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ADAPTIVE TRACKING VIA PINNING IN NETWORKS OF NONIDENTICAL NODES

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We investigate the control of dynamical networks for the case of nodes, that although different, can be make passive by feedback. The so-called V-stability characterization allows for a simple set of stabilization conditions even in the case of nonidentical nodes. This is due to the fact that under V-stability characterization the dynamical difference between node of a network reduces to their different passivity degrees, that is, a measure of the required feedback gain necessary to make the node stable at a desired solution. We propose a pinning control strategy that extends this approach to solve the tracking problem, furthermore using an adaptive controller approach we provide a methodology to impose a common reference trajectory to a network of different nodes by pinning only a few of them to the desired solution. We illustrate our results with numerical simulation of well-known benchmark systems.

Keywords: complex networks, tracking problem, pinning control, adaptive control

Classification: 05C82, 34H05, 49K15

1. INTRODUCTION

Many complex systems of interest can be modeled as networks, including the Internet, WWW, genetic regulation, social groups and metabolic reactions, amount many other interesting examples [3, 10]. The dynamical analysis of the different collective behaviors that can occur in networks have become of great interest in recent years. The analysis of networks differs from that of general dynamical systems in the fact that the behavior is determine by two components: The rules governing the evolution of the individual nodes; and the information flows traveling along the connecting structure of the network. In other words, isolated dynamics of its nodes and its network topology [1, 13, 14]. This is further complicated in the case of networks with different node dynamics [6, 18]. In the literature the issue of nonidentical nodes has been addressed in different ways. One can see, for example the synchronization problem in almost identical or structurally different systems as both part of the nonidentical problem setting, yet they are significantly different problems [2]. In this contribution, we focus on nonidentical nodes that although different are of the same dimension and more significantly can be make passive by feedback.

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Conventionally the first thing one does when analyzing the dynamics of a system is to determine its stability. Therefore, we start by analyzing the stability properties of the nodes in isolation, then the effect of the structural characteristics of its connections are determine. To this end, we use a basic concept in nonlinear dynamical systems, e. g., its energy. Consider that there is an amount of feedback control that makes the node passive. Then, there is an among of coupling required to preserve the overall dissipative nature of the nodes once they are interconnected. Briefly, we look for a common Lyapunov function $V(x)$ for all nodes in the network, which is constructed such that for each node one can determine a passivity degree, that is, a scalar parameter indicating the effort needed to stabilize the node by feedback, which allows to construct a Lyapunov function for the coupled nodes that has its derivative along the node's trajectories negative definitive. Describing the network dynamics in this way we obtain its so-called V-stability characterization of the network [4, 17]. Then, the effect of the network topology can be derived from the eigenspectrum of its Laplacian matrix. In particular, for nonidentical nodes, the V-stability characterization of the network has the advantage that replaces the actual node dynamics by its degree of passivity independently of its actual dynamical, under mild conditions of existence. Despite the conservativeness associated with Lyapunov stability analysis, using the V-stability characterization of the network has the marked advantage of providing simple conditions for stabilization of network. Furthermore, using the pinning control strategy, stabilization can be achieved even when only a small fraction of the nodes are controlled [8, 15, 16]. The pinning strategy has been used in the synchronization of networks [11, 19] and even for the lag synchronization between two distinct networks [12]. In this contribution we propose an adaptive version of the pinning strategy design to stabilize a network of nonidentical nodes, and show that the proposed adaptive pinning strategy can be use to force the network to track a desired solution.

The remainder of this paper is organized as follows: In Section 2, a detail description of the V-stability characterization of a networks with nonidentical nodes is presented. The pinning control problem for networks along with some of its variants including our proposed adaptive pinning strategy are described in Section 3, where previous results are extended to the tracking problem, that is imposition of a desired trajectory in the network. A numerical illustration of the approach proposed is presented in Section 4. Finally, the contribution is concluded with final comments in Section 5.

