Applications of Mathematics

Peixin Zhang; Mingxuan Zhu

Local-in-time existence for the non-resistive incompressible magneto-micropolar fluids

Applications of Mathematics, Vol. 67 (2022), No. 2, 199–208

Persistent URL: `http://dml.cz/dmlcz/149566`

LOCAL-IN-TIME EXISTENCE FOR THE NON-RESISTIVE INCOMPRESSIBLE MAGNETO-MICROPOLAR FLUIDS

Peixin Zhang, Quanzhou, Mingxuan Zhu, Qufu

Received April 20, 2020. Published online May 20, 2021.

Abstract. We establish the local-in-time existence of a solution to the non-resistive magneto-micropolar fluids with the initial data $u_0 \in H^{s-1+\varepsilon}$, $w_0 \in H^{s-1}$ and $b_0 \in H^s$ for $s > \frac{3}{2}$ and any $0 < \varepsilon < 1$. The initial regularity of the micro-rotational velocity w is weaker than velocity of the fluid u.

Keywords: non-resistive magneto-micropolar fluid; local existence

MSC 2020: 35A01, 35Q30, 76B03

1. Introduction

The non-resistive incompressible magneto-micropolar equations in three dimensions can be written as follows:

$$\begin{cases} u_t + u\cdot\nabla u + \nabla(p + \frac{1}{2}|b|^2) - (\mu+\zeta)\Delta u - (b\cdot\nabla)b - \zeta\nabla\times w = 0, \\ w_t + u\cdot\nabla w + 2\zeta w - \nu\Delta w - \lambda\nabla\operatorname{div} w - \zeta\nabla\times u = 0, \\ b_t + u\cdot\nabla b - b\cdot\nabla u = 0, \\ \operatorname{div} u = 0,\ \operatorname{div} b = 0, \end{cases} \tag{1.1}$$

with the initial data:

$$(u, w, b)(x, 0) = (u_0, w_0, b_0)(x), \tag{1.2}$$

Zhang was partially supported by the National Natural Science Foundation of China (Grant No. 11701192), the Scientific Research Funds of Huaqiao University (Grant No. 15BS201). Zhu was partially supported by the National Natural Science Foundation of China (Grant No. 11771183).

DOI: 10.21136/AM.2021.0111-20

where $x \in \mathbb{R}^3$ and $t \geqslant 0$. The functions $u = u(x,t)$, $w = w(x,t)$, $b = b(x,t)$, and $p = p(x,t)$ denote the velocity of the fluid, the micro-rotational velocity, the magnetic field, and the pressure, respectively. The constants μ, ζ are the coefficients of kinematic viscosity, vortex viscosity, and ν, λ are angular viscosities.

The magneto-micropolar fluid equations model was first proposed in [1], which describes the motion of electrically conducting micropolar fluids in the presence of a magnetic field. Micropolar fluids represent a class of fluids with nonsymmetric stress tensor (called polar fluids) such as fluids consisting of suspending particles, dumbbell molecules, etc., see [6], [7], [13]. By using the spectral Galerkin method, Rojas-Medar [16] proved the existence and uniqueness of strong solutions for system (1.1) with resistive viscosity in bounded domain. Then, Yuan [18] proved the local existence of the strong solution with initial data in H^s with $s > \frac{3}{2}$ in the whole $\mathbb{R}^3$. The weak solution and global regularity were studied in [14], [17], [15], [16], [19], [20], [5], [4].

When the micro-rotational velocity disappears ($w = 0$), system (1.1) reduces to a non-resistive magneto-hydro-dynamic (MHD) system. There is a great deal of literature on the MHD system. Here we only introduce some studies related to this paper. Jiu and Niu [10] proved the local existence for 2D with the initial data in H^s, for integer $s > 3$. Fefferman et al. [8] established a local existence of non-resistive MHD when $u_0, b_0 \in H^s$ with $s > d/2$ for $d = 2,3$. Recently, Fefferman et al. [9] extended the local existence result in [8] to the non-resistive MHD when $u_0 \in H^{s-1+\varepsilon}$, $b_0 \in H^s$ with $s > d/2$ for $d = 2,3$ and any $0 < \varepsilon < 1$. Chemin et al. [3] established the local existence of weak solutions to 2D and 3D Cauchy problem for non-resistive MHD equations with divergence-free initial data in the Besov space $B^{n/2}_{2,1}$ and they also proved the uniqueness of solutions in 3D case.

