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Kybernetika, Vol. 61 (2025), No. 1, 109–132

Persistent URL: `http://dml.cz/dmlcz/152927`

KYBERNETIKA — VOLUME *61* (2025), NUMBER 1, PAGES 109–132

EFFICIENCY ANALYSIS OF THE RULE-BASED DEFUZZIFICATION APPROACH TO FUZZY INFERENCE SYSTEM FOR REGRESSION PROBLEMS

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A fuzzy inference system (FIS) is an effective prediction method based on fuzzy logic. The performance of this model may vary depending on the defuzzification process. In the Mamdani-type FIS model, the defuzzification process is applied to the fuzzy output of the system only once at the last stage. In the FIS with rule-based defuzzification (FIS-RBD) model, the defuzzification process is applied to the fuzzy consequent part of each rule and the overall result of the system is calculated as the weighted average of the separately defuzzified results of the rules. Note that, the original shapes of the combined rule results are lost in the aggregated fuzzy result of the classical Mamdani-type system and the effect of each rule on the system result decreases when aggregated. However, rule results can affect the overall result more significantly in the FIS-RBD approach.

In this study, a comparative analysis was made on the effectiveness of the classical Mamdani-type FIS and FIS-RBD models for regression problems. Five datasets from different domains and various defuzzification methods were used in comparisons. In the results obtained, it was observed that the The FIS-RBD model gave better results than the classical Mamdani-type FIS model. To carry out calculation experiments, a new Python package called Fuzlab was developed by modifying the existing Python library called FuzzyLab. In addition to creating the FIS-RBD model, the developed package also allows the use of the Weighted Average Based on Levels (WABL) defuzzification method in fuzzy logic-based calculations.

Keywords: fuzzy inference system (FIS), defuzzification, rule-based defuzzification (RBD), regression, Python library

Classification: 93C42, 68T05, 68N30

1. INTRODUCTION

In our daily lives, we constantly must make predictions about various issues, and the accuracy of our predictions affects our quality of life. While making these predictions, we evaluate a lot of uncertainty and make plans. For this reason, in recent years, researchers have developed different prediction and planning algorithms in various fields [5, 14, 20, 26, 50, 54, 59, 61]. Modeling deterministic uncertainties with fuzzy logic and fuzzy sets gives realistic results. Fuzzy logic theory is important in many areas because it enables

DOI: 10.14736/kyb-2025-1-0109

reasoning and decision-making in situations where there is uncertainty, ambiguity, or incomplete information. Unlike classical binary logic, which is based on absolute values of true or false, fuzzy logic handles varying degrees of truth or membership, making it is useful in complex and real-world applications. Today, fuzzy logic theory is used in various areas such as decision-making, inference, recommendation systems, image segmentation, supply chain, path planning, etc. [10, 13, 19, 27, 53, 63].

Regression models are another important prediction and inference model among the artificial intelligence approaches. Regression models can be divided into two large groups: data-driven and rule-based models. Examples of successful data-driven models include random forest and boosting regression models. There are various developments and applications of these models in the current literature [4, 15, 17, 18, 22, 23, 34, 55, 56, 60]. Regression models can also be grouped in terms of containing fuzzy logic. Various regression models that contain fuzzy information are available in the literature [12, 41, 42, 49, 62]. There are also various types of these models according to the fuzziness of the input and/or output variables. Detailed analysis of various fuzzy regression models is given in [45]. In systems containing fuzzy information, giving linguistic variables facilitates the creation of the model and offers the opportunity to expand the solution space [40, 44, 46, 51, 52]. In the study [46], an optimization model for the evaluation of student performances using decision-maker opinions as fuzzy linguistic terms is proposed and solved. In the paper [52], a fuzzy classification algorithm on linguistic data as fuzzy numbers is developed. The Weighted Averaging Based on Levels (WABL) method is used as the defuzzification method.

In addition to data-driven regression models, rule-based regression models also provide successful application results. In rule-based models, fuzzy inference systems (FIS) containing fuzzy logic are especially widely used [24, 25, 30, 31]. In the study [30], a fuzzy inference system is constructed to diagnose metabolic syndrome (MetS). MetS diagnostic criteria are used as a reference to establish the rules. In the study [31], a multiple linear regression model was created using Sugeno's fuzzy inference system approach. An alternative model for estimating linear regression with quite a few independent variables but not too many data sets is presented. The proposed FIS model was used for the prediction of serum iron, and Fuzzy C-Means (FCM) clustering was used in the study to produce fuzzy sets and fuzzy rules.

In the study by Ansarifar et al. [3] an interaction regression model for crop yield prediction is presented. In the study by Zhu et al. [62], fuzzy rule-based regression models based on decision trees were designed and implemented. A two-stage design of the rule-based model is proposed to provide an alternative for dealing with high-dimensional data. In studies [16, 37], collision risk prediction and avoidance models are constructed based on fuzzy logic rules. Yazid et al. [58] proposed a position control of a quadcopter drone using a first-order Sugeno-type fuzzy inference system.

In the study [9], the fuzzy regression functions approach was proposed to overcome the difficulty of creating rules in the fuzzy inference system. Parameter estimates of the regression functions were obtained by robust regression. In the study [8], the Type-1 fuzzy regression functions approach was used instead of fuzzy rules. The Gustafson-Kessel clustering algorithm was used to calculate the membership degrees of the input values. The study [29] discussed the fuzzy automatic control process in the municipal solid waste pyrolysis process (MSW) with variable composition and moisture content.

With the developed fuzzy control method, it was possible to determine the optimal ratio of air/MSW for various waste types and to realize appropriate automatic control of a pyrolysis plant with different moisture content values to ensure high temperature. In the study [57], rock brittleness was estimated using a fuzzy inference system and nonlinear regression analysis models. It has been observed that the prediction performance of the non-linear multiple regression model is higher than the fuzzy inference system model. However, according to the prediction values obtained, it was concluded that both models exhibited high performance. In the study [1], a fuzzy inference system was created to predict the risk level of COVID-19 in diabetic patients. Eight input parameters, which were found to be the most effective symptoms in diabetic patients, were taken.

In the article [49], fuzzy regression approaches that use and do not use the clustering method to estimate production income is examined and compared. Li et al. [32] proposed a grammatical evolution-based fuzzy regression approach to eliminate the non-linearity and fuzziness of holiday load behaviors. The proposed hybrid approach is based on the theorem that fuzzy polynomial regression can model all fuzzy functions. In studies [6, 7], a fuzzy logic controller was built with the Robot Operating System (ROS) for autonomous navigation of the TurtleBot3 robot in a simulated and real environment. For this purpose, an open-access Python FuzzyLab library was developed and used.

Nasiboglu [43] recently proposed a new Mamdani-type FIS with the rule-based defuzzification (FIS-RBD) model. The main feature of this model is that instead of applying the defuzzification operator to the system's overall output in Mamdani-type fuzzy inference models, the defuzzification operator is applied to the result of each rule. Subsequently, the defuzzification of each rule is combined with the weighted average method based on the firing degrees of the rules.

