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COLEMAN AUTOMORPHISMS OF FINITE GROUPS WITH A SELF-CENTRALIZING NORMAL SUBGROUP

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Abstract. Let G be a finite group with a normal subgroup N such that $C_G(N) \leqslant N$. It is shown that under some conditions, Coleman automorphisms of G are inner. Interest in such automorphisms arose from the study of the normalizer problem for integral group rings.

Keywords: Coleman automorphism; integral group ring; the normalizer property

MSC 2020: 20C05, 16S34, 20C10

1. Introduction

Let G be a finite group and $\mathbb{Z}G$ be its integral group ring over $\mathbb{Z}$. Denote by $\mathrm{U}(\mathbb{Z}G)$ the group of units of $\mathbb{Z}G$. The normalizer problem (see [18], problem 43) of integral group rings asks whether $\mathrm{N}_{\mathrm{U}(\mathbb{Z}G)}(G) = G \cdot \mathrm{Z}(\mathrm{U}(\mathbb{Z}G))$ for any finite group G, where $\mathrm{N}_{\mathrm{U}(\mathbb{Z}G)}(G)$ and $\mathrm{Z}(\mathrm{U}(\mathbb{Z}G))$ denote the normalizer of G in $\mathrm{U}(\mathbb{Z}G)$ and the center of $\mathrm{U}(\mathbb{Z}G)$, respectively. If the equality is valid for G, then we say that the normalizer property holds for G.

For any $u \in \mathrm{N}_{\mathrm{U}(\mathbb{Z}G)}(G)$, we write φ_u to denote the automorphism of G induced by u via conjugation, i.e., $g^{\varphi_u} = u^{-1}gu$ for all $g \in G$. All such automorphisms of G form a subgroup of $\mathrm{Aut}(G)$, denoted by $\mathrm{Aut}_{\mathbb{Z}}(G)$. Obviously we have $\mathrm{Inn}(G) \leqslant \mathrm{Aut}_{\mathbb{Z}}(G)$. Question 3.7 in [10] asks whether $\mathrm{Aut}_{\mathbb{Z}}(G) = \mathrm{Inn}(G)$ for any finite group G. It is easy to see that this question is equivalent to the normalizer problem.

Coleman automorphisms of finite groups have an intimate connection with the normalizer problem. Recall that an automorphism σ of a finite group G is called a *Coleman automorphism* if the restriction of σ to each Sylow subgroup of G equals

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the restriction of some inner automorphism of G. All such automorphisms of G form a subgroup of $\mathrm{Aut}(G)$, denoted by $\mathrm{Aut}_{\mathrm{Col}}(G)$. Set $\mathrm{Out}_{\mathrm{Col}}(G) = \mathrm{Aut}_{\mathrm{Col}}(G)/\mathrm{Inn}(G)$, $\mathrm{Out}_{\mathbb{Z}}(G) = \mathrm{Aut}_{\mathbb{Z}}(G)/\mathrm{Inn}(G)$. It is known by Coleman's lemma (see [1]) that $\mathrm{Out}_{\mathbb{Z}}(G) \leqslant \mathrm{Out}_{\mathrm{Col}}(G)$. Thus, if we can show that $\mathrm{Out}_{\mathrm{Col}}(G) = 1$ under some conditions, then $\mathrm{Out}_{\mathbb{Z}}(G) = 1$. Recently, a large number of positive results on the normalizer problem have been obtained by several authors. For instance, Hertweck and Kimmerle in [8] proved that $\mathrm{Out}_{\mathrm{Col}}(G) = 1$ for a quasinilpotent group G. It follows that the normalizer property holds for finite quasinilpotent groups. Van Antwerpen in [19] proved that all Coleman automorphisms of a finite group with a self-central normal p-subgroup are inner. Petit Lobão and Sehgal in [16] showed that the normalizer property holds for the wreath product $G = N\mathrm{wr}S_m$ of a finite nilpotent group N by symmetric group S_m. In addition, other affirmative results concerning this problem can also be found in [1], [2], [3], [4], [7], [11], [12], [13], [14], [15].