Motivated by Fefferman-McCormick-Robinson-Rodrigo's approach (see [9]), we study the local-in-time existence of system (1.1)–(1.2). More precisely, we have the following result:

Theorem 1.1. *Suppose that the initial data* (u_0, w_0, b_0) *satisfy* $u_0 \in H^{s-1+\varepsilon}$, $w_0 \in H^{s-1}$, *and* $b_0 \in H^s$ *for* $s > \frac{3}{2}$ *and any* $0 < \varepsilon < 1$. *Then there exists* $T^* > 0$ *such that system* (1.1)–(1.2) *has a solution* (u, w, b) *satisfying*

$$u \in L^\infty(0,T^*; H^{s-1+\varepsilon}) \cap L^2(0,T^*; H^{s+\varepsilon}),$$
$$w \in L^\infty(0,T^*; H^{s-1}) \cap L^2(0,T^*; H^{s}),$$

and

$$b \in L^\infty(0,T^*; H^s).$$

R e m a r k 1.1. Comparing with the MHD system, the existence of micro-rotational velocity makes it more difficult. It is worth to point out that the initial regularity of w is weaker than u. When $w = 0$, Theorem 1.1 reduces to the result in [9]. Theorem 1.1 is an improvement of the result in [18].

2. Preliminaries

In this section, we will give some elementary facts which will be used later. Throughout the paper we use the notation Λ^s to denote the fractional derivative of order s, given in terms of the Fourier transform by $\widehat{\Lambda^s f} = |\xi|^s \hat{f}$. We write

$$\|u\|_{H^s}^2 = \|\Lambda^s u\|^2 + \|u\|^2, \quad s > 0,$$

which is equivalent to the standard H^s norm when s is a positive integer, where $\|\cdot\| \triangleq \|\cdot\|_{L^2}$. Before the proof of Theorem 1.1, we present the maximum regularity-type result of heat equation.

Lemma 2.1 ([9]). *If $f \in L^r(0,T;H^{s-1})$, $1 < r < \infty$, $s > 1$, and*

$$\partial_t u - \Delta u + \nabla p = f, \quad \operatorname{div} u = 0, \quad u(0) = u_0 \in H^{s-1+\varepsilon},$$

where u_0 is divergence free, then for $T \leqslant 1$

$$\int_0^T \|u\|_{H^{s+1}}\,\mathrm{d}t \leqslant C_\varepsilon T^{\varepsilon/2}\|u_0\|_{H^{s-1+\varepsilon}} + C_r T^{1-1/r}\|f\|_{L^r(0,T;H^{s-1})}. \tag{2.1}$$

Lemma 2.2 ([8]). *Given $s > \frac{3}{2}$, there is a constant $C = C(s)$ such that for all u, b with $\nabla u, b \in H^s$,*

$$\|\Lambda^s[(u\cdot\nabla)b] - (u\cdot\nabla)(\Lambda^s b)\|_{L^2} \leqslant c\|\nabla u\|_{H^s}\|b\|_{H^s}.$$

In [11], [12], the following useful inequalities are proved in the Sobolev spaces.

Lemma 2.3 (Kato-Ponce inequality [11], [12]). *Let $s > 0$, $1 < p < \infty$. If $f \in W^{1,p_1} \cap W^{s,q_2}$, $g \in L_{p_2} \cap W^{s,q_1}$, then*

$$\|\Lambda^s(fg) - f\Lambda^s g\|_{L^p} \leqslant C(\|\nabla f\|_{L^{p_1}}\|\Lambda^{s-1}g\|_{L^{q_1}} + \|g\|_{L^{p_2}}\|\Lambda^s f\|_{L^{q_2}})$$

and

$$\|\Lambda^s(fg)\|_{L^p} \leqslant C(\|f\|_{L^{p_1}}\|\Lambda^s g\|_{L^{q_1}} + \|g\|_{L^{p_2}}\|\Lambda^s f\|_{L^{q_2}})$$

with $1/p = 1/p_1 + 1/q_1 = 1/p_2 + 1/q_2$.

