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# PROFESSOR ALEXANDER ŽENÍŠEK PASSED AWAY

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Alexander Ženíšek (photo Jan Franců)

On December 30, 2020, an outstanding Czech mathematician and physicist, Prof. RNDr. Alexander Ženíšek, DrSc., passed away in Brno after a long illness. He was a world expert in numerical solution of problems of mathematical physics and one of the founders of the mathematical theory of the finite element method to approximate solutions of partial differential equations.

Alexander Ženíšek was born in Brno on January 29, 1936. During the period 1954–1959, he studied physics at the Faculty of Science of Jan Evangelista Purkyně University in Brno. After that he studied mathematics at the same faculty until 1964.

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In 1967 he obtained the academic title RNDr. and a year later he defended the scientific degree Candidate of Sciences ($\approx$ PhD). In 1969 he passed habilitation to become Docent ($\approx$ Associcate Professor), but due to political reasons he did not receive this degree until 1978. In 1981 he got another scientific degree Doctor of Science (DrSc.) in the field *Approximate and Numerical Methods*. He then became Professor in the same field in 1986.

From 1959 to 1972, he worked as Assistant (later Assistant Professor) at the Department of Physics of the Faculty of Mechanical Engineering at the Brno University of Technology (VUT). Then he moved to the position of Researcher at the Laboratory of Computing Machines at VUT, which was later renamed to the Regional Computing Center at the Brno University of Technology. From 1981 to 1990, he held the position of Chief Researcher there. In February 1990, however, he got the position of Professor of Mathematics at the Department of Mathematics of the Faculty of Mechanical Engineering at VUT and from 1994 until 2003 he was the director there. In 1993, he founded and subsequently developed a five-year professional study of Mathematical Engineering at the Faculty of Mechanical Engineering.

The scientific activities of our colleague Alexander Ženíšek were extremely rich. Already in 1969, he became famous for his article [8] on the convergence of the finite element method for solving boundary value problems of a system of elliptic equations, including the linear elasticity problem. Here, among other claims, he proved that the linear interpolant $\pi_T v$ of a smooth function $v$ on a triangle $T$ generally does not approximate $v$ well in the Sobolev $H^1$-norm if the largest angle $\gamma$ in the triangle approaches the straight angle (180°). A similar problem was also addressed by the Irish mathematician and physicist John L. Synge as early as 1957. He proved the following upper estimate, which can be written in the current standard notation as

$$\|v - \pi_T v\|_{1,T} \leqslant \frac{C(T)}{\sin \gamma}$$

if $\gamma \leqslant \gamma_0 < 180°$ for a fixed $\gamma_0$ (see [7], p. 212). However, this does not imply that the linear interpolant converges to a given function $v$ when the largest angle tends to 180°. On the other hand, Ženíšek found a lower estimate of the above norm, which diverges to infinity for a suitable quadratic function for $\gamma \to 180°$.

In the 1970s, Ženíšek (see e.g. [9], [10]) focused on the construction of finite elements for the numerical solution of partial differential equations of order $2k$, such as the fourth order biharmonic equation with given boundary conditions or elasticity of shells. His polynomial elements of class $C^{(k-1)}$ match up in such a way that at the boundaries of individual adjacent triangles in a given triangulation all derivatives up to the order $k$ are continuous, see Figure 1.

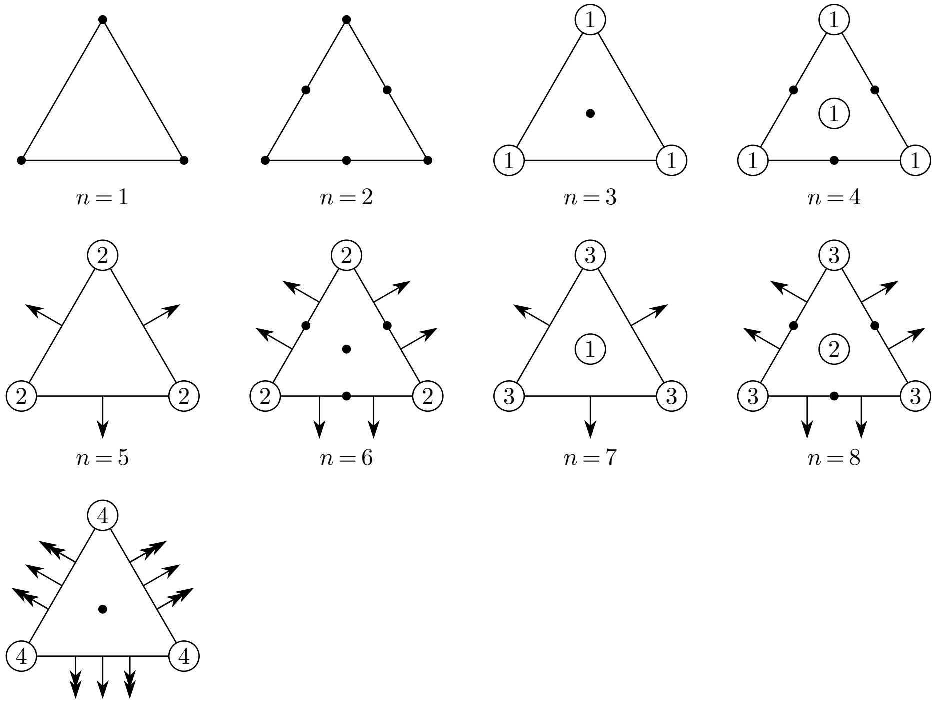


Figure 1. Ženíšek's finite elements of a polynomial degree $n$. The case of $n \in \{5, 6, 7, 8\}$ corresponds to $C^1$ finite elements, while $n = 9$ corresponds to $C^2$ finite elements. The associated degrees of freedom are denoted in the standard way.