However, the study [43] did not investigate a comprehensive analysis of FIS-RBD and its difference from the classical Mamdani-type FIS on large datasets. In this study,

- we proposed different defuzzification operators for the Mamdani-type FIS-RBD model,
- we performed a comprehensive analysis by comparing the FIS-RBD model with the Mamdani-type FIS model on 5 different data sets from various domains,
- we created a new Python library called FuzLab by developing the FuzzyLab library [6, 7], which is an open-source Python Library.

The rest of the paper is organized as follows. In the second section, the preliminaries used in model-building and rule-building approaches are examined. In section 3, the FIS-RBD model that forms the basis of the study is reviewed. Section 4 provides information about the computational experiments on the data sets and software used to compare the models. The results of the computational experiments and comparative analysis of the results are given in Section 5. Finally, in the conclusion section, the general evaluation and conclusion of the study are included.

2. PRELIMINARIES

2.1. Defuzzification methods

In the fuzzy inference system, fuzzy numbers are used to create the inputs and outputs of the rules. Among the fuzzy numbers used for this purpose, triangular and trapezoidal are the most commonly used parametric fuzzy numbers. The membership function of a triangular fuzzy number is as follows:

Definition 1. Triangular fuzzy number $A = (a, b, c)$ is a a fuzzy number whose membership function is as follows:

$$A(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x < b, \\ \frac{c-x}{c-b}, & b \le x \le c, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Here, b is the peak point of the fuzzy number, a and c are the left and right boundaries, respectively.

Definition 2. Trapezoidal fuzzy number $A = (a, b, c, d)$ is a fuzzy number whose membership function is as follows:

$$A(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x < b, \\ 1, & x \in [b, c], \\ \frac{d-x}{d-c}, & c \le x \le d, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

If we denote the membership function of any fuzzy number A with $A(x)$, then its COG (Centroid of Gravity) defuzzified value can be calculated as follows:

$$COG(A) = \frac{\int_{-\infty}^{\infty} xA(x)\, \mathrm{d}x}{\int_{-\infty}^{\infty} A(x)\, \mathrm{d}x} \tag{3}$$

Defuzzified values of fuzzy numbers such as the Mean of Maxima (MOM), the The smallest of Maxima (SOM) and the Largest of maxima (LOM) are also frequently used ones:

$$MOM(A) = \mathrm{mean}\{x : A(x) = 1\} \tag{4}$$

$$SOM(A) = \min\{x : A(x) = 1\} \tag{5}$$

$$LOM(A) = \max\{x : A(x) = 1\} \tag{6}$$

As can be seen, the defuzzification methods mentioned above are defined based on values of x. On the other hand, there are also defuzzification methods based on the membership values of the fuzzy number. The most universal of these is the WABL method. The WABL method is an adjustable method and can be adjusted according to the decision maker's strategy [35, 42, 47]. By adjusting the parameters appropriately in the WABL method, results such as COG, MOM, etc. can also be produced [47].

The WABL method can be formally defined by using the LR-representation of a fuzzy number $A = (L_A, R_A)$ as follows. The left side and the right side functions are

$L_A : [0,1] \to (-\infty, \infty)$ and $R_A : [0,1] \to (-\infty, \infty)$, respectively. Then the weighted average based on the levels' value of the fuzzy number A is calculated as in eq.(7):

$$\mathrm{WABL}\,(A) = \int_0^1 \left((1-c)\,L_A\,(t) + cR_A\,(t)\right) p\,(t)\;\mathrm{d}t \tag{7}$$

The parameter $c \in [0,1]$ in eq.(7) indicates the importance of the maximum value on the level set (the optimism index). The importance of the level sets is reflected by a function $p : [0,1] \to (-\infty, \infty)$, satisfying the following conditions:

$$\int_0^1 p\,(t)\;\mathrm{d}t = 1, \quad p\,(t) \geq 0 \tag{8}$$

Definition 3. The height h of a fuzzy number A is defined by the following formula:

$$h = \mathrm{height}\,(A) = \sup\{A(x) |\; x \in (-\infty, \infty)\} \tag{9}$$

A fuzzy number with a height equal to 1 is called a normal fuzzy number, otherwise, it is called a subnormal fuzzy number.

Usually, eq. (7) is used to calculate the WABL value of a normal fuzzy number. However, FIS results can often be in the form of subnormal fuzzy numbers. In these cases, the following eq. (10), which is a more general formula, can be used to calculate the WABL value:

$$\mathrm{WABL}\,(A) = \frac{\int_0^h \left((1-c)L_A\,(t) + cR_A\,(t)\right) p(t)\,\mathrm{d}t}{\int_0^h p(t)\,\mathrm{d}t} \tag{10}$$

The h value in eq. (10) is the height of the fuzzy number A.

When the levels of the fuzzy number are not continuous but discretely divided into levels $\{t_i\},\; i = 0, \ldots, n$ formula (10) can be applied as follows:

$$\mathrm{WABL}\,(A) = \frac{\sum\limits_{i=0}^{n} \left((1-c)L_A\,(t_i) + cR_A\,(t_i)\right) p(t_i)}{\sum\limits_{i=0}^{n} p(t_i)} \quad , \tag{11}$$

where $p(t_i)$ can be any positive value reflecting the weight of the level $t_i \in [0,1]$.

2.2. Construction of the rules

In this study, fuzzy inference systems with two inputs and one output have been designed for comparison purposes. To create fuzzy rules, each input variable is divided into 3 clusters. For this, the k-means clustering algorithm was used. In total, the input space is divided into a $3 \cdot 3 = 9$ grid. By calculating the average of the output (or target) variable for each cell of this grid, triangular fuzzy numbers in the form of $A = (a, b, c)$ were created (Figure 1). The centers of the clusters were considered as the centers of the relevant fuzzy numbers. Then, the center values were sorted by one of the sorting algorithms [2], and the distance between the mean values of the neighboring clusters after sorting was taken as the width of the fuzzy number. The detailed rules' construction process is given in the Algorithm 1.

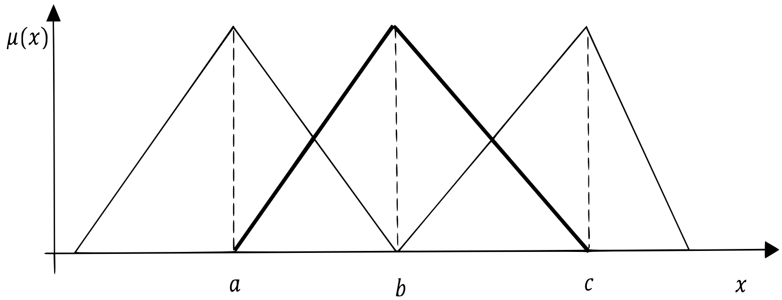

Fig. 1. Creating fuzzy numbers according to neighbor cluster centers.

Algorithm 1.