3. Proof of Theorem 1.1

Multiplying $(1.1)_1$, $(1.1)_2$ and $(1.1)_3$ by u, w and b, respectively, integrating by parts yields

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^2 + (\mu+\zeta)\|\nabla u\|^2 = \langle (b\cdot\nabla)b, u\rangle + \zeta\langle \nabla\times w, u\rangle, \tag{3.1}$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|w\|^2 + \nu\|\nabla w\|^2 + \lambda\|\operatorname{div} w\|^2 + 2\zeta\|w\|^2 = \zeta\langle \nabla\times u, w\rangle, \tag{3.2}$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|b\|^2 = \langle (b\cdot\nabla)u, b\rangle. \tag{3.3}$$

Combining (3.1)–(3.3) together, noting that $\langle (b\cdot\nabla)b, u\rangle = -\langle (b\cdot\nabla)u, b\rangle$ and $\langle \nabla\times w, u\rangle = \langle \nabla\times u, w\rangle$, using the Cauchy inequality $|2\zeta\langle \nabla\times u, w\rangle| \leqslant \zeta\|\nabla u\|^2 + \zeta\|w\|^2$, we have

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|u\|^2 + \|w\|^2 + \|b\|^2) + \mu\|\nabla u\|^2 + \nu\|\nabla w\|^2 + \lambda\|\operatorname{div} w\|^2 + \zeta\|w\|^2 \leqslant 0.$$

It follows that

$$(\|u\|^2 + \|w\|^2 + \|b\|^2) + 2\int_0^t (\mu\|\nabla u\|^2 + \nu\|\nabla w\|^2 + \lambda\|\operatorname{div} w\|^2 + \zeta\|w\|^2)\,\mathrm{d}s \leqslant (\|u_0\|^2 + \|w_0\|^2 + \|b_0\|^2) \triangleq M_0. \tag{3.4}$$

We also can obtain from (3.1)–(3.3)

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|u\|^2 + \|w\|^2) &+ (\mu+\zeta)\|\nabla u\|^2 + \nu\|\nabla w\|^2 + \lambda\|\nabla w\|^2 + 2\zeta\|w\|^2 \\ &= \langle (b\cdot\nabla)b, u\rangle + 2\zeta\langle \nabla\times u, w\rangle \leqslant C\|b\|\|\nabla u\|\|b\|_{L^\infty} + C\|w\|\|\nabla u\| \\ &\leqslant \frac{\mu}{2}\|\nabla u\|^2 + C\|b\|_{H^s}^4 + CM_0\end{aligned} \tag{3.5}$$

and

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|b\|^2 \leqslant \|b\|^2\|\nabla u\|_{L^\infty} \leqslant C\|b\|^2\|\nabla u\|_{H^s}. \tag{3.6}$$

We apply the operator Λ^s to $(1.1)_2$ and take the inner product with $\Lambda^s b$ in L^2. By Lemma 2.2 and the same argument as in [9], one has

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\Lambda^s b\|^2 \leqslant |\langle \Lambda^s[(b\cdot\nabla)u], \Lambda^s b\rangle| + |\langle \Lambda^s[(u\cdot\nabla)b], \Lambda^s b\rangle| \leqslant C\|b\|_{H^s}^2\|\nabla u\|_{H^s}. \tag{3.7}$$

Combining (3.6) and (3.7) together, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|b\|_{H^s}^2 \leqslant C_1\|b\|_{H^s}^2\|\nabla u\|_{H^s},$$

which implies that

$$\|b(t)\|_{H^s}^2 \leqslant \|b_0\|_{H^s}^2 \exp\Big\{C_1\int_0^t \|\nabla u\|_{H^s}\,\mathrm{d}\tau\Big\}. \tag{3.8}$$

Next, we get the estimates on u in the space $L^1(0,T;H^{s+1})$, using Lemma 2.1. We rewrite $(1.1)_1$ as follows:

$$u_t-(\mu+\zeta)\Delta u+\nabla\Big(p+\frac{1}{2}|b|^2\Big) = f \triangleq -(u\cdot\nabla)u+(b\cdot\nabla)b+\zeta\nabla\times w,\ \operatorname{div} u = 0,\ u(0) = u_0.$$

By (2.1), we have

$$\int_0^T \|u\|_{H^{s+1}}\,\mathrm{d}t \leqslant C_\varepsilon T^{\varepsilon/2}\|u_0\|_{H^{s-1+\varepsilon}} + C_r T^{1-1/r}\|f\|_{L^r(0,T;H^{s-1})}. \tag{3.9}$$