Recall that the wreath product of N by H is the regular wreath product and denoted by $N\mathrm{wr}H$, where N and H are two finite groups. The aim of the present paper is to investigate Coleman automorphisms of extensions of some finite groups. The motivation for our study arises from the metabelian group constructed by Hertweck in [5] for which the normalizer property fails to hold. However, it is known that the normalizer property holds for any abelian group. In addition, Hertweck in [6] constructed a group $G = (C_3 \times C_5) \rtimes C_2$ for which $\mathrm{Out}_{\mathrm{Col}}(G) \cong C_2$. This example also illustrates that if G is an extension of a finite nilpotent group by an abelian group, then in general it is not the case that $\mathrm{Out}_{\mathrm{Col}}(G) = 1$. However, in this paper we shall prove the following results.

Theorem 1.1. *Let G be a finite group with a normal subgroup N such that $C_G(N) \leqslant N$. Assume that $H^1(G/N, Z(N)) = 1$ and regard $H/\mathrm{Inn}(N)$ as the image of G/N in $\mathrm{Out}(N)$. If $H/\mathrm{Inn}(N)$ is self-normalizing in $\mathrm{Out}(N)$, then every Coleman automorphism of G is inner. In particular, the normalizer property holds for G.*

Theorem 1.2. *Let G be a finite group with a nontrivial nilpotent normal subgroup N. Assume that the center of G/N is trivial and G/N acts faithfully on the center of each Sylow subgroup of N. Then every Coleman automorphism of G is inner. In particular, the normalizer property holds for G.*

As a direct consequence of Theorem 1.2, we have:

Corollary 1.1. *Let $G = N\mathrm{wr}H$ be the wreath product of a nontrivial finite nilpotent group N by a centerless finite group H. Then every Coleman automorphism of G is inner. In particular, the normalizer property holds for G.*

Finally, we fix some notation used in this paper. Let σ be an automorphism of a finite group G and H be a subgroup of G. Denote by $\sigma|_H$ the restriction of σ to H. Let N be a normal subgroup of G. If σ fixes N, i.e., $N^\sigma = N$, then σ induces an automorphism of G/N, which is denoted by $\sigma|_{G/N}$. Let $x \in G$ be a fixed element. Denote by $\mathrm{conj}(x)$ the automorphism of G induced by x via conjugation, i.e., $g^{\mathrm{conj}(x)} = g^x$ for any $g \in G$. Denote by $\pi(G)$ the set of all primes dividing the order of G. For any $p \in \pi(G)$, we use $O_p(G)$ to denote the largest normal p-subgroup of G and $O_{p'}(G)$ to denote the largest normal p'-subgroup of G, respectively. Other notation used will be mostly standard, refer to [8], [17], [18].

2. Preliminaries

In this section, some lemmas needed in the sequel are presented.

Lemma 2.1 ([8], Proposition 1)**.** *Let G be a finite group. Then the prime divisors of $|\mathrm{Aut}_{\mathrm{Col}}(G)|$ lie in $\pi(G)$, the set of prime divisors of $|G|$.*

Lemma 2.2. *Let $\sigma \in \mathrm{Aut}_{\mathrm{Col}}(G)$ and N be a normal subgroup of G. Then*
(1) $\sigma|_N \in \mathrm{Aut}(N)$,
(2) $\sigma|_{G/N} \in \mathrm{Aut}_{\mathrm{Col}}(G/N)$.

P r o o f. These results are derived from the proof of Corollary 3 (i) in [8]. □

Lemma 2.3 ([9], Satz I.17.1)**.** *Let G be a group and let N be a nontrivial normal subgroup with $C_G(N) \leqslant N$ and $H^1(G/N, Z(N)) = 1$. Then any automorphism of G which fixes N element-wise is inner.*

Lemma 2.4. *Let G be a finite group with a nilpotent normal subgroup N. Assume that P is an arbitrary Sylow subgroup of N and G/N acts faithfully on $Z(P)$. Then $C_G(P) \leqslant N$. In particular, $C_G(N) \leqslant N$.*

P r o o f. Let $g \in C_G(P)$. We may set $g = xh$ with $x \in N$ and $h \in G$. Since N is a finite nilpotent group, thus $N = \times_{p\in\pi(N)} P$ and $Z(P) \neq 1$, where $P \in \mathrm{Syl}_p(N)$. For any $y \in Z(P)$, on the one hand, we have $y^g = y$. On the other hand, we have $y^g = y^{xh} = y^h$. Consequently, we obtain $y^h = y$. By assumption G/N acts faithfully on $Z(P)$. This implies that $h \in N$ and thus $g \in N$, i.e., $C_G(P) \leqslant N$. In particular, $C_G(N) \leqslant C_G(P) \leqslant N$, and we are done. □