Step 1. The values of each input variable x and y are divided into 3 clusters separately using the k-means clustering algorithm. Let the clusters be $\{X_i\}, i = 1, 2, 3$, and $\{Y_j\}$, $j = 1, 2, 3$, with cluster centers x_i, $i = 1, 2, 3$, and y_j, $j = 1, 2, 3$, respectively.

Step 2. For each input variable, the cluster centers are listed in ascending order separately. Let them be:
Cluster centers for input X: $x_1 \leq x_2 \leq x_3$
Cluster centers for input Y: $y_1 \leq y_2 \leq y_3$

Step 3. Fuzzy numbers $\tilde{X}_i$ and $\tilde{Y}_i$ for $i = 1, 2, 3$ are created using the centers x_1, x_2, x_3 and y_1, y_2, y_3, respectively, as follows:
$\tilde{X}_1 = (a_1, b_1, c_1, d_1)$, where $a_1 = x_{\min},\ b_1 = x_{\min},\ c_1 = x_1,\ \ d_1 = x_2$,
$\tilde{X}_2 = (a_2, b_2, c_2)$, where $a_2 = x_1,\ b_2 = x_2,\ c_2 = x_3$,
$\tilde{X}_3 = (a_3, b_3, c_3, d_3)$, where $a_3 = x_2,\ b_3 = x_3,\ c_3 = x_{\max},\ \ d_3 = x_{\max}$
Similarly,
$\tilde{Y}_1 = (a_1, b_1, c_1, d_1)$, where $a_1 = y_{\min},\ b_1 = y_{\min},\ c_1 = y_1,\ \ d_1 = y_2$,
$\tilde{Y}_2 = (a_2, b_2, c_2)$, where $a_2 = y_1,\ b_2 = y_2,\ c_2 = y_3$,
$\tilde{Y}_3 = (a_3, b_{3,} c_3, d_3)$, where $a_3 = y_2,\ b_3 = y_3,\ c_3 = y_{\max},\ \ d_3 = y_{\max}$.
Here, the $x_{\min}$ and $x_{\max}$, are the minimum and the maximum values of the variable x. The notations $y_{\min}, y_{\max}, z_{\min}, z_{\max}$ also have similar meanings.

Step 4. For each $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3\}$, the set Z_{ij} of output values corresponding to the input variables X_i and Y_j is created as follows:
$Z_{ij} = \{z_l |\, (x_l, y_l, z_l)$ is a data of the dataset such that $x_l \in X_i$ and $y_l \in Y_j;\ 1 \leq l \leq N\}$,
where N is the total number of data in the dataset.
Then the average values of Z_{ij} is calculated for each $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3\}$:

$$z_{ij} = \text{mean}\ Z_{ij},$$

Step 5. The values of z_{ij}, are sorted in ascending order as follows:

$$z_1 \leq z_2 \leq \cdots \leq z_9$$

Moreover, we record which z_{ij} corresponds to which z_t, where $t \in \{1, 2, 3, \ldots, 9\}$, $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3\}$. In other words, we make a one-to-one mapping ν that for a given pair (i, j) determines the corresponding new index t.

Step 6. The fuzzy numbers $\tilde{Z}_t$ are constructed according to the sorted values of z_t, where $t \in \{1, 2, 3, \ldots, 9\}$. The first and the last fuzzy numbers $\tilde{Z}_t = (a_t, b_t, c_t, d_t)$ are trapezoidal ones as follows:
$a_1 = z_{\min}$, $b_1 = z_{\min}$, $c_1 = z_1$, $d_1 = z_2$
and
$a_9 = z_8$, $b_9 = z_9$, $c_9 = z_{\max}$, $d_9 = z_{\max}$.

The fuzzy numbers in between are created as triangular fuzzy numbers $\tilde{Z}_t = (a_t, b_t, c_t)$, where
$a_t = z_{t-1}$, $b_t = z_t$, $c_t = z_{t+1}$ for $1 < t < 9$.

Step 7. 9 rules are created as follows:
if $x\,is\,\tilde{X}_i$ **and** $y\,is\,\tilde{Y}_j$ **then** $z\,is\,\tilde{Z}_t$, where $t = \nu(i, j)$.

3. FUZZY INFERENCE SYSTEM WITH RULE-BASED DEFUZZIFICATION

The fuzzy inference system is one of the models used successfully in regression and classification problems. Fuzzy inference systems have two basic forms: Mamdani-type and Sugeno-type systems. In Mamdani-type fuzzy inference systems, the outputs are like fuzzy sets. The output of each rule is "truncated" by subjecting it to the "and" operation with the firing level of that rule. Then, the "truncated" results of all rules are aggregated to create the overall fuzzy output of the system as shown in Figure 2. Finally, this general fuzzy output of the system is defuzzified and converted into a precise value. The general scheme of the classical Mamdani-type fuzzy inference system is given in Figure 3.

In the study by Nasiboglu [43], a new type of Fuzzy Inference system, Mamdani-type FIS with Rule-Based Defuzzification was proposed. In the proposed FIS-RBD model, the output of each rule is defuzzified in situ and converted into exact values, and the overall output of the system is calculated as the weighted average of these values. The weights are treated as equal to the firing degrees of the rules. A rule-based detailed working example of the FIS-RBD system is given in Figure 4. In this respect, the FIS-RBD model can be seen as a hybrid of the classical Mamdani-type FIS and Sugeno-type FIS models. The general scheme of the FIS-RBD model is given in Figure 5 [43].

In the classical Mamdani-type FIS model, since the overall output of the system is an aggregated fuzzy number, the effect of the rule results in separation affects the overall result less. However, in the Mamdani-type FIS-RBD model, since the output of each rule is defuzzified before aggregation, it affects the overall result more. The amount of this effect may vary depending on the defuzzification method. There are different defuzzification methods in the literature [11, 21, 33]. Among these methods, studies

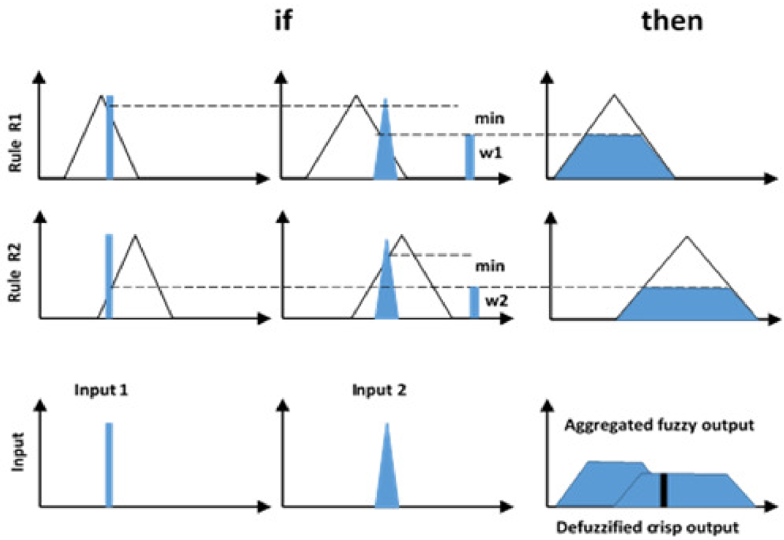

Fig. 2. Working principle of the rules in classical Mamdani-type FIS [24].