Since

$$\begin{aligned}\|f\|_{H^{s-1}} &= \|\operatorname{div}(b\otimes b) - \operatorname{div}(u\otimes u) + \zeta\nabla\times w\|_{H^{s-1}}\\ &\leqslant \|b\otimes b\|_{H^s} + \|u\otimes u\|_{H^s} + C\|w\|_{H^s}\\ &\leqslant C\|b\|_{H^s}^2 + C\|u\|_{H^s}^2 + C\|w\|_{H^s}\\ &\leqslant C\|b\|_{H^s}^2 + C\|u\|^{2\varepsilon/(s+\varepsilon)}\|u\|_{H^{s+\varepsilon}}^{2s/(s+\varepsilon)} + C\|w\|_{H^s}\\ &\leqslant C\|b\|_{H^s}^2 + CM_0^{2\varepsilon/(s+\varepsilon)}\|u\|_{H^{s+\varepsilon}}^{2s/(s+\varepsilon)} + C\|w\|_{H^s},\end{aligned}$$

combining (3.4) and choosing $r = (s+\varepsilon)/s > 1$, from (3.9) we have

$$\begin{aligned}\int_0^T \|u\|_{H^{s+1}}\,\mathrm{d}t &\leqslant C_\varepsilon T^{\varepsilon/2}\|u_0\|_{H^{s-1+\varepsilon}} + C_\varepsilon T^{(s+\varepsilon)/s}\\ &\quad\times\Big(\int_0^T(\|b\|_{H^s}^{2(s+\varepsilon)/s} + M_0^{\varepsilon/s}\|u\|_{H^{s+\varepsilon}}^2 + \|w\|_{H^s}^{(s+\varepsilon)/s})\,\mathrm{d}t\Big)^{s/(s+\varepsilon)}.\end{aligned} \tag{3.10}$$

We now estimate the norm of u,w in $H^{s-1+\varepsilon}$ and $H^{s+\varepsilon}$. We apply the operator $\Lambda^{s-1+\varepsilon}$ to $(1.1)_1$ and take the inner product with $\Lambda^{s-1+\varepsilon}u$. Then

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|\Lambda^{s-1+\varepsilon}u\|^2) + \|\Lambda^{s+\varepsilon}u\|^2 &\leqslant C|\langle\Lambda^{s-1+\varepsilon}[(u\cdot\nabla)u],\Lambda^{s-1+\varepsilon}u\rangle|\\ &\quad + C|\langle\Lambda^{s-1+\varepsilon}[(b\cdot\nabla)b],\Lambda^{s-1+\varepsilon}u\rangle|\\ &\quad + C|\langle\Lambda^{s-1+\varepsilon}(\nabla\times w),\Lambda^{s-1+\varepsilon}u\rangle|\\ &\triangleq \sum_{i=1}^3 I_i.\end{aligned} \tag{3.11}$$

By the Sobolev interpolation and the Young inequality, we have

$$\begin{aligned}I_1+I_2&=C|\langle\Lambda^{s-1}\mathrm{div}(u\otimes u),\Lambda^{s-1+2\varepsilon}u\rangle|\\&\quad+C|\langle\Lambda^{s-1}\mathrm{div}(b\otimes b),\Lambda^{s-1+2\varepsilon}u\rangle|\\&\leqslant C\|u\|_{H^s}^2\|u\|_{H^{s-1+2\varepsilon}}+C\|b\|_{H^s}^2\|u\|_{H^{s-1+2\varepsilon}}\\&\leqslant C(\|u\|_{H^{s-1+\varepsilon}}^{\varepsilon}\|u\|_{H^{s+\varepsilon}}^{1-\varepsilon})^2\|u\|_{H^{s-1+\varepsilon}}^{1-\varepsilon}\|u\|_{H^{s+\varepsilon}}^{\varepsilon}\\&\quad+C\|b\|_{H^s}^2\|u\|_{H^{s-1+\varepsilon}}^{1-\varepsilon}\|u\|_{H^{s+\varepsilon}}^{\varepsilon}\\&\leqslant C\|u\|_{H^{s-1+\varepsilon}}^{2(1+\varepsilon)/\varepsilon}+C\|b\|_{H^s}^{2(1+\varepsilon)}+\frac{1}{6}\|u\|_{H^{s+\varepsilon}}^2\end{aligned}$$

and

$$I_3=\frac{1}{6}\|\Lambda^{s+\varepsilon}u\|_{L^2}^2+C\|\Lambda^{s-1+\varepsilon}w\|_{L^2}^2.$$

