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TWO-STAGE STOCHASTIC PROGRAMMING APPROACH TO A PDE-CONSTRAINED STEEL PRODUCTION PROBLEM WITH THE MOVING INTERFACE

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The paper is concerned with a parallel implementation of the progressive hedging algorithm (PHA) which is applicable for the solution of stochastic optimization problems. We utilized the Message Passing Interface (MPI) and the General Algebraic Modelling System (GAMS) to concurrently solve the scenario-related subproblems in parallel manner. The standalone application combining the PHA, MPI, and GAMS was programmed in C++. The created software was successfully applied to a steel production problem which is considered by means of the two-stage stochastic PDE-constrained program with a random failure. The numerical heat transfer model for the steel production was derived with the use of the control volume method and the phase changes were taken into account with the use of the effective heat capacity. Numerical experiments demonstrate that parallel computing facility has enabled a significant reduction of computational time. The quality of the stochastic solution was evaluated and discussed. The developed system seems computationally effective and sufficiently robust which makes it applicable in other applications as well.

Keywords: stochastic programming, progressive hedging, parallel computing, steel production, heat transfer, phase change

Classification: 90C15, 90C06, 80A20, 80A22, 49M27, 93C20

1. INTRODUCTION

A number of decision-making engineering problems lead to optimization models constrained with ordinary or partial differential equations. Such models are frequently large-scale as they consist of physical models derived by means of the finite difference, finite volume, or finite element methods [5, 8, 33]. Applications in various engineering areas can be found; e. g. in systems control [2, 19], in optimal design [30, 37], or in scheduling problems [13, 27]. Moreover, the mentioned problems often include uncertain parameters [1, 6, 26]. Stochastic programming that uses random variables for the uncertainty modelling is a suitable solution approach to such kind of problems. The further analysis of the problems revealed their stage-related decision structure and the possibility to apply the assumption of the discrete probability distribution of random

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variables. This leads to the scenario-based approach and multi-stage decision making problems.

In the paper we consider a large-scale steel production problem under uncertainty which is modelled by a two-stage PDE-constrained programme. A computationally effective parallel implementation of the progressive hedging algorithm (PHA) that has been modified for the considered problem by the authors has been adopted for an efficient solution of the problem. The PHA, originally proposed by Rockafellar and Wets [25, 34], decomposes the extensive form of the problem into the scenario-based sub-problems which are linked together by means of penalties. The decomposition also enables a straightforward parallel solution of individual scenario-based sub-problems, and the computational time can significantly be reduced. The PHA therefore mitigates computational problems which can particularly arise in large-scale problems exceeding the computational capability. The PHA is an iterative method which gradually produces a series of aggregated solutions which are implementable. Though the convergence to a globally optimal solution is not guaranteed in case of non-convex non-linear and mixed-integer problems, computational results presented in several studies have demonstrated the applicability of the PHA to those kinds of problems with good results. Carpentier et al. [7] presented the use of the PHA in the solution of the management of the hydroelectric multireservoir systems under uncertainty. The CPLEX solver without parallel computing was used for the solution of scenario-related sub-problems in sequential order. Gul et al. [12] built a stochastic multi-stage mixed-integer model for the surgery planning problem under uncertainty. The authors applied the PHA, they compared it to a heuristic method and evaluated the value of the stochastic solution. Veliz et al. [32] investigated a forest planning problem. A multi-stage stochastic problem was formulated and solved by means of the PHA. The authors demonstrated that the PHA is well applicable and competitive to a direct solution of the extensive form, even in case of non-parallel implementation of the PHA. Goncalves et al. [11] applied the PHA to the solution of the operation scheduling problem of a hydrothermal system. They utilized the CPLEX solver and parallel computing on several processors. The authors reported that they achieved the reduction of 80% of the computational time when comparing the parallel implementation to the serial implementation. Gade et al. [10] reported the assessment of the quality of solutions generated by the PHA by means of the lower bounds. The authors presented a method for the determination of lower bounds for multi-stage mixed-integer problems and they demonstrated computational results in stochastic unit commitment and server location problems.

