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# The Truth About Numerology

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# The Truth About Numerology

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## Introduction

This problem arose during one of my Spanish classes at FNSPE CTU. [1)] While studying the future tense, we encountered an exercise that supposedly predicted one's future for the coming year based on their date of birth and current year. Specifically, the task was as follows:

*Take your birth date and sum the day with the month (e.g.* $3+9=12$ *for September 3). Then sum the digits of the current year (e.g.* $2+0+2+3=7$ *for 2023). Sum the two numbers (*$12+7=19$*) and take the sum of the digits (*$1+9=10$*). Repeat doing so until you reach a number between* 1 *and* 9 *(*$1+0=1$*). Read what awaits you next year based on your resulting number.*

The problem appeared when our teacher described the algorithm differently: she first took the sums of the digits of the first two numbers ($1+2=3$ and $7=7$) and only then summed the numbers up ($3+7=10$). Surprisingly, everyone arrived at the same result as with the original algorithm! ($1+0=1$) The destiny seemed inevitable.

**Proposition.** *Let* $a, b \in \mathbb{N}$. *Denote* $R\colon \mathbb{N} \to \{1, 2, \ldots, 9\}$ *the function that takes a natural number and by taking the sum of its digits (in decimal representation) repeatedly, it eventually returns a number between* 1 *and* 9. *Then*

$$R(a+b) = R(R(a)+R(b)).$$

**Remark.** It is easy to see that $R$ is well-defined since the sum of the digits of $a \in \mathbb{N}$ is always strictly lower than $a$ and never zero.

## Solution

Our problem eventually had an elementary solution using congruence modulo 9. The following theorem is a simple corollary of basic rules of modular arithmetic.

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**Theorem 1.** *Let $f$ be a polynomial with integer coefficients, $a, b \in \mathbb{Z}$, $m \in \mathbb{N}$. If $a \equiv b \mod m$, then $f(a) \equiv f(b) \mod m$.*

If we denote $a = a_n \ldots a_1 a_0$ and $b = b_m \ldots b_1 b_0$ the decimal representations of $a$ and $b$, let $f(x) := \sum_{i=0}^{n} a_i x^i$, $g(x) := \sum_{j=0}^{m} b_j x^j$, then $a = f(10)$ and $b = g(10)$.

Now since $10 \equiv 1 \mod 9$, we have $a = f(10) \equiv f(1) \mod 9$ and similarly $b = g(10) \equiv g(1) \mod 9$. At the same time, $f(1)$ is nothing else then the sum of the digits of $a$ (and analogously for $g(1)$ and $b$). Therefore, by taking the sum of the digits of a given number, one preserves its remainder mod 9. We conclude that $R(a) \equiv a \mod 9$. Finally, we arrive at

$$R(a + b) \equiv a + b \equiv R(a) + R(b) \equiv R(R(a) + R(b)) \mod 9. \quad (1)$$

And since both $R(a + b)$ and $R(R(a) + R(b))$ are between 1 and 9, our proposition is proven. We can see now that no matter at which point of the algorithm we sum the two branches up, the result is always the lowest positive remainder of the sum $a + b$ mod 9.

**Corollary 2.** *Let $a \in \mathbb{N}$ and let $R$ be the function defined above. Then $R(a)$ is the remainder of $a$ mod 9, redefined as 9 for $a \equiv 0 \mod 9$, i.e. the lowest positive remainder of $a$ mod 9.*

## Vocabulary

- proposition – tvrzení
- digit – cifra
- decimal representation (of a number) – reprezentace (čísla) v desítkové soustavě
- remark – poznámka
- theorem – věta
- integer – celé číslo
- remainder modulo 9 – zbytek modulo 9
- corollary – důsledek