Operating with Λ^{s-1} on $(1.1)_2$ and taking the inner product with $\Lambda^{s-1}w$,

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|\Lambda^{s-1}w\|^2)+\|\Lambda^{s}w\|^2+\|\Lambda^{s-1}w\|^2&\leqslant C|\langle\Lambda^{s-1}[(u\cdot\nabla)w],\Lambda^{s-1}w\rangle|\\&\quad+C|\langle\Lambda^{s-1}(\nabla\times u),\Lambda^{s-1}w\rangle|\\&\triangleq\sum_{i=4}^{5}I_i.\end{aligned}\tag{3.12}$$

Now, we estimate the two terms on the right-hand side by using the Kato-Ponce inequality [11], [12]. Next, we estimate I_4 in three cases. For the case $s+\varepsilon=5/2$ we have

$$\begin{aligned}I_4&=C|\langle\Lambda^{s-1}\mathrm{div}(u\otimes w),\Lambda^{s-1}w\rangle|\\&=C|\langle\Lambda^{s-1}(u\cdot\nabla w),\Lambda^{s-1}w\rangle-\langle(u\cdot\nabla\Lambda^{s-1}w),\Lambda^{s-1}w\rangle|\\&\leqslant C(\|\nabla u\|_{L^3}\|\Lambda^{s-1}w\|_{L^6}+\|\Lambda^{s-1}u\|_{L^{6/(3-2\varepsilon)}}\|\nabla w\|_{L^{3/\varepsilon}})\|\Lambda^{s-1}w\|\\&\leqslant C\|u\|_{H^{s-1+\varepsilon}}^{4/(2s+2\varepsilon-3)}\|\Lambda^{s-1}w\|^2+\frac{1}{4}\|\Lambda^{s}w\|^2.\end{aligned}$$

For the case $3/2<s+\varepsilon<5/2$,

$$\begin{aligned}I_4&=C|\langle\Lambda^{s-1}\mathrm{div}(u\otimes w),\Lambda^{s-1}w\rangle|\\&\leqslant C\|\Lambda^{s-1}(u\otimes w)\|\|\Lambda^{s}w\|\\&\leqslant C(\|u\|_{L^{6/(5-2s-2\varepsilon)}}\|\Lambda^{s-1}w\|_{L^{6/(2s+2\varepsilon-2)}}\\&\quad+\|\Lambda^{s-1}u\|_{L^{6/(3-2\varepsilon)}}\|w\|_{L^{3/\varepsilon}})\|\Lambda^{s}w\|\\&\leqslant C\|u\|_{H^{s-1+\varepsilon}}\|\Lambda^{s-1}w\|^{s+\varepsilon-3/2}\|\Lambda^{s}w\|^{7/2-s-\varepsilon}\\&\leqslant C\|u\|_{H^{s-1+\varepsilon}}^{4/(2s+2\varepsilon-3)}\|\Lambda^{s-1}w\|^2+\frac{1}{4}\|\Lambda^{s}w\|^2.\end{aligned}$$

For the case $s+\varepsilon > 5/2$,

$$\begin{aligned}
I_4 &= C|\langle \Lambda^{s-1}\mathrm{div}(u\otimes w), \Lambda^{s-1}w\rangle| \\
&\leqslant C\|\Lambda^{s-1}(u\otimes w)\|\|\Lambda^s w\| \\
&\leqslant C(\|u\|_{L^\infty}\|\Lambda^{s-1}w\| + \|\Lambda^{s-1}u\|_{L^{6/(3-2\varepsilon)}}\|w\|_{L^{3/\varepsilon}})\|\Lambda^s w\| \\
&\leqslant C\|u\|_{H^{s-1+\varepsilon}}^{4/(2s+2\varepsilon-3)}\|\Lambda^{s-1}w\|^2 + \frac{1}{4}\|\Lambda^s w\|^2 + \|u\|_{H^{s-1+\varepsilon}}^2\|\Lambda^{s-1}w\|^2. \\
I_5 &= C|\langle \Lambda^{s-1}(\nabla\times u), \Lambda^{s-1}w\rangle| \\
&\leqslant \frac{1}{6}\|\Lambda^{s+\varepsilon}u\|^2 + C\|\Lambda^{s-1-\varepsilon}w\|^2.
\end{aligned}$$