2. V-STABILITY OF DYNAMICAL NETWORKS

Consider a network of N linearly and diffusively coupled n-dimensional dynamical systems, its state equation is given by

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1, j\neq i}^{N} c_{ij}\Gamma[x_j(t) - x_i(t)], \text{ for } i = 1, 2, \dots, N \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^\top \in \mathbf{R}^n$ is the state variable of the ith node, and $f_i : \mathbf{R}^n \to \mathbf{R}^n$ is at least locally Lipschitz and describes the dynamics of the ith node in isolation. The internal coupling between nodes is given by the zero-one diagonal matrix $\Gamma \in \mathbf{R}^{n\times n}$, while the weighted connection between nodes is described by the external coupling matrix $C = \{c_{ij}\} \in \mathbf{R}^{N\times N}$ in the following way: if the ith node is connected to the jth node ($j \neq i$), then $c_{ij} = c_{ji} > 0$; otherwise $c_{ij} = c_{ji} = 0$. The diagonal elements satisfy the diffusive condition: $c_{ii} = -\sum_{j=1,j\neq i}^{N} c_{ij}$, for $\forall i$. That is C is zero-sum by rows and columns $\sum_{j=1}^{N} c_{ij} = \sum_{j=1}^{N} c_{ji} = 0$. Under these conditions, the network can be rewritten as:

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{N} c_{ij}\Gamma x_j(t), \text{ for } i = 1, 2, \ldots, N \tag{2}$$

Our first assumption is that there is a common equilibrium state.

Assumption 1. For each node in the network there is an common equilibrium state, $\bar{x} \in \mathbf{R}^n$, satisfying $f_i(\bar{x}) = 0$, for $i = 1, 2, \ldots, N$, such that, for the entire network there is a stationary homogeneous equilibrium state

$$\bar{X} = (\bar{x}^\top, \bar{x}^\top, \ldots, \bar{x}^\top)^\top \in \mathbf{R}^{nN} \tag{3}$$

Locally the stability of equilibrium state solution of the network can be determine linearizing its dynamics around $\bar{X}$ [16]. To characterize the stability of the equilibrium solution we make the following assumption.

Assumption 2. There is a continuously differentiable Lyapunov function $V(x(t)) : D \subseteq \mathbf{R}^n \to \mathbf{R}_+$ satisfying $V(\bar{x}) = 0$ with $D = \bigcup_{i=1}^{N} D_i$, $D_i = \{x_i(t) : \|x_i(t) - \bar{x}\| < \alpha\}$, $\alpha > 0$ and $\bar{x} \in D$, such that for each node function $f_i(x_i(t))$, there is a scalar θ_i guaranteeing

$$\frac{\partial V(x_i(t))}{\partial x_i(t)}(f_i(x_i(t)) - \theta_i\Gamma(\bar{x} - x_i(t))) < 0 \tag{4}$$

for all $x_i(t) \in D_i$, $x_i(t) \neq \bar{x}$, $i = 1, 2, \ldots, N$. Where for each node the value θ_i is called its *passivity degree* [17].

For simplicity let $\bar{X} = 0$. Then, a Lyapunov function for the entire network is given by:

$$V_{all}(X(t)) = \sum_{i=1}^{N} V(x_i(t)) \tag{5}$$

with $X(t) = (x_1(t)^\top, x_2(t)^\top, \ldots, x_N(t)^\top)^\top \in \mathbf{R}^{nN}$. Its time derivative along the trajectories of $X(t)$ is given by

$$\dot{V}_{all}(X(t)) = \sum_{i=1}^{N} \frac{\partial V(x_i(t))}{\partial x_i} f_i(x_i(t)) + \sum_{i=1}^{N} \frac{\partial V(x_i(t))}{\partial x_i} \sum_{j=1}^{N} c_{ij}\Gamma x_j(t). \tag{6}$$

From Assumption 2 one has $V_{all}(\bar{X}) = 0$ and $\dot{V}_{all}(\bar{X}) = 0$. Then, for $X(t) \neq \bar{X} = 0$, the following inequality is found

$$\begin{aligned} \dot{V}_{all}(X(t)) &< -\textstyle\sum_{i=1}^{N} \frac{\partial V(x_i(t))}{\partial x_i}\theta_i\Gamma x_i(t) + \sum_{i=1}^{N} \frac{\partial V(x_i(t))}{\partial x_i} \sum_{j=1}^{N} c_{ij}\Gamma x_j(t) \\ \dot{V}_{all}(X(t)) &< M(X(t)) \end{aligned} \tag{7}$$

where

$$M(X(t)) = \sum_{i=1}^{N} \frac{\partial V(x_i(t))}{\partial x_i} \left(-\theta_i \Gamma x_i(t) + \sum_{j=1}^{N} c_{ij} \Gamma x_j(t) \right). \tag{8}$$