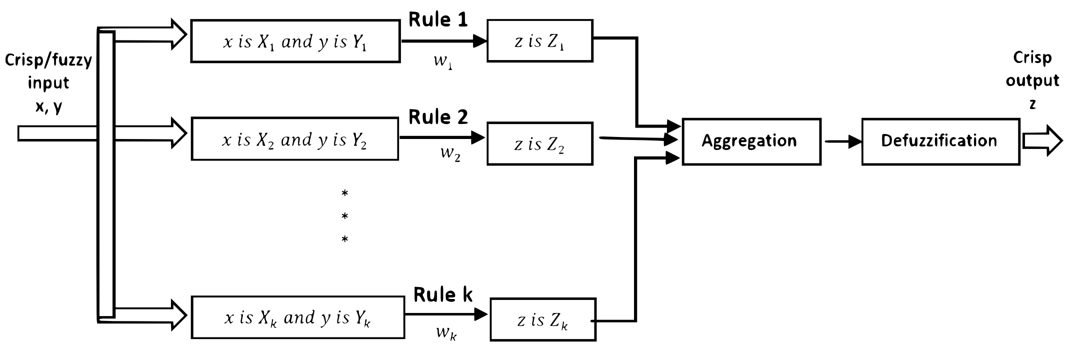

Fig. 3. General structure of the classic Mamdani-type FIS [24].

are showing that the adjustable WABL method gives more effective results [36, 38, 42, 48]. In this respect, it can be concluded that the adjustability feature of the WABL defuzzification method is more advantageous than other defuzzification methods also in the FIS-RBD model.

In both the classical Mamdani-type and FIS-RBD models, calculating the firing degrees of the rules by the values of the input variables is one of the most important stages of the system. The firing degree of any ith rule is calculated as given below:

$$w_i = \min\{\mu\,(x\,is\,X)\,,\mu\,(y\,is\,Y)\} \quad i = 1, \ldots, k \tag{12}$$

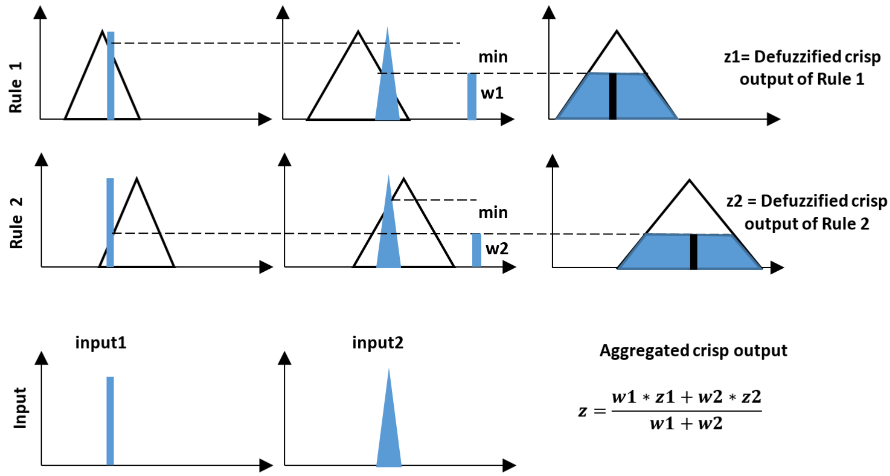

Fig. 4. An example of the FIS-RBD with detailed rule-based defuzzification.

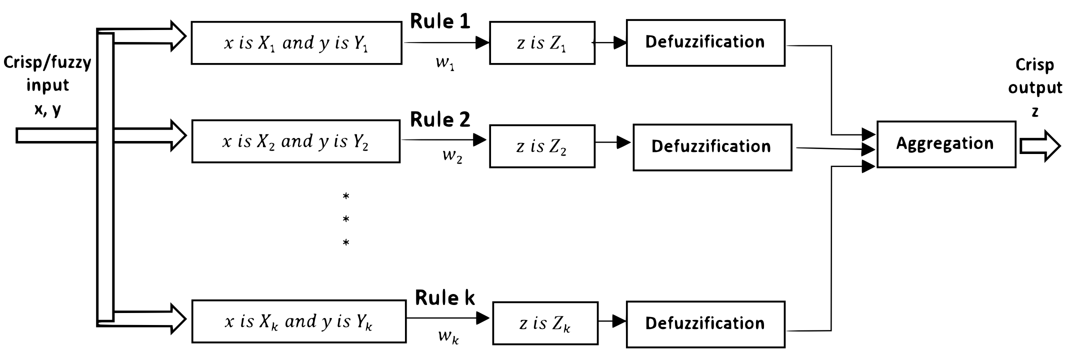

Fig. 5. FIS with rule-based defuzzification (FIS-RBD).

Consequently, the fuzzy output of the ith rule truncated by firing degree can be calculated as given below:

$$\tilde{Z}_i\,(z) = \min\{w_i, Z_i\,(z)\}, \quad i = 1, \ldots, k \tag{13}$$

The truncated fuzzy output of the ith rule is defuzzified using any defuzzification function such as WABL, COG, MOM, etc. instead of the following $Defuzzy(.)$ function:

$$z_i = \mathrm{Defuzzy}(\tilde{Z}_i\,(z)), \quad i = 1, \ldots, k \tag{14}$$

Finally, the overall crisp output of the fuzzy inference system can be calculated as the weighted averaging given below:

$$z = \frac{\sum_{i=1}^{k} w_i z_i}{\sum_{i=1}^{k} w_i} \quad . \tag{15}$$

4. COMPUTATIONAL EXPERIMENTS

The open source *FuzzyLab* library in Python was used to perform the calculation experiments [6, 7][1]. A new library called *Fuzlab* was created by adding the new Mamdani-type FIS-RBD model and the WABL defuzzification method to the basic *FuzzyLab* library in the Git repository. A new software code based on the formula (11) has been added to the *Fuzlab* library to calculate WABL values of the fuzzy numbers. The source code of the *Fuzlab* library can be accessed on the GitHub platform[2].

A new class called *mamfisRBD*, which can create the FIS-RBD model, has been added to the newly developed *Fuzlab* library. To create this FIS-RBD model, a command such as the following must be used:

\>>>fis = mamfisRBD()

In the *Fuzlab* library, the WABL defuzzification method is calculated based on the formula (11) and the following commands can be used as examples to set the WABL parameters:

\>>>fis.DefuzzificationMethod = 'wabl'
\>>>fis.WablOptimism = 0.5
\>>>fis.WablDegrees = 21
\>>>fis.WablImportances = np.ones(fis.WablDegrees)

Here *fis.WablOptimism* parameter is the c-optimism index used in the WABL method, *fis.WablDegrees* is the number of discrete levels of the fuzzy number, and *fis.WablImportances* is the level weight coefficients list (discrete function $[p(i), i = 0, \ldots, fisWablDegrees{-}1]$) used in formula (11). More detailed information about the use of the *mamfisRBD()* model can be found on `https://github.com/enasibov/FIS-RBD-model/tree/main`

Using the *Fuzlab* library, prediction models were built on five different datasets [28, 39, 41, 42]. The datasets used in the computational experiments are as follows: "California housing dataset" (housing.csv), "Second-hand car prices" (cars2.csv), "Auto-Mpg Data" (auto-mpg.csv), "Diabetes" (diabetes.csv) and "Diamonds" (diamonds.csv). Detailed information about the data sets used is given below.