As for optimization studies related to the control of the continuous casting process, most research papers are related to deterministic optimization with no randomness. A number of studies aim at optimization of operational parameters of the casting machine, of the scheduling, and of the secondary cooling zone, see e. g. [21, 35, 36]. However, only a limited number of papers have been published on the topic of stochastic optimization related to the continuous casting, e. g. [38]. To the best knowledge of the authors, there is no paper related to the application of the PHA to the steel production problem or to a heat transfer problem with the moving interface. The present paper builds on previous research of the authors on the mathematical programming approach and random failures in the continuous casting process [23, 24]. A basis of presented results was conducted in

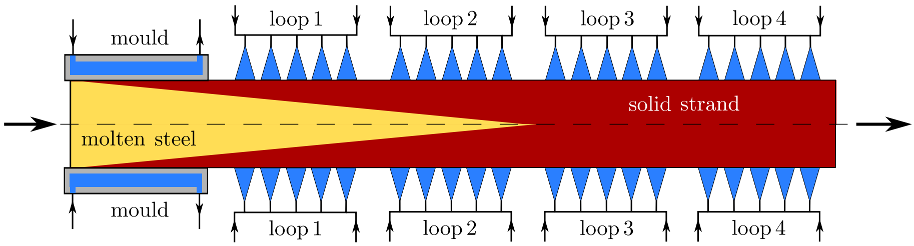

Fig. 1. The schematic of the continuous steel casting method.

the master thesis [15] and some partial results have already been published at scientific conferences [16, 17, 18].

The present paper summarizes the complete application of the stochastic approach to the continuous casting process. A detailed description of the improved heat transfer model based on the effective heat capacity is provided. A computationally efficient parallel implementation of the progressive hedging algorithm is presented and the Message Passing Interface (MPI) was used to solve independent scenario-related sub-problems in the parallel manner. Individual sub-problems are formulated in the General Algebraic Modelling System (GAMS) and solved by the non-linear CONOPT solver. The use and the applicability of the proposed implementation are demonstrated in the two-stage steel production problem with a random failure.

2. STEEL PRODUCTION CONTROL PROBLEM UNDER UNCERTAINTY

2.1. Underlying principle of the steel production problem

The continuous steel casting is a modern production method of steel. Nowadays, more than 95% of the total world steel production is cast by means of the continuous casting [22]. The schematic of the horizontal continuous casting method is shown in Figure 1. The molten steel enters the continuous casting machine through the water-cooled mould. In the mould the initial amount of heat is withdrawn from the steel and this causes the formation of the solid shell at the surface of the cast semi-product, the so-called strand. The steel strand continues from the mould to the secondary cooling zone. The secondary cooling zone consists of several independent cooling loops where the water nozzles producing water sprays are installed. The heat withdrawal in the secondary cooling zone therefore occurs due to the heat convection and radiation. The intensity of the heat convection from the strand is driven by the heat transfer coefficient which is directly dependent on water flow rates through the cooling loops, see e. g. [29].

2.2. Uncertainty in the steel production problem

There are several different categories of parameters under uncertainty which influence the steel production problem. These mainly include the uncertainty in the chemical composition of steel, random faults in parts of the casting machine, and clogging of cooling nozzles. It is well reported [4] that the strand cooling, its distribution and intensity have a major influence on the steel quality and mechanical properties. Steelmakers therefore pay attention mainly to the secondary cooling and they aim at the gradual cooling in which the temperature of the strand smoothly decreases with no reheating, temperature shocks or zig-zag temperature paths. One of serious faults in the secondary cooling zone is an abrupt stop of water spray cooling within a cooling loop and we investigate this case in the paper. Such a problem is typically caused by a fault in the electric motor of the water pump. In that case there is no cooling of the strand which leads to undesirable steep overheating of the strand and, as a consequence, to irreversible steel quality problems.

2.3. Simplifications and assumptions

Technology. Casting machines can accommodate even more than 20 cooling loops in the secondary cooling zone, particularly in case of large cross-section steel strands. In the paper we consider the problem with a casting machine having four cooling loops as shown in Figure 1. The reason is that requirements for the computational hardware increase rapidly with higher number of cooling loops. Moreover, the goal of the paper is to demonstrate the applicability of the PHA to the steel making problem rather than to solve a huge realistic problem from industry. Nevertheless, the model presented in the paper is formulated without the loss of generality and it can be used for modelling of a problem with the casting machine having an arbitrary number of cooling loops.