Then, it follows that the network (2) is asymptotically stable about its equilibrium point if $M(X(t)) < 0$ for all $x(t) \in \mathcal{D}$, with $\mathcal{D} = D_1 \times D_2 \times \ldots \times D_N \subseteq \mathbf{R}^{nN}$. That is, the network (2) is locally exponentially stable about its equilibrium point if

$$\begin{aligned} &M(X(t)) \leq -\mu_1 \|X(t)\|^2 \\ &\mu_2 \|X(t)\|^2 \leq V_{all}(X(t)) \leq \mu_3 \|X(t)\|^2 \end{aligned} \tag{9}$$

for some constants $\mu_1,\ \mu_2,\ \mu_3 > 0$ for all $X(t) \in \mathcal{D}$. Moreover, the region of attraction is given by

$$\Omega = \{X(t) : V_{all}(X(t)) < r\} \tag{10}$$

with $r = \inf_{X(t) \in \mathcal{D}} V_{all}(X(t))$. In the case of $\Omega = \mathbf{R}^{nN}$, the stability becomes global.

Lets consider that there is a common monomial quadratic Lyapunov function $V(x(t)) = \frac{1}{2} x(t)^\top Q x(t)$ where $Q = Q^\top > 0$, that satisfies Assumption 2, with θ_i the passivity degree value of the ith node in the network. Then, $\frac{\partial V(x_i(t))}{\partial x_i} = x_i(t)^\top Q$, and (8) becomes

$$M(X(t)) = \sum_{i=1}^{N} x_i(t)^\top \left(-\theta_i Q \Gamma x_i(t) + \sum_{j=1}^{N} c_{ij} Q \Gamma x_j(t) \right) \tag{11}$$

which in Kronecker product notation becomes

$$M(X(t)) = X(t)^\top [(-\Theta + C) \otimes Q\Gamma] X(t) \tag{12}$$

where $\Theta = Diag(\theta_1, \theta_2, \ldots, \theta_N) \in \mathbf{R}^{N \times N}$. For the time derivative of the Lyapunov function to be definitive negative $(-\Theta + C) \otimes Q\Gamma$ must be definitive negative, or equivalently the following inequalities must hold [17]:

$$Q\Gamma + \Gamma^\top Q \geq 0 \tag{13}$$

$$-\Theta + C \leq 0. \tag{14}$$

An important remark is that using this approach the stability of the equilibrium solution of the network can be determine from the passivity degree of the nodes. In particular, if all the passivity degree values are positive $\theta_i > 0$, then the network is V-stable about its equilibrium point. In particular, for nodes that don't have a positive passivity degree a pinning control strategy can be use to make the entire network V-stable [4, 17, 11].

3. V-STABILITY BY PINNING CONTROL

Without loss of generality, assume that the first l nodes in the network are pinning to the desired equilibrium solution ($\bar{x} = 0$) via controllers of the form

$$u_i(t) = -K_i x_i(t), \text{ for } i = 1, 2, \ldots, l. \tag{15}$$

In [5] it was shown that a network of identical nodes can be pinned to a common equilibrium point by a single pinning controller, with the shortcoming of requiring an extremely large gain. In the case of nonidentical nodes, this result has not been establish, as the difference between nodes can be arbitrary and these differences may require control actions beyond what a single controller can provide, even at very large gain values. Using the V-stability approach in [17] a bound on the number of controllers related to the passivity degree is argued to be the number of eigenvalues that need to change to made $M(X(t))$ negative definitive. However, since the performance of the controlled network depends on the coupling structure and node dynamics, to determine the optimal control gains becomes a multifactoral problem, which must additionally take into account the amount of energy required to achieved the desired control objective.

Under the action of (15) the dynamics of an isolated node in close-loop becomes

$$\dot{x}_i(t) = f_i(x_i(t)) - B_i K_i x_i(t) \tag{16}$$

where $K_i \in \mathbf{R}^{m\times n}$ and $B_i \in \mathbf{R}^{n\times m}$ are the pinning gain and the input matrices of the ith node, respectively.