Housing dataset details:
`https://github.com/enasibov/FIS-RBD-model/blob/main/Data/housing.csv`;
used dataset shape: (20640, 3); used train data size: 18576; input variables: Latitude, Longitude; predicted value: Median house value.

Cars2 dataset details:
`https://github.com/enasibov/FIS-RBD-model/blob/main/Data/cars2.csv`;
used dataset shape: (571, 3); used train data size: 513; input variables: Year, Km; predicted value: Price.

[1] `https://github.com/ITTcs/fuzzylab`
[2] `https://github.com/enasibov/FIS-RBD-model/tree/main`

Mpg dataset details:
`https://github.com/enasibov/FIS-RBD-model/blob/main/Data/auto-mpg.csv`; used dataset shape: (392, 3); used train data size: 352; input variables: Horsepower, Weight; Predicted value: Mpg.

Diabetes dataset details:
`https://github.com/enasibov/FIS-RBD-model/blob/main/Data/diabetes.csv`; used dataset shape: (768, 3); used train data size: 691; input variables: Glucose, Insulin; predicted value: DiabetesPedigreeFunction.

Diamonds dataset details:
`https://github.com/enasibov/FIS-RBD-model/blob/main/Data/diamonds.csv`; used dataset shape: (53940, 3); used train data size: 48546; input variables: depth, table; predicted value: price.

Note that the train-test split parameters for all datasets were adjusted as follows: test_size=0.1, random_state=0. Furthermore, k-means algorithm's parameters used to create rules for all datasets were adjusted as: n_clusters = 3, random_state = 0, n_init=3.

5. RESULTS AND DISCUSSION

For each data set, calculations were repeated in 10 tests. In each of the tests, predictions were made for 100 randomly selected with replacement sampling and the average root mean square error (RMSE) values calculated in accordance with eq. (16) were recorded.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{m} \left(z_{pred,i} - z_{act,i}\right)^2}{m}} \tag{16}$$

In this formula, z_{pred} is the value predicted by the model for a certain input, and z_{act} is the actual value corresponding to that input in the dataset. The averages of the RMSE values of the $m = 100$ samples used in each test are given in Tables 1 – 5 for each dataset and different defuzzification methods. COG, MOM, and WABL methods were used as defuzzification methods. In the WABL defuzzification method, the value of the optimism parameter is adjusted for the best result and the following uniform weighting function was used for discretized levels $i = 0, \ldots, n$:

$$p\,(i) = 1. \tag{17}$$

Here n is the number of discrete levels into which the interval [0,1] is divided. In the calculations, $n = 21$ were used.

In Tables 1 – 5, the cases with lower average RMSE values in the comparisons based on each defuzzification method are highlighted in bold. The results obtained for different values of the optimism parameter in the WABL method are also given in Tables 6 – 10. In the tables, level weights are determined uniformly as in equation 17. RMSE values are high in some datasets in the tables due to the large numbers in these datasets. For this, normalization of the data could have been done or other metrics could have been used. However, since our aim was only to compare two models, it was not necessary to use normalization or other metrics. It was enough for us that the RMSE values of one model were lower than the other.

Test No.	COG		MOM		WABL adjusted (c=0.5)	
	Mamdani FIS	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**
1	102973	**100237**	**97477**	100505	**98458**	99606
2	128164	**125261**	126689	**124886**	124832	**124587**
3	106203	**103076**	**100450**	102931	**101144**	101539
4	97123	**94552**	95590	**93442**	**93674**	93778
5	120206	**116744**	**112066**	116422	**114831**	115429
6	124910	**120496**	**115978**	121397	**118457**	118648
7	93591	**92929**	98341	**93122**	**93519**	93733
8	94205	**91533**	**87957**	91441	**91039**	92204
9	109944	**106504**	**104233**	105714	**104861**	105328
10	110269	**108300**	**103743**	107957	**106867**	108140
Average	108759	**105963**	**104252**	105782	**104768**	105299

Tab. 1. RMSE results for Housing dataset.

Test No.	COG		MOM		WABL adjusted (c=0.0)	
	Mamdani FIS	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**
1	259310	**172780**	205484	**175166**	22954	**21720**
2	271102	**174192**	231048	**176449**	21592	**19932**
3	303755	**216096**	256389	**219046**	22146	**21154**
4	260831	**182618**	216131	**185277**	22683	**21272**
5	258335	**159163**	199926	**161310**	20592	**19948**
6	286130	**201051**	223645	**204070**	24532	**23671**
7	306157	**205068**	254090	**207702**	19403	**19059**
8	278978	**192180**	216999	**194724**	18265	**16399**
9	252777	**164508**	194365	**166515**	18667	**17440**
10	278090	**209456**	248755	**212645**	24298	**22695**
Average	275547	**187711**	224683	**190290**	21513	**20329**

Tab. 2. RMSE results for Cars2 dataset.

Test No.	**COG**		**MOM**		**WABL adjusted (c=0.25)**	
	Mamdani FIS	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**
1	5.52	**4.36**	4.98	**4.01**	4.76	**4.68**
2	6.12	**5.17**	5.36	**4.33**	4.72	**4.64**
3	6.41	**5.18**	5.55	**4.26**	4.73	**4.63**
4	5.66	**4.78**	5.26	**4.02**	4.51	**4.45**
5	5.44	**4.55**	4.82	**3.74**	4.26	**4.23**
6	6.06	**4.78**	5.30	**4.09**	4.56	**4.54**
7	5.79	**4.50**	5.15	**4.09**	**4.55**	4.57
8	5.67	**4.48**	5.22	**3.77**	3.97	**3.82**
9	6.11	**4.88**	5.32	**4.25**	4.68	**4.55**
10	6.42	**4.91**	5.35	**3.95**	4.26	**4.10**
Average	5.92	**4.76**	5.23	**4.05**	4.50	**4.42**

Tab. 3. RMSE results for Mpg dataset.

Test No.	**COG**		**MOM**		**WABL adjusted (c=0.0)**	
	Mamdani FIS	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**
1	0.76	**0.45**	0.66	**0.45**	0.42	**0.34**
2	0.73	**0.51**	0.65	**0.51**	0.50	**0.46**
3	0.79	**0.49**	0.71	**0.50**	0.35	**0.28**
4	0.74	**0.46**	0.57	**0.46**	0.48	**0.44**
5	0.74	**0.48**	0.68	**0.49**	0.36	**0.31**
6	0.74	**0.46**	0.64	**0.46**	0.38	**0.34**
7	0.74	**0.46**	0.58	**0.47**	0.43	**0.38**
8	0.76	**0.49**	0.68	**0.49**	0.41	**0.38**
9	0.71	**0.39**	0.59	**0.40**	0.32	**0.26**
10	0.80	**0.48**	0.61	**0.49**	0.39	**0.34**
Average	0.76	**0.47**	0.64	**0.47**	0.40	**0.35**

Tab. 4. RMSE results for Diabetes dataset.