The number of degrees of freedom $N$ of Ženíšek's finite elements is equal to triangular numbers, namely,

$$N = \frac{1}{2}(n + 1)(n + 2),$$

where $n$ stands for the degree of a polynomial in two variables. His elements are unisolvent which means that for $N$ prescribed values of degrees of freedom there exists exactly one polynomial of degree $n$ attaining these values.

Professor Ženíšek published 7 papers in Numerische Mathematik. For instance, in 1987 and 1988 he wrote together with Prof. Miloslav Feistauer from the Faculty of Mathematics and Physics of Charles University two fundamental articles [2], [3] on variational crimes in solving nonlinear second order elliptic boundary value problems. The term *variational crime* was introduced by the famous American mathematician Gilbert Strang. The point is that the solution of partial differential equations is usually converted to the minimization of a certain integral functional that contains derivatives of lower order than the original problem. However, when solving such a variational problem numerically, we commit several transgressions, the so-called variational crimes, being

1. numerical integration on individual elements in a given triangulation,
2. approximation of a curved boundary by a polytopic one,
3. interpolation of boundary conditions.

Feistauer and Ženíšek derived the corresponding a priori error estimates and proved that the finite element method modified in this way converges. All the above-mentioned results are cited in the famous Handbook of Numerical Analysis, vol. II (eds. P. G. Ciarlet and J. L. Lions), 1991.

In [11], Ženíšek investigated discrete forms of Friedrichs' inequalities which are needed in various existence proofs. In particular, he proved that the seminorm in finite element spaces $V_h$ with some mixed boundary conditions is uniformly equivalent to the Sobolev $H^1$-norm.

Professor Ženíšek also dealt with many other issues of numerical solution of problems in mathematical physics. Let us mention, for example, calculations of a steady-state magnetic field, evolution problems of parabolic type, problems with discontinuous coefficients [13], and regions whose boundaries contain turning points of cusp type [14]. For this purpose, he introduced special degenerate Lagrange finite elements. Later he also investigated together with his PhD student Jana Zlámalová degenerate Hermite $C^0$-tetrahedral finite elements [17].

Alexander Ženíšek published about 20 papers in the journal Applications of Mathematics. For example, in his last paper, together with Pavel Doktor, he proved the density of infinitely differentiable functions in special Sobolev spaces with mixed boundary conditions on three-dimensional domains [1].

Professor Ženíšek also published a number of excellent monographs. We briefly mention only the most important ones. In 1971, together with V. Kolář, J. Kratochvíl, and M. Zlámal, he wrote one of the first theoretical books about the finite element method, see [5]. A year later, A. Ženíšek with B. Klimeš and J. Kracík wrote another book [4]. Together with V. Kolář, J. Kratochvíl, and F. Leitner he then published further important monograph [6] that became a practical guide on how to calculate deformations and mechanical stresses for specific real-life problems from technical practice. Alexander Ženíšek is the sole author of another very extensive monograph *Nonlinear Elliptic and Evolution Problems and Their Finite Element Approximations* [12]. Its introduction was written by the famous French mathematician Pierre A. Raviart from the University of Pierre and Marie Curie in Paris. Further monograph by A. Ženíšek on Sobolev spaces [15] won the Josef Hlávka Award for Scientific Literature. He wrote a four-hundred-page book on bridge theory and enriched the bridge literature with precise analyzes of his opponents' games. In addition to several poetry collections (e.g. *The Fourth Side of a Triangle* under his artistic pseudonym

Vendelín Verpán), we must also mention Ženíšek's book *Relativity in the Pocket* [16], which familiarizes the reader with the foundations of Einstein's theory of relativity in an accessible form. For instance, little is known that the Doppler effect may produce opposite effects for relativistic speeds than those predicted by the Special Theory of Relativity, namely, time dilatation and length contraction, see [16].

In May 1994, Professor Ženíšek became one of the founding members of the Learned Society of the Czech Republic. In 2001, he received the Gold Medal of the Faculty of Mechanical Engineering at the Brno University of Technology for his lifelong contributions to the development of science. After retiring in 2005 he became Professor Emeritus at the VUT.

With the departure of Professor Alexander Ženíšek, our mathematics and physics community loses a respected colleague and an expert with a wide range of scientific interests. He also had many hobbies. For instance, he was a charismatic bass singer, a passionate bridge player, and a gifted poem composer. Personally, I appreciate the most that I could discuss with him various paradoxes in the special and general theory of relativity and practical applications of the finite element method at several international conferences. The mathematical foundations of this method were laid mainly in Brno in the sixties and he was one of the leading persons from this well-known school. We will all dearly miss his gentle humor, profound erudition, and enthusiasm for science.

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