Following a similar derivation as above, the passivity degree of a controlled nodes can be determine from Assumption 2, then we have

$$\frac{\partial V(x_i(t))}{\partial x_i}\left(f_i(x_i(t)) - B_i K_i x_i(t) - \theta_i \Gamma(\bar{x} - x_i(t)) + \kappa_i \Gamma x_i(t)\right) < 0 \tag{17}$$

for all $x_i(t) \in \mathcal{D}_i \subseteq \mathcal{D}$, $x_i(t) \neq 0$, with $i = 1, 2, \ldots, l$ and $\bar{x} = 0$. T Where the additional constants $\kappa_i \geq 0$ represent the effect of the local controller on the level of passivity of ith node. Then, we have

$$\frac{\partial V(x_i(t))}{\partial x_i}\left(f_i(x_i(t)) - B_i K_i x_i(t) + (\theta_i + \kappa_i)\Gamma x_i(t)\right) < 0. \tag{18}$$

Assuming that the Lyapunov function is quadratic, the previous derivation holds and the V-stability condition becomes that the characteristic matrix of the controlled network be negative definitive

$$\mathcal{P} = -\Theta + C - \mathcal{K} \tag{19}$$

where $\mathcal{K} \in \mathbf{R}^{N\times N}$ is a diagonal matrix with l elements given by κ_i, $i = 1, 2, \ldots, l$, and its remaining $N - l$ diagonal elements zeros.

The contribution of the pinning controllers can make the entire network V-stable. The questions of which and how many nodes must be chosen to make the network V-stable are of great interest and remain open problems that can be addressed from different perspectives like control energy and structural features [4, 11, 9]. Using simple linear algebra arguments, in [17] an argument is made to chose as many nodes as nonnegative eigenvalues of the matrix $-\Theta + \mathcal{C}$, provided that the coupling strength be sufficiently large.

The above approach can be extended to address the tracking problem in networks of nonidentical nodes. To that end, we first assume the following:

Assumption 3. The control objective for the entire network is that each node tracks the reference dynamics

$$\dot{s}(t) = f(s(t)) \tag{20}$$

where $f : \mathbf{R}^n \to \mathbf{R}^n$ is at least locally Lipschitz.

It is further assume that:

Assumption 4. For each node in the network there are real constants γ_i such that

$$\|f(s(t)) - f_i(x_i(t))\| \leq \gamma_i \|s(t) - x_i(t)\|, \text{ for } \forall t \text{ and } i = 1, 2, \ldots, N. \tag{21}$$

The inequality (21) is perhaps restrictive, yet is satisfied by different benchmark chaotic systems like the ones used in the numerical simulation. Furthermore, this assumption can be replace by the QUAD condition as in [7] with similar results.

The dynamics of the tracking errors $e_i(t) = s(t) - x_i(t)$ is obtained from (2) and (20) to be

$$\dot{e}_i(t) = f(s(t)) - \left[f_i(x_i(t)) + \sum_{j=1}^{N} c_{ij} \Gamma e_j + \nu_i(t) \right] \text{ for } i = 1, 2, \ldots, N \tag{22}$$

where the controllers $\nu_i(t)$ are design such that the entire network be V-stable about the solution $S(t) = \left[s(t)^\top, \ldots, s(t)^\top\right]^\top \in \mathbf{R}^{nN}$. As shown above, this can be achieved by applying the controllers only to l nodes in the network. In particular, we propose to use the following feedback controller

$$\nu_i(t) = -K_i(t) \Gamma e_i(t), \tag{23}$$

where the control gains are generated by the adaptive law:

$$\dot{K}_i(t) = \alpha_i e_i(t)^\top e_i(t), \text{ for } i = 1, 2, \ldots, l \tag{24}$$

note that the controller gains for $l+1, l+2, \ldots, N$ are set to zero.

We endeavor to use the V-stability approach to design the controllers ν_i, therefore we make the additional assumption that each tracking error has a passivity degree. That is:

Assumption 5. Consider that for each tracking error (22) there exist a Lyapunov function such that

$$\frac{\partial V(x_i(t))}{\partial x_i} (F_i(e_i(t), s(t)) - \hat{\Theta}_i \Gamma e_i(t)) < 0, \text{ for } i = 1, 2, \ldots, N \tag{25}$$

where $F_i(e_i(t), s(t)) = f(s(t)) - \left[f_i(x_i(t)) + \sum_{j=1}^{N} c_{ij} \Gamma e_j + \nu_i(t) \right]$ and $\hat{\Theta}_i$ is the passitivy degree of the ith tracking error.

The main restriction to solve the tracking problem using the V-stability approach is in fact the existence of the passivity degree $\hat{\Theta}_i$ for the tracking errors. The existence of $\hat{\Theta}_i$ is directly related to the reference solution, in the case of identical nodes if the reference solution is a solution of the nodes, this problem is similar to pinning synchronization as presented in [11].