Test No.	COG		MOM		WABL adjusted (c=0.0)	
	Mamdani FIS	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**	**Mamdani FIS**	**FIS-RBD**
1	7784.23	**5385.04**	**5124.14**	5378.57	6240	**5651**
2	6193.07	**4820.39**	4837.70	**4830.24**	**5027**	5191
3	6512.54	**4477.68**	**4350.48**	4480.54	4637	**5026**
4	6934.11	**5350.88**	**5284.30**	5357.23	**5599**	5662
5	6495.23	**5420.25**	5462.38	**5420.67**	5724	**5635**
6	7437.15	**5381.17**	5574.16	**5381.40**	5954	**5578**
7	6903.50	**5055.97**	5680.32	**5059.10**	5960	**5340**
8	7932.35	**5529.37**	**5204.37**	5525.41	6200	**6008**
9	8491.71	**5563.00**	5662.67	**5562.25**	6442	**5902**
10	8342.55	**6020.04**	6594.76	**6023.65**	6684	**6296**
Average	7302.64	**5300.38**	5377.53	**5301.91**	5847	**5629**

Tab. 5. RMSE results for Diamonds dataset.

Test	C=0.0		C=0.25		C=0.5		C=0.75		C=1.0	
	FIS	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**
1	113528	110673	103208	103302	98458	99606	100075	99995	107773	104421
2	131031	131316	125811	126364	124832	124587	128191	126118	135565	130842
3	117361	114153	106393	106070	101144	101539	102496	101041	110205	104632
4	105064	102239	96054	95793	93674	93778	98405	96470	109328	103503
5	128916	127853	119727	120099	114831	115429	114777	114223	119573	116587
6	135964	133407	124897	124399	118457	118648	117409	116637	121891	118557
7	100446	96805	93249	93101	93519	93733	101196	98618	114803	107176
8	102872	97678	93151	92698	91039	92204	97035	96265	109817	104350
9	116134	114399	107736	108013	104861	105328	107950	106624	116530	111761
10	127931	119747	114088	112212	106867	108140	107609	107926	116161	111590
Ave.	117925	114827	108431	108205	104768	105299	107514	106392	116165	111342

Tab. 6. RMSE results using WABL defuzzification for Housing dataset.

Test	C=0.0		C=0.25		C=0.5		C=0.75		C=1.0	
	FIS	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**
1	22954	21720	101206	87753	211457	185754	322399	284400	433503	383193
2	21592	19932	112519	89570	233536	189741	355107	290403	476810	391177
3	22146	21154	129293	110438	265975	229678	403199	349448	540552	469342
4	22683	21272	107269	90182	227948	194807	349106	299839	470374	404961
5	20592	19948	100633	82232	210936	175406	321752	269098	432688	362907
6	24532	23671	111198	99172	235723	213530	360836	328391	486083	443363
7	19403	19059	126666	104385	264173	221329	402002	338577	539907	455894
8	18265	16399	107805	98314	223086	204863	338778	311735	454566	418682
9	18667	17440	100206	84731	209783	179427	319768	274496	429848	369650
10	24298	22695	116550	101873	248186	220296	380300	339076	512521	457934
Ave.	21513	20329	111335	94865	233080	201483	355325	308546	477685	415710

Tab. 7. RMSE results using WABL defuzzification for Cars2 dataset.

Test	C=0.0		C=0.25		C=0.5		C=0.75		C=1.0	
	FIS	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**
1	7.16	**6.83**	4.76	**4.68**	4.85	**4.24**	7.34	**5.90**	10.66	**8.51**
2	6.45	**6.34**	4.72	**4.64**	5.58	**4.90**	8.27	**6.90**	11.57	**9.62**
3	6.98	**6.51**	4.73	**4.63**	5.72	**4.88**	8.94	**7.05**	12.79	**9.96**
4	6.57	**6.41**	4.51	**4.45**	5.19	**4.52**	7.95	**6.56**	11.36	**9.36**
5	6.50	**6.38**	4.26	**4.23**	4.76	**4.22**	7.48	**6.36**	10.86	**9.27**
6	6.96	**6.47**	4.56	**4.54**	5.29	**4.60**	8.38	**6.59**	12.13	**9.36**
7	7.08	**6.75**	**4.55**	4.57	5.07	**4.35**	8.08	**6.29**	11.81	**9.11**
8	5.89	**5.58**	3.97	**3.82**	5.03	**4.28**	7.96	**6.52**	11.40	**9.32**
9	6.38	**6.27**	4.68	**4.55**	5.57	**4.77**	8.25	**6.73**	11.53	**9.41**
10	6.36	**5.91**	4.26	**4.10**	5.52	**4.59**	8.78	**6.92**	12.58	**9.85**
Ave.	6.63	**6.35**	4.50	**4.42**	5.26	**4.54**	8.14	**6.58**	11.67	**9.38**

Tab. 8. RMSE results using WABL defuzzification for Mpg dataset.

Test	C=0.0		C=0.25		C=0.5		C=0.75		C=1.0	
	FIS	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**
1	0.42	**0.34**	0.43	**0.36**	0.67	**0.49**	0.98	**0.67**	1.31	**0.87**
2	0.50	**0.46**	0.48	**0.45**	0.64	**0.54**	0.88	**0.68**	1.15	**0.85**
3	0.35	**0.28**	0.43	**0.35**	0.71	**0.54**	1.04	**0.77**	1.39	**1.02**
4	0.48	**0.44**	0.46	**0.42**	0.61	**0.49**	0.85	**0.62**	1.13	**0.79**
5	0.36	**0.31**	0.41	**0.36**	0.66	**0.53**	0.97	**0.75**	1.29	**0.98**
6	0.38	**0.34**	0.41	**0.35**	0.64	**0.49**	0.94	**0.69**	1.26	**0.90**
7	0.43	**0.38**	0.43	**0.38**	0.62	**0.50**	0.88	**0.68**	1.17	**0.88**
8	0.41	**0.38**	0.45	**0.40**	0.68	**0.52**	0.97	**0.70**	1.28	**0.90**
9	0.32	**0.26**	0.37	**0.32**	0.60	**0.45**	0.87	**0.62**	1.16	**0.79**
10	0.39	**0.34**	0.44	**0.38**	0.68	**0.53**	0.98	**0.72**	1.30	**0.93**
Ave.	0.40	**0.35**	0.43	**0.38**	0.65	**0.51**	0.94	**0.69**	1.24	**0.89**

Tab. 9. RMSE results using WABL defuzzification for Diabetes dataset.