From the above estimates and (3.11)–(3.12) it follows

$$\begin{aligned}
(3.13)\qquad & \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|\Lambda^{s-1+\varepsilon}u\|^2 + \|\Lambda^{s-1}w\|^2) + \|\Lambda^{s+\varepsilon}u\|^2 + \|\Lambda^s w\|^2 + \|\Lambda^{s-1}w\|^2 \\
&\leqslant C\|u\|_{H^{s-1+\varepsilon}}^{2(1+\varepsilon)/\varepsilon} + C\|u\|_{H^{s-1+\varepsilon}}^{4/(2s+2\varepsilon-3)}\|\Lambda^{s-1}w\|^2 \\
&\quad + \frac{1}{2}\|u\|_{H^{s+\varepsilon}}^2 + \frac{1}{4}\|\Lambda^s w\|^2 \\
&\quad + C\|\Lambda^{s-1+\varepsilon}w\|^2 + C\|\Lambda^{s-1-\varepsilon}w\|^2 \\
&\leqslant C\|u\|_{H^{s-1+\varepsilon}}^{2(1+\varepsilon)/\varepsilon} + C\|u\|_{H^{s-1+\varepsilon}}^{8/(2s+2\varepsilon-3)} + \|w\|_{H^{s-1}}^4 \\
&\quad + \frac{1}{2}\|u\|_{H^{s+\varepsilon}}^2 + C\|w\|^2 + \frac{1}{2}\|\Lambda^s w\|^2.
\end{aligned}$$

Combining (3.5) and (3.13), we have

$$\begin{aligned}
(3.14)\qquad & \frac{\mathrm{d}}{\mathrm{d}t}(\|u\|_{H^{s-1+\varepsilon}}^2 + \|w\|_{H^{s-1}}^2) + \|u\|_{H^{s+\varepsilon}}^2 + \|w\|_{H^s}^2 \\
&\leqslant C\|u\|_{H^{s-1+\varepsilon}}^{2(1+\varepsilon)/\varepsilon} + C\|u\|_{H^{s-1+\varepsilon}}^{8/(2s+2\varepsilon-3)} + \|w\|_{H^{s-1}}^4 \\
&\quad + C\|b\|_{H^s}^{2(1+\varepsilon)} + C\|b\|_{H^s}^4 + C(\|u\|^2 + \|w\|_{L^2}^2 + M_0) \\
&\leqslant C_2(\|u\|_{H^{s-1+\varepsilon}}^2 + \|w\|_{H^{s-1}}^2)^{\max\{(1+\varepsilon)/\varepsilon, 4/(2s+2\varepsilon-3), 2\}} \\
&\quad + C_3\|b\|_{H^s}^{2(1+\varepsilon)} + C_4\|b\|_{H^s}^4 + C_5 M_0.
\end{aligned}$$

We will choose $T^* > 0$ such that $\|b\|_{H^s} \leqslant 2\|b_0\|_{H^s}$ for all $t\in[0,T^*]$. Set

$$\begin{aligned}
M_1 &\triangleq \|u_0\|_{H^{s-1+\varepsilon}} + \|w_0\|_{H^{s-1}}, \\
M_2 &\triangleq 2^{2(1+\varepsilon)}C_3\|b_0\|_{H^s}^{2(1+\varepsilon)} + 2^4 C_4\|b_0\|_{H^s}^4 + C_5 M_0, \\
\chi &= \max\Big\{\frac{1+\varepsilon}{\varepsilon}, \frac{4}{2s+2\varepsilon-3}, 2\Big\}
\end{aligned}$$

and choose T^* sufficiently small so that

$$(3.15)\qquad 0 < (1-(\chi-1)C_2T(M_1^2+TM_2)^{\chi-1})^{-1/(\chi-1)} < 2 \quad \forall T\in(0,T^*)$$

and

$$
(3.16)\quad C_\varepsilon T^{\varepsilon/2}M_1 + C_\varepsilon T^{(s+\varepsilon)/s}[2^{2(s+\varepsilon)/s}T\|b_0\|_{H^s}^{2(s+\varepsilon)/s} + (M_0^{\varepsilon/s}+1)(2^\chi C_2 T(M_1^2+TM_2)^\chi + M_2T) + T]^{s/(s+\varepsilon)} < \frac{\ln 2}{C_1}
$$

for all $0 < T < T^*$.