Lemma 2.5 ([6], Lemma 2)**.** *Let p be a prime, and σ an automorphism of G of p-power order. Assume further that there is $N \trianglelefteq G$ such that σ fixes all elements of N, and that σ induces the identity on G/N. Then σ induces the identity* *on $G/O_p(Z(N))$. If σ fixes in addition a Sylow p-subgroup of G element-wise, then σ is an inner automorphism.*

Lemma 2.6. *Let $\sigma \in \mathrm{Aut}(G)$ be of p-power order and let N be a normal subgroup of G. Assume that $N^\sigma = N$ and σ induces an inner automorphism of G/N. Then there is $\gamma \in \mathrm{Inn}(G)$ such that $\sigma\gamma|_{G/N} = \mathrm{id}|_{G/N}$ and $\sigma\gamma$ is still an automorphism of p-power order.*

Proof. Let $o(\sigma) = p^m$ with $m \in \mathbb{N}$. Since σ induces an inner automorphism of G/N, there is $x \in G$ such that $\sigma|_{G/N} = \mathrm{conj}(x)|_{G/N}$. Let $\beta := \mathrm{conj}(x)$. Then we have $\sigma\beta^{-1}|_{G/N} = \mathrm{id}|_{G/N}$. Let $(\sigma\beta^{-1})^n$ be the p-part of $\sigma\beta^{-1}$ with $n \in \mathbb{N}$ and $(n,p) = 1$. Then there are $a, b \in \mathbb{Z}$ such that $an + bp^m = 1$. It is easy to see that $(\sigma\beta^{-1})^{an}$ is of p-power order and $(\sigma\beta^{-1})^{an}|_{G/N} = \mathrm{id}|_{G/N}$. Since $\mathrm{Inn(G)} \trianglelefteq \mathrm{Aut}(G)$, there is $\gamma \in \mathrm{Inn}(G)$ such that $(\sigma\beta^{-1})^{an} = \sigma^{an}\gamma = \sigma^{1-bp^m}\gamma = \sigma\gamma$. The assertions follow immediately. $\square$

Lemma 2.7 ([8], Lemma 6)**.** *Let $\sigma \in \mathrm{Aut}(G)$ and $N \trianglelefteq G$ with $N^\sigma = N$, and suppose that for some Sylow subgroup Q of N, there is $h \in G$ such that $\sigma|_Q = \mathrm{conj}(h)|_Q$. Then σ stabilizes $M = NC_G(Q) \trianglelefteq G$, and $\sigma|_{G/M} = \mathrm{conj}(h)|_{G/M}$.*

3. Proof of the theorems

Proof of Theorem 1.1. Let $\gamma \in N_{\mathrm{Aut}(N)}(H)$, then $H^\gamma = H$, which implies that

$$(H/\mathrm{Inn}(N))^\gamma = H^\gamma/\mathrm{Inn}(N) = H/\mathrm{Inn}(N).$$

Since $H/\mathrm{Inn}(N)$ is self-normalizing in $\mathrm{Out}(N)$, we have that

$$\gamma\mathrm{Inn}(N) \in N_{\mathrm{Out}(N)}(H/\mathrm{Inn}(N)) = H/\mathrm{Inn}(N).$$

Consequently, we obtain that

$$H = N_{\mathrm{Aut}(N)}(H). \tag{3.1}$$

Let σ be an arbitrary Coleman automorphism of G. By Lemma 2.2, $\sigma|_N \in \mathrm{Aut}(N)$. However, for every $g \in G$ we have

$$(\mathrm{conj}(g)|_N)^{\sigma|_N} = \mathrm{conj}(g^\sigma)|_N. \tag{3.2}$$

Additionally, G acts on N, by conjugation with kernel $C_G(N) = Z(N)$. Hence there is a homomorphism from $G/Z(N)$ into $\mathrm{Aut}(N)$, mapping $g \in G$ to $\mathrm{conj}(g)|_N$.