Uncertainty and scenario tree. In the paper we focus on the case described above: a cooling system in a loop can fail and as a result the heat withdrawal from the strand stops causing overheating and quality issues. In general, the fail of cooling can occur in an arbitrary cooling loop, even in several loops simultaneously. Hence, such uncertain failures are represented by a random vector and its outcomes form the scenario tree that is suitable for further computations.

3. HEAT AND MASS TRANSFER MODEL FOR CONTINUOUS STEEL CASTING

In the section the mathematical description of the model for the continuous steel casting is provided. Further, the mathematical model is numerically reformulated with the use of the control volume method to the form suitable for the implementation in the modelling system GAMS; see [17, 18] for further details.

3.1. Physical model

The heat and mass transfer model of the continuous steel casting is based on the Fourier-Kirchhoff equation [14]. The governing equation incorporates the heat conduction within the cast strand while the direct modelling of the fluid flow in the melt is omitted. As the continuous steel casting relies on the phase transformation of steel from the liquid phase to the solid phase, the model accounts for phase changes. There are several approaches for phase change modelling, see e. g. [28]. We employ the effective heat capacity method (see [28] for its details) as the method is quite simple and, in particular, easily implementable into the optimization model. Only one half of the strand as shown in Figure 2 is considered in the model since the spatial domain is symmetrical with respect to the horizontal axis, cf. Figures 1 and 2.

In the paper the casting process of so-called slabs is considered. A slab is a kind of the strand having the rectangular cross-section with a high aspect ratio (usually 8 or more); the width of the slab is therefore several times larger than its height. For such slabs it is justifiable to consider a 2D heat transfer model which neglects interactions in the perpendicular direction of the 2D domain. In the model explained below, the 2D domain is considered as a longitudinal vertical cross-section of the slab in the middle of the width of the slab.

The governing equation describing the 2D heat and mass transfer and phase transformations within the cast strand (the spatial domain Ω) is [28]

$$\varrho c_{\mathrm{eff}} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + v_{\mathrm{z}} \varrho c_{\mathrm{eff}} \frac{\partial T}{\partial z} \qquad \text{in } \Omega \times (0, \tau\rangle \tag{1}$$

where $T(x,z,t)$ is the temperature in a spatial point (x,z) at time $t \in (0,\tau\rangle$, ϱ is the density, c_{eff} is the effective heat capacity, k is the thermal conductivity, v_{z} is the casting speed and τ is the final time.

The model is completed with the initial (2) condition specifying the initial temperature $T_0(x,z)$ in Ω and boundary conditions (3) – (6)

$$T(x,z,t) = T_0(x,z) \qquad \text{in } \Omega \times \{0\}, \tag{2}$$

$$T(x,z,t) = T_{\mathrm{casting}} \qquad \text{in } \Gamma_{\mathrm{in}} \times (0,\tau\rangle, \tag{3}$$

$$-k\frac{\partial T}{\partial \boldsymbol{n}} = 0 \qquad \text{in } \Gamma_{\mathrm{out}} \times (0,\tau\rangle \text{ and } \Gamma_{\mathrm{sym}} \times (0,\tau\rangle, \tag{4}$$

$$-k\frac{\partial T}{\partial \boldsymbol{n}} = \dot{q}_{\mathrm{mould}} \qquad \text{in } \Gamma_{\mathrm{mould}} \times (0,\tau\rangle, \tag{5}$$

$$-k\frac{\partial T}{\partial \boldsymbol{n}} = h_m\left(T(x,z,t) - T_\infty\right) + \sigma\varepsilon\left(T(x,z,t)^4 - T_\infty^4\right) \qquad \text{in } \Gamma_m \times (0,\tau\rangle, \quad m = 1,\dots,n_{\mathrm{CC}} \tag{6}$$

which are added to the governing equation (1). In Eq. (4) – (6), the vector $\boldsymbol{n}$ is the normal vector to the surface. The boundary condition (3) prescribes the constant pouring temperature T_{casting} at the meniscus in the mould (see Γ_{in} in Figure 4) the boundary condition (4) simulates the physical symmetry at the plane of symmetry and the zero heat flux at the end of the strand, see both Γ_{out} and Γ_{sym} boundaries in Figure 4. Further, the defined heat flux $\dot{q}_{\mathrm{mould}}$ is prescribed in the mould (see Γ_{mould} boundary part) according to the boundary condition (5). The convective and radiative heat transfer is included in the boundary condition (6) and applied at surfaces of the strand. Here, the cooling occurs due to the forced (in the spraying zones) and the natural (at free surfaces) convection and radiation where h_m is the heat transfer coefficient in the mth cooling loop (see Γ_m in Figure 4), σ is the Stefan-Boltzmann constant, ε is the emissivity, and T_∞ is the ambient temperature. As already mentioned in Section 2 the number of cooling loops n_{CC} in the secondary cooling can vary according to dimensions of cast steel strands and according to the configuration of a casting machine. In the paper we consider, without the loss of generality, the number of cooling loops $n_{\mathrm{CC}} = 4$.