Denote

$$
T = \sup\{T_0 \in [0,T^*];\ \|b\|_{H^s} \leqslant 2\|b_0\|_{H^s}\ \forall\, t \in [0,T_0]\}.
$$

Suppose that $T < T^*$. Then from (3.4) and (3.14) we have for all $t \in [0,T]$,

$$
\begin{aligned}
(3.17)\qquad \frac{\mathrm{d}}{\mathrm{d}t}(\|u\|_{H^{s-1+\varepsilon}}^2 + \|w\|_{H^{s-1}}^2) + \|u\|_{H^{s+\varepsilon}}^2 + \|w\|_{H^s}^2 &\leqslant C_2(\|u\|_{H^{s-1+\varepsilon}}^2 + \|w\|_{H^{s-1}}^2)^\chi + 2^{2(1+\varepsilon)}C_3\|b_0\|_{H^s}^{2(1+\varepsilon)} \\
&\quad + 2^4C_4\|b_0\|_{H^s}^4 + C_5M_0 \\
&\leqslant C_2(\|u\|_{H^{s-1+\varepsilon}}^2 + \|w\|_{H^{s-1}}^2)^\chi + M_2.
\end{aligned}
$$

Using standard ODE comparison techniques (see Theorem 6 in [2]) and (3.15), we obtain

$$
\begin{aligned}
(3.18)\qquad (\|u\|_{H^{s-1+\varepsilon}}^2 + \|w\|_{H^{s-1+\varepsilon}}^2) &\leqslant (M_1^2+TM_2)(1-(\chi-1)C_2T(M_1^2+TM_2)^{\chi-1})^{-1/(\chi-1)} \\
&\leqslant 2(M_1^2+TM_2).
\end{aligned}
$$

Inserting (3.18) into (3.17) and integrating over $[0,T]$ yields

$$
(3.19)\qquad \int_0^T (\|u\|_{H^{s+\varepsilon}}^2 + \|w\|_{H^s}^2)\,\mathrm{d}t \leqslant 2^\chi C_2T(M_1^2+TM_2)^\chi + M_2T.
$$

Substituting (3.19) and $\|b\|_{H^s} \leqslant 2\|b_0\|_{H^s}$ into (3.10), by (3.16), we get

$$
\begin{aligned}
(3.20)\quad \int_0^T (\|u\|_{H^{s+1}})\,\mathrm{d}t &\leqslant C_\varepsilon T^{\varepsilon/2}M_1 + C_\varepsilon T^{(s+\varepsilon)/s} + C_\varepsilon T^{(s+\varepsilon)/s}[2^{2(s+\varepsilon)/s}T\|b_0\|_{H^s}^{2(s+\varepsilon)/s} \\
&\quad + (M_0^{\varepsilon/s}+1)(2^\chi C_2T(M_1^2+TM_2)^\chi + M_2T) + T]^{s/(s+\varepsilon)} \\
&< \frac{\ln 2}{C_1}.
\end{aligned}
$$

Substituting this into (3.8) ensures that $\|b\|_{H^s} \leqslant 2\|b_0\|_{H^s}$ for all $t \in [0,T]$, contradicting the maximality of T. It follows that $T = T^*$ and hence,

$$
\|b\|_{H^s} \leqslant 2\|b_0\|_{H^s}, \quad t \in [0,T^*].
$$

Theorem 1.1 now follows from (3.18), (3.19) and (3.20). □

A c k n o w l e d g m e n t s. The authors are indebted to anonymous referees for their helpful comments.