Note that H is the image of $G/Z(N)$ in $\mathrm{Aut}(N)$. Thus $H = \{\mathrm{conj}(g)|_N\colon g \in G\}$, and (3.2) implies that $\sigma|_N \in N_{\mathrm{Aut}(N)}(H)$. By (3.1), there exists some element $y \in G$ such that $\sigma|_N = \mathrm{conj}(y)|_N$, that is

$$\sigma\mathrm{conj}(y^{-1})|_N = \mathrm{id}|_N.$$

By Lemma 2.3, $\sigma\,\mathrm{conj}(y^{-1}) \in \mathrm{Inn}(G)$, i.e., $\sigma \in \mathrm{Inn}(G)$. This completes the proof of Theorem 1.1. $\square$

As immediate consequences of Theorem 1.1, we have:

Corollary 3.1. *Let G be a finite group with a normal subgroup N such that $C_G(N) \leqslant N$. Assume that $Z(N) = 1$ and regard $H/\mathrm{Inn}(N)$ as the image of G/N in $\mathrm{Out}(N)$. If $H/\mathrm{Inn}(N)$ is self-normalizing in $\mathrm{Out}(N)$, then every Coleman automorphism of G is inner. In particular, the normalizer property holds for G.*

Corollary 3.2. *Let G be a finite group with a normal subgroup N such that $C_G(N) \leqslant N$. Assume that $(|G/N|, |N|) = 1$ and regard $H/\mathrm{Inn}(N)$ as the image of G/N in $\mathrm{Out}(N)$. If $H/\mathrm{Inn}(N)$ is self-normalizing in $\mathrm{Out}(N)$, then every Coleman automorphism of G is inner. In particular, the normalizer property holds for G.*

P r o o f of Theorem 1.2. By Lemma 2.1, we may assume that $\sigma \in \mathrm{Aut}_{\mathrm{Col}}(G)$ is an arbitrary Coleman automorphism of p-power order. We have to show that σ is an inner automorphism of G. The proof of Theorem 1.2 splits into two cases:

Case 1: $|\pi(N)| = 1$. In this case, the group N is a normal q-subgroup of G for some prime $q \in \pi(G)$. By Lemma 2.4, $C_G(N) \leqslant N$, thus the assertion follows from Theorem 2.2 in [19].

Case 2: $|\pi(N)| > 1$. If $p \in \pi(N)$, then $G/O_{p'}(N)$ satisfies the conditions of Theorem 1.2. By Case 1, we have $\mathrm{Out}_{\mathrm{Col}}(G/O_{p'}(N)) = 1$. Note further that $\sigma \in \mathrm{Aut}_{\mathrm{Col}}(G)$ implies $\sigma|_{G/O_{p'}(N)} \in \mathrm{Aut}_{\mathrm{Col}}(G/O_{p'}(N))$. Consequently, we have $\sigma|_{G/O_{p'}(N)} \in \mathrm{Inn}(G/O_{p'}(N))$. Thus there exists some $x \in G$ such that $\sigma|_{G/O_{p'}(N)} = \mathrm{conj}(x)|_{G/O_{p'}(N)}$. Without loss of generality, by Lemma 2.6, we may assume that

$$\sigma|_{G/O_{p'}(N)} = \mathrm{id}|_{G/O_{p'}(N)}. \tag{3.3}$$

Next we shall show that $\sigma|_{O_{p'}(N)} \in \mathrm{Aut}_{\mathrm{Col}}(O_{p'}(N))$. For this purpose, let Q be an arbitrary Sylow subgroup of $O_{p'}(N)$. By the definition of Coleman automorphisms, there exists $y \in G$ such that

$$\sigma|_Q = \mathrm{conj}(y)|_Q. \tag{3.4}$$

Write $M := O_{p'}(N)C_G(Q)$. Then, on the one hand, by Lemma 2.7, we have

$$\sigma|_{G/M} = \mathrm{conj}(y)|_{G/M}. \tag{3.5}$$

On the other hand, note that $M \geqslant O_{p'}(N)$. Then, by (3.3), we have

$$\sigma|_{G/M} = \mathrm{id}|_{G/M}. \tag{3.6}$$

Consequently, (3.5) and (3.6) yield that $\mathrm{conj}(y)|_{G/M} = \mathrm{id}|_{G/M}$, which implies that $yM \in Z(G/M)$. By Lemma 2.4, $C_G(Q) \leqslant N$. Note further that N is nilpotent. Consequently, we must have $M = O_{p'}(N)C_G(Q) = N$. Thus $G/M = G/N$. Since the center of G/N is trivial, $Z(G/M) = 1$. Consequently, $yM \in Z(G/M)$ implies $y \in M$. Recall that $M = O_{p'}(N)C_G(Q) = C_G(Q)O_{p'}(N)$, so we may set $y = ch$ with $c \in C_G(Q)$ and $h \in O_{p'}(N)$. Then, by (3.4), we have

$$\sigma|_Q = \mathrm{conj}(y)|_Q = \mathrm{conj}(ch)|_Q = \mathrm{conj}(h)|_Q. \tag{3.7}$$