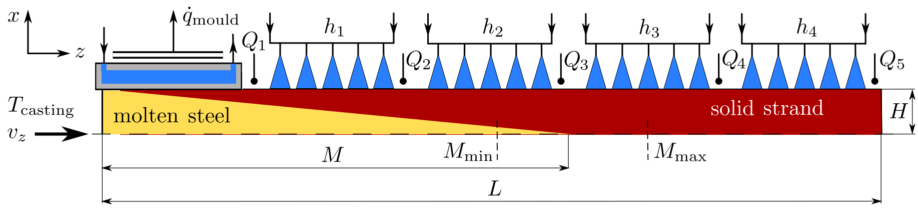

Fig. 2. The spatial domain for the steel production problem.

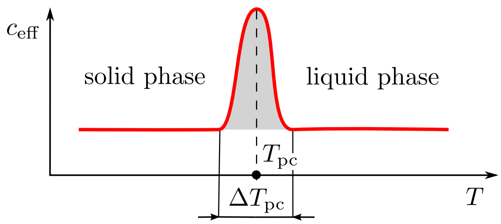

Fig. 3. The idea of the effective heat capacity method.

As mentioned above, the effective heat capacity method is applied to the phase change modelling [28]. The principle of the method relies on the artificial increase of the physical heat capacity in the temperature range of the phase change. The amount of the latent heat accompanying the phase change is then proportional to the shaded area in Figure 3. A typical bell shaped effective heat capacity as a function of the temperature is illustrated in Figure 3. The effective heat capacity can be defined [28] as

$$c_{\text{eff}} = \frac{\partial H}{\partial T} = \varrho c - \varrho L_{\text{f}} \frac{\partial f_{\text{S}}}{\partial t} \frac{\partial t}{\partial T} \tag{7}$$

where H is the enthalpy, c is a specific heat, L_{f} is the latent heat of the phase change and f_{S} is the solid fraction expressing the ratio between the solid and liquid phases during the solidification.

3.2. Numerical continuous casting model and its deterministic formulation

The mathematical model presented in the foregoing section was discretized with the use of the control volume method. The discretization enables an implementation of the model into the modelling system GAMS, and thus it makes possible to find the numerical solution in discrete points within the domain. First, the spatial domain Ω is partitioned into smaller elements, so-called control volumes specified by indices $i = 1, \ldots, N_x$ and $j = 1, \ldots, N_z$ and denoted as $[i, j]$. The union of all the control volumes is often referred to as the mesh. Similarly, the time domain $(0, \tau\rangle$ is divided into the time steps indexed by $n = 1, \ldots, N_\tau$. The energy conservation balance is then applied to each control volume. The balances for all mesh elements form the system of equations and its solution is considered as the numerical solution to the model described by Eqs. (1)–(7).

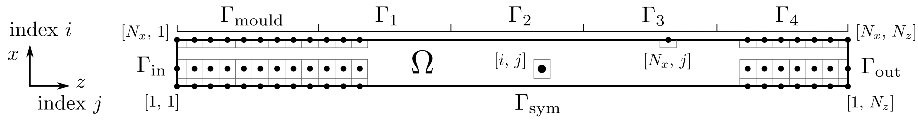

Fig. 4. The discretization of the spatial domain into control volumes.

