing the following chapters:

Pars Prima. *De Natura Arithmeticae et Numerorum Integrorum.*

Pars Secunda. *De Numeris Fractis.*

Pars Tertia. *De Progressionibus et Combinationibus Integrorum.*

Pars Quarta. *De Analogia et Proportione Numerorum.*

Pars Quinta. *De Algebra et Extractione Radicum. Algebra vel Analysis.*

Pars Sexta. *De Arithmetica Geometrica Astronomica et Politica.*

Pars Septima. *Proponit Arithmeticam Curiosam & nonulla exempla ad superiores partes.*

The author begins with integers, fractions, then he lectures progressions (arithmetical and geometrical), proportions (including *Regula de Tri*, etc.), algorithms for calculation arithmetical roots. Finally, he gives applications of arithmetic to geometry, astronomy and politics (i.e. elements of combinatorics). The last part of the book contains examples. The anonymous author uses symbols $=, <, >$. Suddenly the pages 164–168 are written in Polish and contain measure and monetary systems in Lithuania. The end of the book *Arithmetica calculatoria* explains how to use the abacus.

There is no essential difference between the texbooks [1] and [10]. The book [10] contains only much more information about decimal system and decimal expansions of the rationals. It is based on Latin books from the XVII century, presenting only the fundamental algorithms on examples, without proofs.

The book [11] is a free translation, and rightly speaking, a summary of CHRISTIAN WOLFF's work, made by NAKCYANOWICZ. The text was university textbook at Vilnius. We know two of its editions: from 1759 and 1761. In the XVII century, Latin was still the language of science, and perhaps for this reason NAKCYANOWICZ prepared the text in Latin. The text contains arithmetic, elements of geometry (planimetry and stereometry) and plane trigonometry. Arithmetic was written precisely and exactly, contains definitions and theorems, e.g. definitions of irrational numbers, primes, composed and relatively prime numbers. Properties of numbers are defined by nine axioms, putting together properties of equalities and inequalities between numbers, e.g.:

Axiom III: $A = B$ *implies* $A + C = B + C$.

Axiom IV: $A > B + C$ implies $A > B$ and $A > C$ [all numbers are positive]

Axiom V: $A > B$ gives $A + C > B + C$.

Axiom VIII: $A = B$ follows $AC = BC$.

The book contains logarithms, decimal and hexagonal fractions, as well as many algorithms (*Arithmetica calculatoria*). The absence of exercises is compensated by many numerical examples illustrating the introduced algorithms. Geometry is lectured exactly as in EUCLID's *Elements*, but some algebraical notations are used in the text. Then it was not too popular in textbooks. The range of mathematics in [11] is similar as in [8].

MARQUART's textbook [9] for high schools, very similar to [1], presents arithmetic in elementary way without any proof.

The first edition of the univeristy textbook [5] appeared in 1762. The text contains:

Pars Prima. Arithmetica. Pars Secunda. Elementa Geometriae. Tractatus Analyticus de Sectionibus Conicis. De Calculo Differentiali.

I shall not discuss beginning parts of the treatise [5]. It is simaliar to other described books. I only say few words on its last part *De Calculo Differentiali.* It is based on Newton's work [13]. The careful lecture uses differentials, introduces definite integral and gives many examples of applications of differential calculus and expansions of functions into power series. Following NEWTON the author presents approximate formulae for the roots from numbers of degree up to seven, based on the Newton's formula for $(1+x)^\alpha$ (exponent $\alpha > 0$) as the power series of x. The formula above is exactly Newton's *binomial theorem.* The author writes *that there are the following values:* $1.2.3.4.5.\ \ldots\infty$, and next he writes formulae like $\infty + 1 = \infty$, $1 + \frac{1}{\infty} = 1$, $\infty + \infty = 2\infty$, $\frac{2}{\infty}$ *divis:per* $\frac{\alpha}{\infty} = \frac{2}{\alpha}$. This was the state of mathematics of the XVIII century.

Programmes of lectures at Vilnius University, similarly as at Cracow Academy, can be found in *Prospectus Lectionum in Alma Universitatae et Academia Vilnensi* [...]. (see [18]).