Test	C=0.0		C=0.25		C=0.5		C=0.75		C=1.0	
	FIS	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**	**FIS**	**FIS-RBD**
1	6240	**5651**	6240	**5651**	6240	**5651**	6240	**5651**	6240	**5651**
2	**5027**	5191	**5027**	5191	**5027**	5191	**5027**	5191	**5027**	5191
3	4637	**5026**	4637	**5026**	4637	**5026**	4637	**5026**	4637	**5026**
4	**5599**	5662	**5599**	5662	**5599**	5662	**5599**	5662	**5599**	5662
5	5724	**5635**	5724	**5635**	5724	**5635**	5724	**5635**	5724	**5635**
6	5954	**5578**	5954	**5578**	5954	**5578**	5954	**5578**	5954	**5578**
7	5960	**5340**	5960	**5340**	5960	**5340**	5960	**5340**	5960	**5340**
8	6200	**6008**	6200	**6008**	6200	**6008**	6200	**6008**	6200	**6008**
9	6442	**5902**	6442	**5902**	6442	**5902**	6442	**5902**	6442	**5902**
10	6684	**6296**	6684	**6296**	6684	**6296**	6684	**6296**	6684	**6296**
Ave.	5847	**5629**	5847	**5629**	5847	**5629**	5847	**5629**	5847	**5629**

Tab. 10. RMSE results using WABL defuzzification for Diamonds dataset.

As can be seen in Tables 6 – 10, the FIS-RBD model using the WABL defuzzification method gave results with lower RMSE values compared to the classical Mamdani-type FIS model at every value of the optimism parameter in the [0,1] range. When the WABL defuzzification method is used, graphics reflecting the average RMSE values for various data at different values of the optimism parameter are given in Figures 6, 7, 8. As can be seen from the graphics, the FIS-RBD model gives lower RMSE values than the classical Mamdani-type FIS model at almost all values of the optimism parameter for all datasets.

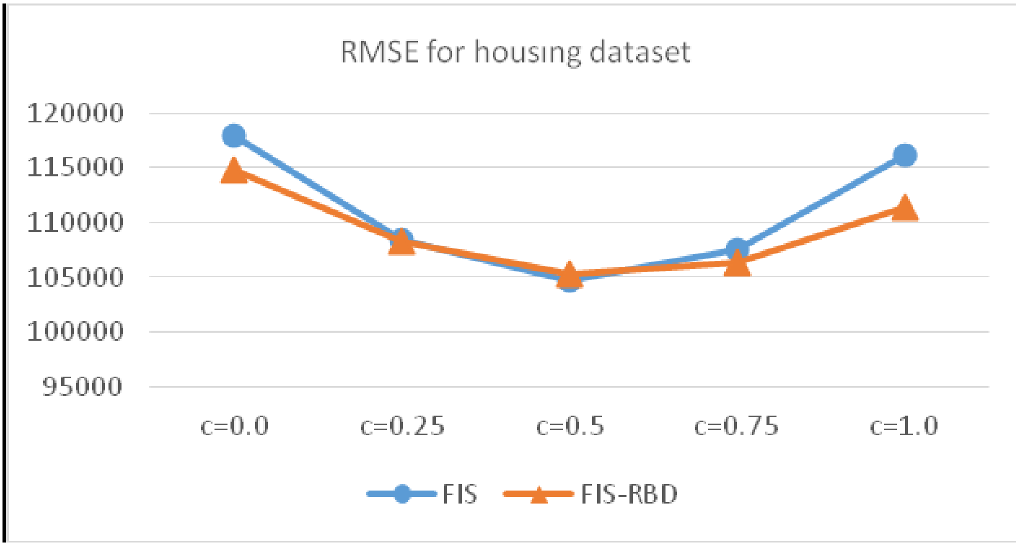

Fig. 6. Average RMSE results for Housing dataset using different values of the WABL optimism parameter.

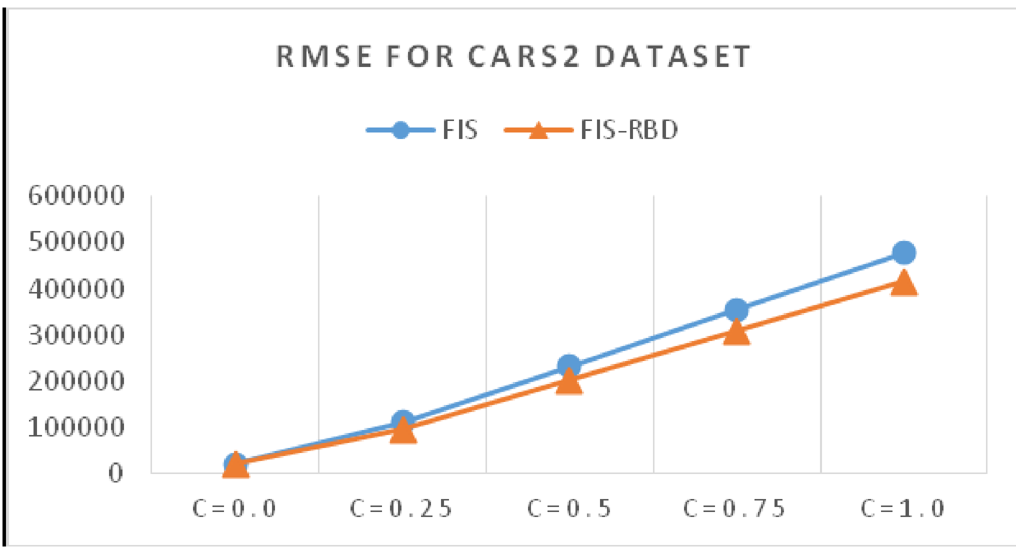

Fig. 7. Average RMSE results for Cars2 dataset using different values of the WABL optimism parameter.

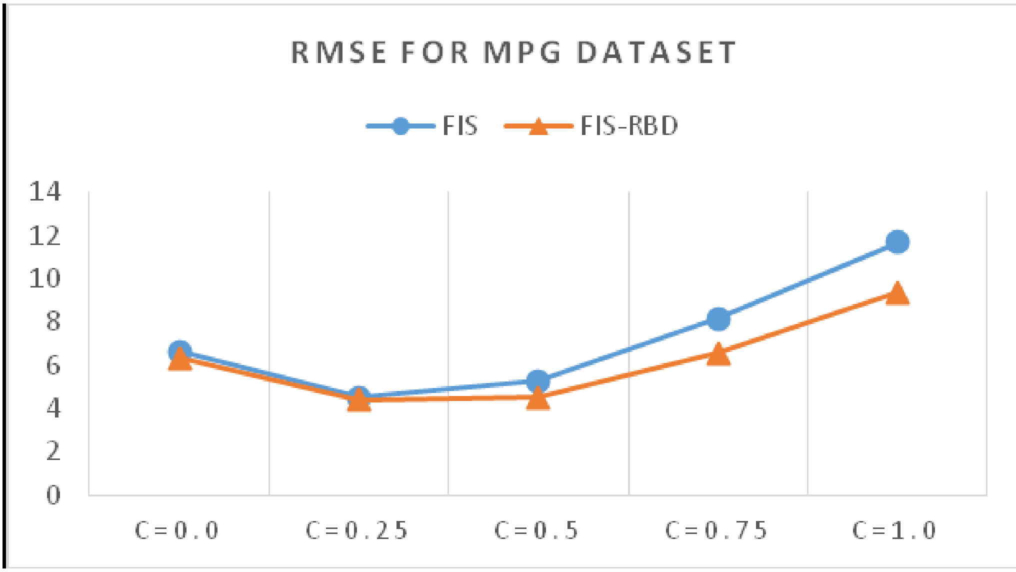

Fig. 8. Average RMSE results for Mpg dataset using different values of the WABL optimism parameter.