Let us assume that the time domain $(0, \tau\rangle$ is partitioned into N_τ time intervals, each with the length of Δt (thus $\tau = N_\tau \Delta t$). The energy conservation law [14] states that the sum of energies $E_{[i,j]}$ transferred inward and outward a control volume through its boundaries during any time interval of length Δt must be equal to the change of the internal energy U of that control volume: $\sum_{[i,j]} E_{[i,j]} = \Delta U$. In the considered case, the transferred energy is heat transferred due to heat transfer mechanisms (heat conduction in interior control volumes and also heat convection and radiation in boundary control volumes) and due to the mass transfer as the strand continuously moves through the casting machine. Let us consider the interior control volume $[i, j]$ shown in Figure 4 and the implicit discretization approach for the time scale. The energy conservation law applied to the interior control volume specified by $[i, j]$ and nth time interval where $i = 1, \ldots, N_x$, $j = 1, \ldots, N_z$, and $n = 1, \ldots, N_\tau$ yields to

$$k\Delta z \frac{T^n_{i-1,j} - T^n_{i,j}}{\Delta x} + k\Delta z \frac{T^n_{i+1,j} - T^n_{i,j}}{\Delta x} + k\Delta x \frac{T^n_{i,j-1} - T^n_{i,j}}{\Delta z} + k\Delta x \frac{T^n_{i,j+1} - T^n_{i,j}}{\Delta z} + \\ + v_{\mathrm{z}} \varrho c_{\mathrm{eff}} \Delta x \left(T^n_{i,j-1} - T^n_{i,j}\right) = \varrho c_{\mathrm{eff}} \Delta x \Delta z \frac{T^n_{i,j} - T^{n-1}_{i,j}}{\Delta t} \tag{8}$$

where Δx and Δz are the dimensions of the control volume in the x-axis and z-axis, respectively and $T^n_{i,j}$ values denote unknown temperature distribution. The four terms on the first line of the left-hand side in Eq. (8) are the heat transfer rates that represent the conduction heat transfer from the neighbouring control volumes. The fifth term on the left-hand side of Eq. (8) is the energy input due to the movement of the strand through the casting machine. The right-hand side of Eq. (8) represents the change of the internal energy during the time interval Δt. The energy conservation law is similarly applied to boundary control volumes. For instance, let us consider the boundary control volume $[N_x, j]$ exposed to the water spray cooling in a cooling loop $m = 3$, see Figure 4. The energy conservation law applied to the boundary control volume yields to

$$h_m \Delta z (T_{\text{ambient}} - T^n_{N_x,j}) + \sigma\varepsilon\Delta z \left(T^4_{\text{ambient}} - (T^n_{N_x,j})^4\right) + \\ + k\Delta z \frac{T^n_{N_x-1,j} - T^n_{N_x,j}}{\Delta x} + k\left(\frac{\Delta x}{2}\right) \frac{T^n_{N_x,j-1} - T^n_{N_x,j}}{\Delta z} + k\left(\frac{\Delta x}{2}\right) \frac{T^n_{N_x,j+1} - T^n_{N_x,j}}{\Delta z} + \\ + v_{\mathrm{z}} \varrho c_{\mathrm{eff}} \frac{\Delta x}{2} \left(T^n_{N_x,j-1} - T^n_{N_x,j}\right) = \varrho c_{\mathrm{eff}} \left(\frac{\Delta x \Delta z}{2}\right) \frac{T^n_{N_x,j} - T^{n-1}_{N_x,j}}{\Delta t}. \tag{9}$$

The application of the energy conservation law to all control volumes leads to the set of algebraic equations for the unknown temperature distribution $T^n_{i,j}$ with a specified temperature T_{ambient}. Since the spatial domain is partitioned into $N_x N_z$ control volumes and the time domain into N_τ time intervals, the set consists of the $N_x N_z N_\tau$ algebraic equations for the identical number of the unknown temperatures.

The phase change and the latent heat are implemented in the model by means of the effective heat capacity. A bell-shape function might be used in case of steel having a relatively wide temperature interval of the phase change. The effective heat capacity as the function of the temperature is of the form

$$c_{\text{eff}}(T) = c_0 + \tilde{c}\,\exp\left\{-\frac{(T^n_{i,j} - T_{\text{pc}})^2}{\zeta_T}\right\} \tag{10}$$

where c_0 is the heat capacity outside the temperature range of the phase change, $\tilde{c}$ is the parameter characterising the amount of the latent heat of the phase change, T_{pc} is the mean phase change temperature and ζ_T is a parameter related to the temperature interval of the phase change ΔT_{pc}.