In Prospectus Lectionum (1783/84; also 1988/89) we read, that

> *Tadeusz Kundzicz will lecture Mechanics* [...].
> *Franciszek Narwoysz is going to lecture Treatises of Higher Mathematics, three times a week* [the number of hours is not

> given there], *based mainly on Newton's work, such as Newton's Analytical Geometry* (i.e. [12]), *Newton's book about 'quadratura curvarum'* (i.e. [13]), *Newton's interpolation as well as general properties of geometrical lines by MacLaurin and the curves of degree three and higher, following Newton and Cramer. Finally he will lecture conic sections following the ideas of de la Caille.*

In *Prospectus Lectionum* (1786/87) we read:

> *Tadeusz Kundzicz will lecture Mechanics, Military Architecture and Pyrotechnic.* [. . .]
> *Franciszek Narwoysz is going to lecture Higher Mathematics:*
> *1. On methods of calculating of non-real square roots.* [. . .]
> *2. On transcendental curves.* [. . .]
> *3. On Logarithms and their applications to calculations.*
> *4. On applications of Algebra to Geometry.* [. . .]
> *5. Newton's method of 'Quadratura curvarum', with God's help.* [. . .]
> *6. Solutions of some equations.* [. . .]
> *7. Logistics of power series.* [. . .]
> *8. Selected problems from Newton's Letters* (i.e. from [12]).
> *9. Newton's Analytical Geometry and the first nine exercises.* [. . .]
> *10. Rectification of curves.* [. . .]
> *11. Interpolation methods.* [. . .]
> *12. General properties of [algebraical] curves and classification of curves of degree three.* [. . .]

In *Prospectus Lectionum* (1791/92) we read:

> *Priest Józef Mickiewicz will lecture Hydrostatics.* [. . .]
> *Tadeusz Kundzicz will lecture Statics and Machines.* [. . .]
> *Franciszek Milikont Narwoysz is going to lecture 'Arithmetica Universalis' ([12]). He will discuss the following problems:*
> *1. Eratosthenes method for determination of primes.* [. . .]
> *6. Newton's method of numerical extraction of roots from numbers.* [. . .]
> *9. Descartes' Rule of Signs for an algebraic equation.* [. . .]
> *15. Connection of the trisection of an angle with solutions of the equation of degree three.* [. . .]

> *18. Since direct reading and understanding of Diophantus 'Arithmetica' is difficult, he will apply results from L. Euler's and J. de la Grange's papers [probably Euler's Vollständige Anleitung zur Algebra (1770) and J. Lagrange's Additions aux Élments d'Algebre d'Euler].*
> *19. To obtain qucikly a possibility of applying Calculus, he will lecture Euler's 'Introductio ... '. [...]*
> *20. For students, who listened his last year lecture on curves of degree three, he will give some lectures on curves of degree two, to have the possibility to distinguish them. [...]*

Polish University at Vilnius was finally closed by Russian Government in the year 1832.

References

[1] Alpha Matheseos Arithmetica Theorica et Practica. in Usum et Captum. Tyronum Matheseos Academiae Vilnensis Soc: JESU. recens proposita. [...] VILNAE [1733]

[2] Michał Baliński, *Dawna Akademia Wileńska.* Próba jej historii od założenia w roku 1579 do ostatniego j j przekształcenia w roku 1803. Petersburg 1862. Nakładem i Drukiem Jozefata Ohryzki.

[3] Józef Bieliński. Stan nauk matematyczno-fizycznych za czasów Wszechnicy Wileńskiej. Szkic Bibliograficzny, *Prace Matematyczno-Fizyczne* **2** (1890), 265–432. [The state of physical and mathematical sciences in the times of Vilnius University].

[4] Compendiaria Matheseos Institutio, Quam in usum Auditorum Philosophiae Elucubratus est P.MAKO E S.I., WRATISLAVIAE, 1766.

[5] R.D. De La Caille, Lectiones Elementares Algebrae et Geometriae [...]. VILNAE, Anno 1773.

[6] Jadwiga Dianni. *Studium Matematyki na Uniwersytecie Jagiellońskim do połowy XIX wieku.* Kraków 1963. [School of Mathematics at the Jagiellonian University].

[7] ELEMENTA MATHESIS, AD GEOGRAPHIAM, STATICAM, MECHANICAM, HYDRAULICAM ADPLICATE, In usum Universitatis Vratislaviensis. Typis ejusdem Universitatis 1777.

[8] INSTITUTIONES GEOMETRIAE B.M., [manuscript of lectures at Vilnius Academy in 1707/1708, 246 pp.] Ossolineum, sygn. 6176/I.

[9] Józef Marquart. NAUKA MATEMATYCZNA W CZĘŚCIACH ARYTMETYKI I GEOMETRYI DLA POŻYTKU SZKOLNEY MŁODZI UCZĄCEY SIĘ W SZKOŁACH XIĘŻY SCHOLARUM PIARUM ZEBRANA, W WILNIE [...] Roku 1772.