Following the same procedure as before, the stability of the zero solution of the tracking error dynamics can be determine using the Lyapunov function

$$V_{all}(E(t)) = \frac{1}{2}\sum_{i=1}^{N}(e_i(t)^\top e_i(t) + \frac{1}{\alpha_i}(K_i(t) - K^*)^2 \tag{26}$$

with $E(t) = (e_1(t)^\top, e_2(t)^\top, \ldots, e_N(t)^\top)^\top \in \mathbf{R}^{nN}$. Its time derivative along the trajectories of $E(t)$ is given by

$$\dot{V}_{all}(E(t)) = \sum_{i=1}^{N} e_i(t)^\top \left[\hat{\Theta}_i\Gamma + \sum_{j=1}^{N} c_{ij}\Gamma + (K_i(t) - K^*)\right] e_i(t). \tag{27}$$

Then, following a similar derivation as before, for a sufficiently large $K^* > 0$ the adaptive pinning controller will stabilize the zero fixed point of the tracking error making the characteristic matrix $\hat{\mathcal{P}} = -\hat{\Theta} + \mathcal{C} - \hat{\mathcal{K}}$ negative definitive where the first l diagonal elements of $\hat{\mathcal{K}}$ are the contribution of the pinning controllers and the remaining $N - l$ are zero.

4. NUMERICAL SIMULATIONS

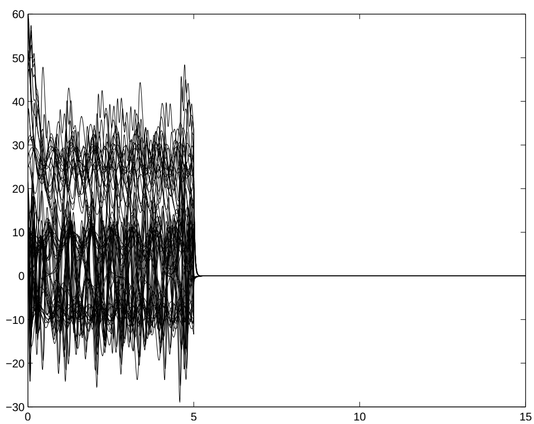

Fig. 1. Stabilizing a network of $N = 30$ nonidentical nodes with one controller per type ($l = 3$) to a common fixed point ($\bar{x} = [0, 0, 0]^\top$).

To illustrate our theoretical results we consider a network with thirty nodes of three different types. In particular, ten are Lorenz dynamical systems [6], ten are Chen systems

[14], and the remaining ten nodes are dimensionless Chua's circuits [13]. The equations that describe these systems are (28), (29), and (30), respectively.

$$\dot{x}_L(t) = \begin{bmatrix} a_L(x_{L2}(t) - x_{L1}(t)) \\ c_L x_{L1}(t) - x_{L2}(t) - x_{L1}(t)x_{L3}(t) \\ x_{L1}(t)x_{L2}(t) - b_L x_{L3}(t) \end{bmatrix} \tag{28}$$

$$\dot{x}_C(t) = \begin{bmatrix} a_C(x_{C2}(t) - x_{C1}(t)) \\ (c_C - a_C)x_{C1}(t) + c_C x_{C2}(t) - x_{C1}(t)x_{C3}(t) \\ x_{C1}(t)x_{C2}(t) - b_C x_{C3}(t) \end{bmatrix} \tag{29}$$

$$\dot{x}_c(t) = \begin{cases} A_{c1}x_c(t) + B_{c1}, & if \quad x_{c1}(t) > 1 \\ A_{c2}x_c(t) + B_{c2}, & if \quad |x_{c1}(t)| \leq 1 \\ A_{c3}x_c(t) + B_{c3}, & if \quad x_{c1}(t) < -1 \end{cases} \tag{30}$$

where the parameter values are chosen to be $a_L = 10$, $b_L = \frac{8}{3}$, and $c_L = 28$ for the Lorenz system; $a_C = 35$, $b_C = 3$, and $c_C = 28$ for the Chen systems; while for the Chua's circuit the parameters are $A_{c1} = A_{c3} = \begin{bmatrix} -\alpha(1+m_0) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$, $A_{c2} = \begin{bmatrix} -\alpha(1+m_1) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$, $B_{c1} = [-\alpha(m_1 - m_0), 0, 0]^\top$, $B_{c2} = [0, 0, 0]^\top$, $B_{c3} = [\alpha(m_1 - m_0), 0, 0]^\top$ with $\alpha = 9$, $\beta = \frac{100}{7}$, $m_0 = -\frac{5}{7}$, and $m_1 = -\frac{8}{7}$. Is important to remark that for these parameter set the solutions of all nodes in isolation are bounded and evolve in their well-known chaotic attractors [6].