Several additional constraints are included to the model due to technological requirements of the continuous casting process. The cooling capacity of the spraying nozzles installed in the secondary cooling zone is limited by the maximum water flow rate provided by the pump and also by the hydrodynamic limits of nozzles. The heat transfer coefficient h_m for the nozzles installed within the cooling loop m is therefore restricted as

$$0 \le h_m \le h_{m,\max} \qquad m = 1, \ldots, n_{\text{CC}} \tag{11}$$

where the value of $h_{m,\max}$ is known. Further, the quality of the produced steel is highly dependent on the thermal history during the casting process, specifically the surface of the strand suffers from defects and cracks due to large thermal gradients. In particular, the surface temperatures of the strand are required to decrease gradually and smoothly in the direction of casting with minimum reheating or temperature shocks. The surface temperatures are therefore required to fit specific temperature ranges in defined control points. Such control points are usually behind the mould and behind each cooling loop as shown in Figure 2 (labelled as Q_m) and thus

$$T_{m,\min} \le T^n_{N_x,q_m} \le T_{m,\max} \qquad m = 1, \ldots, n_{\text{CP}} \tag{12}$$

where q_m is the index of the control volume in the z-axis closest to the control point Q_m and n_{CP} is the number of control points; in our case $n_{\text{CP}} = 5$, see Figure 3. Additionally, $T_{m,\min}$ and $T_{m,\max}$ values are known. Finally, the very important constraint is required for the length of the liquid phase M shown in Figure 2. The distance M must fulfil $M_{\min} \le M \le M_{\max}$. In terms of the temperature, it must hold that

$$T_{1,r_{\min}} \ge T_{\text{liquidus}} \qquad \text{and} \qquad T_{1,r_{\max}} \le T_{\text{solidus}} \tag{13}$$

where $r_{\min}$ and $r_{\max}$ are the indices of the control volumes in the z-axis which are the closest ones to the distances $M_{\min}$ and $M_{\max}$, respectively. The steel is completely liquid at and above the liquidus temperature T_{liquidus} and it is completely solid at and below the solidus temperature T_{solidus}.

Finally, the deterministic optimization problem for the steady-state continuous casting process can be formulated. The steel-maker requires the maximum productivity with the preservation of quality, and thus the relevant parameters of the casting process need to be determined. In terms of the above defined quantities and derived relationships, the deterministic model aims to

$$\begin{cases} \text{maximize} & v_\mathrm{z} \\ \text{subject to} & (8), (9), (10), (11), (12), (13). \end{cases} \tag{14}$$

The steel-maker is mainly interested in optimal values of operational parameters of the continuous casting machine: the casting speed v_z and the heat transfer coefficients h_m in cooling loops of the secondary cooling zone. A solution to the problem (14) also includes the temperature distribution $T^n_{i,j}$ which can be further investigated by metallurgists or materials engineers.

4. TWO-STAGE STOCHASTIC FORMULATION FOR THE STEEL PRODUCTION PROBLEM AND PARALLEL PROGRESSIVE HEDGING ALGORITHM

In this section we transform the deterministic model discussed in the foregoing section into a two-stage stochastic problem. As already mentioned in Section 2.2, there are several sources of uncertainty in the steel production problem. We will focus on the case with a fail of the water pump which is potentially the most dangerous case in practice. Furthermore, we discuss the progressive hedging algorithm and its parallel implementation suitable for computationally effective processing.

4.1. Stochastic optimization and scenario-based approach

Stochastic programming and the use of random variables is a common approach widely applied in optimization problems with uncertainty [31]. A two-stage stochastic problem is considered in the paper which includes a here-and-now first stage decision and a wait-and-see second stage decision [3]. In general, the solution to a stochastic problem searches for the minimum of an objective function $f(\boldsymbol{x}, \boldsymbol{\xi})$ subject to equality and inequality constraints $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\xi}) = \mathbf{0}$ and $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{\xi}) \leq \mathbf{0}$, respectively, where $\boldsymbol{x} \in X \subset \mathbb{R}^N$ and $\boldsymbol{\xi}$ is a K-dimensional random vector on a probability space $(\Omega, \mathscr{A}, P)$.