[10] Mathematyki Polskiey Xięga Pierwsza. [...] ARYTHMETYKA Szkolna, Miernicza, y Cywilna: Tudčież MONETY Panstw Europeyskich [...] UŁOŻONE PRACĄ M.ANTONIEGO LENCZEWSKIEGO, Roku 1757. w WILNIE.

[11] Jakub Nakcyanowicz. *Praelectiones mathematicae ex Wolfianis elementis adornatae* [...]. Vilnae, Anno 1759.

[12] Isaac Newton. *Arithmetica universalis, sive de compositione et resolutione liber.* Londini 1707.

[13] Isaac Newton. *The Method of Fluxions and Infinite Series with its Applications to the Geometry of CURVE-LINES.* London 1736.

[14] Bernard Siruć. *Propositiones ex Analysi Infinitorum selectas publice demonstrat.* MDCCLV, Romae.

[15] Jan Śniadecki. *Rachunku algebraicznego Teorya Przystosowana do Geometryi Linii Krzywych* [...] TOM PIERWSZY Zawieraiący Algebrę na dwie części podzieloną. W Krakowie w Drukarni Szkoły Główny 1783.

[16] Jan Śniadecki. *Rachunku algebraicznego Teorya* [...] TOM II. w którym się przez zrównani nieoznaczon tłómaczą własności LINII i POWIERZCHNI KRZYWYCH. W Krakowie W Drukarni Szkoły Główny Koronny Roku 1783.

[17] *These Mathematicae ex Arithmetica, Algebra, Geometria plana ac solida, Geodaesia, Trigonometria, Cosmographia & Horographia,* Quas Perillustris Magnific: Domin: CAROLUS RAHOZA [...] Ann Dni MDCCLV, Vilnae Typis S.R.M. Academ: Societatis JESU.

[18] Universitas et Academia Vilnensis [...] Prospectus Lectionum in Alma Universitatae et Academia Vilnensi [...] 1781/82; Prospectus Lectionum [...] 1783/84; [...] 1786/87; [...]1788/89; [...] 1789/90; [...] 1790/91; [...] 1791/92; [...] 1792/93; [...] 1793/94; [...] 1798/99; [...] 1801/02.

[19] Witold Więsław. Reforma edukacji w Polsce w XVIII wieku. *Matematyka* **4** (1999), (278), pp. 195–203.

[20] Witold Więsław. Osiemnastowieczne rękopisy matematyczne w Bibliotece Kórnickiej. *Wiadomości Matematyczne* **XXXV** (1999), pp. 113–123.

[21] Witold Więsław. Wpływ przekładów dzieł matematyków francuskich na matematykę polską (In: *XII Szkoła Historii Matematyki.* Krynica 19.–26. 5. 1998, Kraków 1999), 105–124.

[22] Witold Więsław. Nauczanie matematyki w czasach Komisji Edukacji Narodowej (In: *Matematyka XVIII wieku.* XIII Ogólnopolska Szkoła Historii Matematyki, Kołobrzeg 17.–21. 5. 1999) (Uniwersytet Szczeciński, Szczecin 2001, 241–275.)

[23] Witold Więsław. Simon Lhuillier i jego podręczniki, *Matematyka* **1** (2000), 3–11.

[24] Witold Więsław. Osiemnastowieczne popisy z matematyki. *Matematyka* **6** (2000), 324–330.

[25] Wykłady Adama Stefana Jagielskiego i innych na Uniw. Jagiell. wysłuchane i spisane przez doktora filozofii [?], około 1781. (manuscript no 7084, Univ. Library Cracow)

[26] Zadania z Nauk Matematycznych [...], Od Szkoły Główney Podane, w Krakowie 1784/85; [...] 1786/87; [...] 1787/88; [...] 1789/90.

[27] Teofil Żebrawski. *Bibliografija Piśmiennictwa Polskiego z Działu Matematyki i Fizyki oraz ich Zastósowań. W Krakowie 1873. Dodatki do Bibliografii Piśmiennictwa Polskiego* [...]. W Krakowie 1886. [Reprint: IHN PAN 1992]

Witold Więsław
Instytut Matematyczny
Uniwersytetu Wrocławskiego
plac Grunwaldzki 2/4
50-384 Wrocław
Poland
e-mail: wieslaw@math.uni.wroc.pl