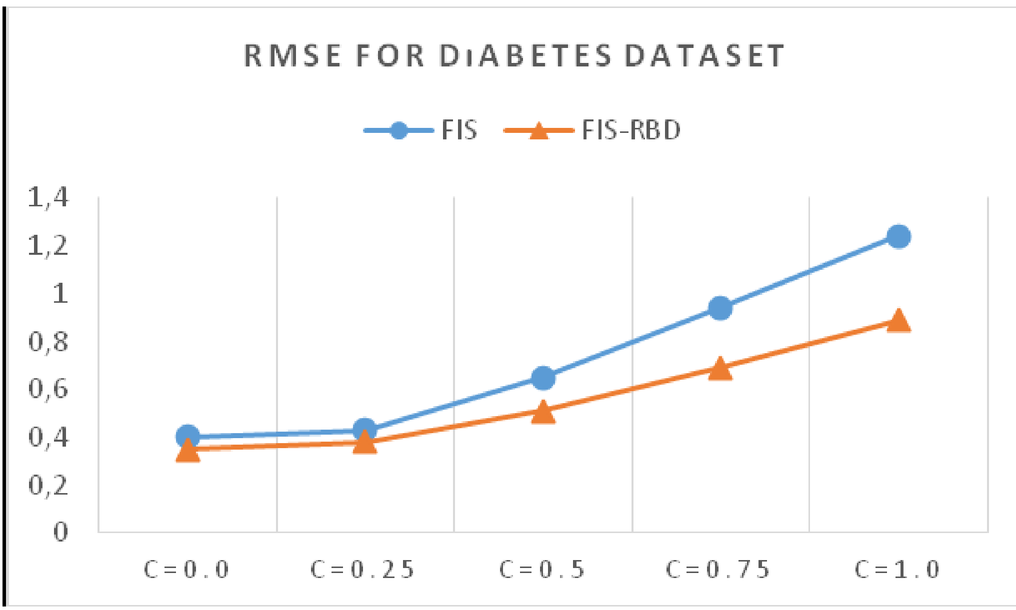

Fig. 9. Average RMSE results for Diabetes dataset using different values of the WABL optimism parameter.

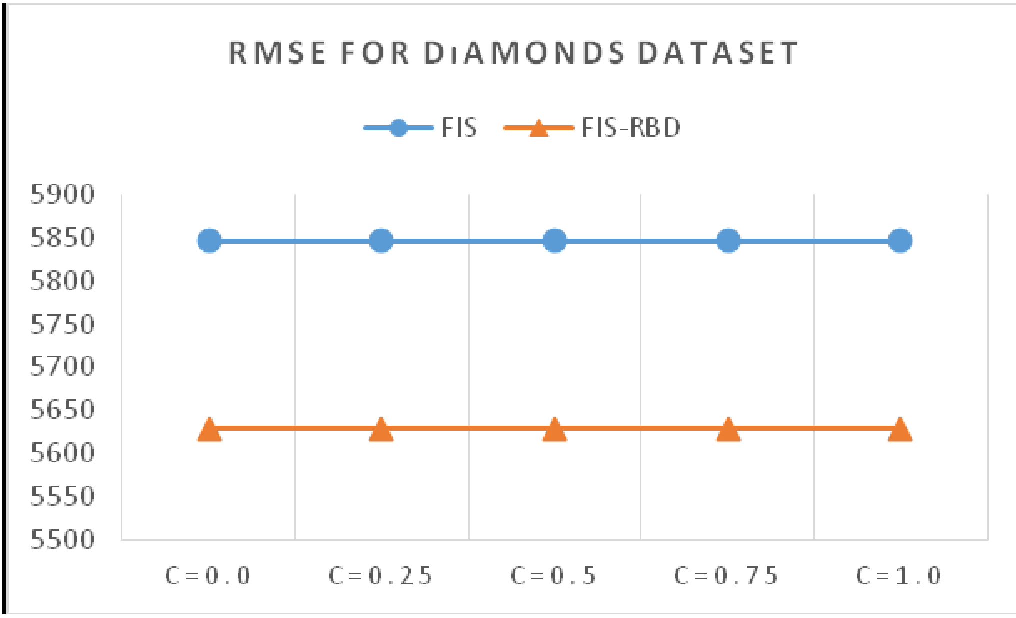

Fig. 10. Average RMSE results for Diamonds dataset using different values of the WABL optimism parameter.

According to the results of Tables 1 to 10, we can see that the FIS-RBD model gave better results than the classical Mamdani-type FIS model based on all defuzzification methods in almost all cases. In the Housing dataset, FIS-RBD gave better results only in the COG defuzzification method, while approximately similar results were observed for the MOM and WABL (with $c = 0.5$) defuzzification methods. Moreover, from the graphics in Figures 6 – 10, it can be seen that the FIS-RBD model also gives lower RMSE average values than the classical FIS model in all datasets for various WABL optimism parameters. Therefore, it can be said that the FIS-RBD model is a more accurate model than the classical Mamdani-type FIS model.

6. CONCLUSION

Since the aim of this study is not to optimize the prediction results, but only to compare the effectiveness of the classical Mamdani-type FIS and FIS-RBD models for regression problems, we did not discuss the construction of the rule set that gives the best results. So, the results of different regression models were compared on the same rule set and the same data set. Comparisons were made using the Housing, Cars2, Auto-mpg, Diabetes and Diamonds datasets available in the literature. Commonly used defuzzification methods such as COG, MOM, and WABL were used to defuzzify the fuzzy outputs. The comparison results showed that the FIS-RBD model gave lower RMSE values than the classical Mamdani-type FIS model in each data set when various defuzzification methods were used. Considering these, it can be said that the FIS-RBD model is a more effective model than the classical Mamdani-type FIS model. In future studies, the accuracy of the FIS-RBD model is planned to increase to give better prediction results by optimizing WABL settings and fuzzy rule sets.

To perform computational experiments, a new Fuzlab Python library was developed based on the existing FuzzyLab library in the literature. A new FIS-RBD model was added to the Fuzlab library. In addition, existing methods were changed or new methods were added to enable the use of the WABL defuzzification method in fuzzy logic-based computations. It is thought that this developed Python Fuzlab library will also be useful in developing other fuzzy logic-based models.

(Received October 18, 2024)

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