References

[1] *G. Ahmadi, M. Shahinpoor*: Universal stability of magneto-micropolar fluid motions. Int. J. Engin. Sci. *12* (1974), 657–663. zbl MR doi

[2] *D. Blömker, C. Nolde, J. C. Robinson*: Rigorous numerical verification of uniqueness and smoothness in a surface growth model. J. Math. Anal. Appl. *429* (2015), 311–325. zbl MR doi

[3] *J.-Y. Chemin, D. S. McCormick, J. C. Robinson, J. L. Rodrigo*: Local existence for the non-resistive MHD equations in Besov spaces. Adv. Math. *286* (2016), 1–31. zbl MR doi

[4] *M. Chen*: Global well-posedness of the 2D incompressible micropolar fluid flows with partial viscosity and angular viscosity. Acta Math. Sci., Ser. B, Engl. Ed. *33* (2013), 929–935. zbl MR doi

[5] *M. Chen, X. Xu, J. Zhang*: The zero limits of angular and micro-rotational viscosities for the two-dimensional micropolar fluid equations with boundary effect. Z. Angew. Math. Phys. *65* (2014), 687–710. zbl MR doi

[6] *S. C. Cowin*: Polar fluids. Phys. Fluids *11* (1968), 1919–1927. zbl doi

[7] *M. E. Erdoğan*: Polar effects in the apparent viscosity of suspension. Rheol. Acta *9* (1970), 434–438. doi

[8] *C. L. Fefferman, D. S. McCormick, J. C. Robinson, J. L. Rodrigo*: Higher order commutator estimates and local existence for the non-resistive MHD equations and related models. J. Funct. Anal. *267* (2014), 1035–1056. zbl MR doi

[9] *C. L. Fefferman, D. S. McCormick, J. C. Robinson, J. L. Rodrigo*: Local existence for the non-resistive MHD equations in nearly optimal Sobolev spaces. Arch. Ration. Mech. Anal. *223* (2017), 677–691. zbl MR doi

[10] *Q. Jiu, D. Niu*: Mathematical results related to a two-dimensional magneto-hydrodynamic equations. Acta Math. Sci., Ser. B, Engl. Ed. *26* (2006), 744–756. zbl MR doi

[11] *T. Kato, G. Ponce*: Commutator estimates and the Euler and Navier-Stokes equations. Commun. Pure Appl. Math. *41* (1988), 891–907. zbl MR doi

[12] *C. E. Kenig, G. Ponce, L. Vega*: Well-posedness of the initial value problem for the Korteweg-de Vries equation. J. Am. Math. Soc. *4* (1991), 323–347. zbl MR doi

[13] *G. Łukaszewicz*: Micropolar Fluids: Theory and Applications. Modeling and Simulation in Science, Engineering and Technology. Birkhäuser, Boston, 1999. zbl MR doi

[14] *E. E. Ortega-Torres, M. A. Rojas-Medar*: Magneto-micropolar fluid motion: Global existence of strong solutions. Abstr. Appl. Anal. *4* (1999), 109–125. zbl MR doi

[15] *M. A. Rojas-Medar*: Magneto-micropolar fluid motion: Existence and uniqueness of strong solution. Math. Nachr. *188* (1997), 301–319. zbl MR doi

[16] *M. A. Rojas-Medar*: Magneto-micropolar fluid motion: On the convergence rate of the spectral Galerkin approximations. Z. Angew. Math. Mech. *77* (1997), 723–732. zbl MR doi

[17] *M. A. Rojas-Medar, J. L. Boldrini*: Magneto-micropolar fluid motion: Existence of weak solutions. Rev. Mat. Complut. *11* (1998), 443–460. zbl MR doi

[18] *J. Yuan*: Existence theorem and blow-up criterion of strong solutions to the magneto-micropolar fluid equations. Math. Methods Appl. Sci. *31* (2008), 1113–1130. zbl MR doi

[19] *B. Yuan, X. Li*: Regularity of weak solutions to the 3D magneto-micropolar equations in Besov spaces. Acta Appl. Math. *163* (2019), 207–223. zbl MR doi

[20] *Z. Zhang*: A regularity criterion for the three-dimensional micropolar fluid system in homogeneous Besov spaces. Electron. J. Qual. Theory Differ. Equ. *2016* (2016), Article ID 104, 6 pages. zbl MR doi

Authors' addresses: *Peixin Zhang*, School of Mathematical Sciences, Huaqiao University, Quanzhou 362021, P. R. China, e-mail: `zhpx@hqu.edu.cn`; *Mingxuan Zhu* (corresponding author), School of Mathematical Sciences, Qufu Normal University, Qufu 273100, P. R. China, e-mail: `mxzhu@qfnu.edu.cn`.