As Q is an arbitrary Sylow subgroup of $O_{p'}(N)$, (3.7) tells us that

$$\sigma|_{O_{p'}(N)} \in \mathrm{Aut}_{\mathrm{Col}}(O_{p'}(N)).$$

Note that $O_{p'}(N)$ is a p'-group. Then, by Lemma 2.1, $\mathrm{Aut}_{\mathrm{Col}}(O_{p'}(N))$ is also a p'-group. But σ is of p-power order, so is $\sigma|_{O_{p'}(N)}$, which forces

$$\sigma|_{O_{p'}(N)} = \mathrm{id}|_{O_{p'}(N)}. \tag{3.8}$$

Now, by Lemma 2.5, (3.3) and (3.8) yield that

$$\sigma|_{G/O_p(Z(O_{p'}(N)))} = \mathrm{id}|_{G/O_p(Z(O_{p'}(N)))}. \tag{3.9}$$

Note again that $O_{p'}(N)$ is a p'-group, so $O_p(Z(O_{p'}(N))) = 1$ and thus (3.9) implies that $\sigma = \mathrm{id}$.

If $p \notin \pi(N)$, let $q \in \pi(N)$. Note that $G/O_{q'}(N)$ satisfies the conditions of Theorem 1.2, so, by Case 1, $\mathrm{Out}_{\mathrm{Col}}(G/O_{q'}(N)) = 1$. Note further that $\sigma \in \mathrm{Aut}_{\mathrm{Col}}(G)$ implies $\sigma|_{G/O_{q'}(N)} \in \mathrm{Aut}_{\mathrm{Col}}(G/O_{q'}(N))$. Consequently, we have $\sigma|_{G/O_{q'}(N)} \in \mathrm{Inn}(G/O_{q'}(N))$. Using an identical argument as in $p \in \pi(N)$, we can show that $\sigma \in \mathrm{Inn}(G)$, so we omit it. In all cases, we have $\mathrm{Out}_{\mathrm{Col}}(G) = 1$. This completes the proof of Theorem 1.2. □

P r o o f of Corollary 1.1. Let $|H| = m$. Then $G = N\mathrm{wr}H = N^m \rtimes H$, where N^m is the direct product of m copies of N. Let $p \in \pi(N)$ and let $P \in \mathrm{Syl}_p(N^m)$. We will show that H acts faithfully on $Z(P)$. Since N is a nilpotent group, thus N^m is also a nilpotent group. Obviously, H acts on $Z(P)$ for any $y \in Z(P)$, if $y^h = y$, where $h \in H$. Since the intersection of $Z(P)$ with each component of N^m is nontrivial, i.e., $Z(P)$ is extensive in N^m, we deduce that $h = 1$, and this shows that H acts faithfully on $Z(P)$. Thus the assertion follows from Theorem 1.2. □

As an immediate consequence of Corollary 1.1, we have:

Corollary 3.3. *Let $G = N\mathrm{wr}S_m$ $(m \geqslant 3)$ be the wreath product of N by S_m, where N is a finite nilpotent group and S_m is a symmetric group of degree m. Then every Coleman automorphism of G is inner, i.e.,* $\mathrm{Out}_{\mathrm{Col}}(G) = 1$.

Remark 3.1. The restriction that H is a centerless finite group in Corollary 1.1 cannot be removed. This fact is illustrated by the following example.

Example 3.1. Let $G = C_6\mathrm{wr}S_2$ be the natural wreath product of a cyclic group C_6 of order 6 by S_2. Then $\mathrm{Out}_{\mathrm{Col}}(G) \cong C_2 \neq 1$.

P r o o f. This is a direct consequence of Theorem 3.2 in [19]. □

Example 3.1 shows that in general it is not the case that $\mathrm{Out}_{\mathrm{Col}}(G) = 1$ for $G = N\mathrm{wr}S_2$ with N being a finite nilpotent group. As a special case of Theorem 1 in [16], we have the following result:

Theorem 3.1. *Let $G = N\mathrm{wr}S_2$ be the wreath product of a finite nilpotent group N by a symmetric group S_2 of degree 2. Then* $\mathrm{Out}_{\mathbb{Z}}(G) = 1$.

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