A common Lyapunov function for all the nodes in the network is given by

$$V(x(t)) = \frac{1}{2}x(t)^\top Qx(t), \text{ with } Q = Diag(1, 1, 1). \tag{31}$$

The passivity degree θ for each system type is easily calculated from (4). In the case of the Lorenz systems, with $\Gamma = I_3$, the inequality becomes

$$38x_{L1}x_{L2} - (10x_{L1}^2 - \theta_L x_{L1}^2 + x_{L2}^2 - \theta L x_{L2}^2 + \frac{8}{3}x_{L3}^2 - \theta_L x_{L3}^2) < 0 \tag{32}$$

which is negative for $\theta_L \leq -15.5$. For the Chen system, using the same $V(x(t))$ and Γ, the inequality becomes:

$$28x_{C1}x_{C1} + 28x_{L2}^2 + \theta_C x_{C2}^2 - 35x_{C1}^2 + \theta_C x_{C1}^2 - 3x_{C3}^2 + \theta_C x_{C3}^2 < 0. \tag{33}$$

For $\theta_C < -28$ is negative. Finally, for the Chua system (30) the inequlity becomes

$$-2.5713x_{c1}^2 + 10x_{c1}x_{c2} - x_{c2}^2 - 13.28x_{c2}x_{c3} + \theta_c x_{c1}^2 + \theta c x_{c2}^2 + \theta_c x_{c3}^2) < 0 \tag{34}$$

in this case, for $\theta_c < -5$ the inequality is satisfied. With these passivity degrees the basin of attraction is given by $\{x(t) \in \mathbf{R}^3 : |x(t)| < r = 10\}$.

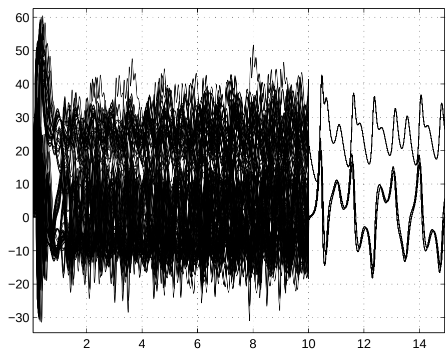

Fig. 2. Tracking the behavior of $x_C(t)$ on a network of $N = 30$ nonidentical nodes with one controller per type ($l = 3$).

Using the Barabási-Albert scale-free algorithm as described in [13], a network form by linear and diffusively coupling ten of each Lorenz, Chen, and Chua systems is constructed. Therefore in all the network has $N = 3$. Without loss of generality, the nodes in the network are ordered from largest to lowest node degree. Additionally, we have one node of each type for the first three indexes. Then, we propose to applied the adaptive controllers of the form (23) only to the first three nodes in the network, that is, $l = 3$ with one controller for each type of node.

In the first case, the pinning objective is $\bar{x} = 0$, which is a common fixed point to all the nodes in the network. With the control action applied at $t = 10$, the results are shown in Figure 1.

In our second case, the reference is to track the isolated behavior of the Chen system (29), that is, $s(t) = x_C(t)$. Again, the control action is applied at $t = 10$. The results are shown in Figure 2.

5. CONCLUSIONS

In this contribution we use the V-stability characterization of a dynamical network with different nodes as the bases from which to proposed a solution to the tracking problem as an extension of the stabilization solution that V-stability provides. The use of pinning controllers and the way this selected actions make the entire network V-stable around a given solution is also shown. The main shortcoming in the application of this approach is the requirement that a passivity degree exist for the systems or error dynamics under consideration, however this restriction is not necessary unsurmountable, as many chaotic benchmarks are dissipative and passiviable by feedback as shown in the numerical examples. An additional restriction of the approach is the inherited conservativeness of the Lyapunov approach, in this sense, using an adaptive control law is proposed as a simple solution that provides a effective way to avoid the need for extremely large feedback gains.

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