The paper is concerned with random variables having a discrete probability distribution and the scenario-based approach is applied to take into account discrete random variables [3]. Assume that the random vector $\boldsymbol{\xi}$ on a probability space $(\Omega, \mathscr{A}, P)$ has a finite discrete distribution with $|\Omega| = L < \infty$. Then, outcomes of the random vector are denoted as particular real-valued vectors $\boldsymbol{\xi}_s$ and referred to as scenarios $s \in \mathcal{S}$ with a probability p_s and $\sum_{s \in \mathcal{S}} p_s = 1$. Then, the general deterministic equivalent based on the expected objective with all constraints satisfied almost surely can be written as follows

$$\begin{cases} \text{minimize} & \sum_{s \in \mathcal{S}} p_s f(\boldsymbol{x}, \boldsymbol{\xi}_s) \\ \text{subject to} & \boldsymbol{x} \in \cap_{s \in \mathcal{S}} C_s \\ & \text{where } C_s = \left\{ \boldsymbol{x} \in X \subset \mathbb{R}^N : \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\xi}_s) \leq \mathbf{0}, \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{\xi}_s) = \mathbf{0} \right\}. \end{cases} \tag{15}$$

4.2. Two-stage stochastic formulation of the steel production problem

In this part, the deterministic formulation of the steel production problem described in Section 3.2 is reformulated into a two-stage stochastic problem. A failure situation within the secondary cooling zone is considered and the stochastic approach is used for the modeling of randomness. In particular, failures of water pumps in the cooling loops are assumed to occur. A failure of the pump means that no water is fed into the cooling nozzles, and therefore there are no water sprays generated by the nozzles. This implies that the heat transfer coefficient of the forced convection induced by cooling nozzles drops to zero. Though there is still the natural convection to the ambient air, the total heat transfer coefficient can be assumed zero (i. e. with the neglected natural convection) as the heat transfer coefficient of the natural convection is much lower than that of the forced convection due to cooling nozzles. The model with the scenario-based approach and scenarios $\boldsymbol{\xi}_s, s \in \mathcal{S}$ is utilized to take into account the uncertainty. One scenario models the failure-free casting process while the other scenarios represent the casting process with failures in the cooling loops of the secondary cooling zone. The steel-maker is primarily interested in the first-stage decision which is the set of operational parameters at the beginning of the casting process. However, the first stage decision needs to take into account the possibility that a failure in the secondary cooling zone can occur with a non-zero probability. And if the failure does occur, the second-stage decision is used to revise the first stage decision and to keep the best conditions for the casting process.

Some modifications to the deterministic model (14) are required for its transformation into the two-stage stochastic model. In particular, the objective function in the model (14) (cf. to f in the model (15)) is reformulated as

$$\text{maximize} \quad \sum_{s\in\mathcal{S}} p_s \left[\sum_{n\in\mathcal{N}} v_{\mathrm{z},n}(\boldsymbol{\xi}_s) \right], \tag{16}$$

where $v_{\mathrm{z},n}(\boldsymbol{\xi}_s)$ represents the casting speed under random circumstances that can be specific for different time periods $n \in \mathcal{N} = \{1, \ldots, N_\tau\}$ and scenarios $s \in \mathcal{S}$. Let the prime be used for variables related to the first-stage decision while the double prime be used for variables related to the second-stage decision. The stage-related heat transfer coefficients are now also dependent on a particular scenario $\boldsymbol{\xi}_s$, $s \in \mathcal{S}$ and such dependency is emphasized by the subscript s. For instance, the second-stage heat transfer coefficient for the scenario s_0 in a cooling loop m is denoted as h''_{m,s_0} and it has to be considered constant for all second stage related time indices. So, they can be omitted. Deterministic constraints to the range of heat transfer coefficients in Eq. (11) are updated with respect to the two-stage structure of heat transfer coefficients as

$$0 \leq h'_{m,s} + h''_{m,s} \leq h_{m,\max} \qquad \text{for } m = 1, \ldots, n_{\mathrm{CC}} \quad \text{and} \quad s \in \mathcal{S}. \tag{17}$$

Let t_{f} be the time when the failure of the pump within the second cooling loop occurs. The set of time related indices $n \in \mathcal{N}$ that identify the period before the failure is denoted by $\mathcal{N}' \subset \mathcal{N}$. Then the heat transfer coefficients h_m in the heat balance equations (9) for the surface control volumes are updated by

$$h_m = h'_{m,s} \qquad \text{for} \quad n \in \mathcal{N}', \quad m = 1, \ldots, n_{\mathrm{CC}}, \quad s \in \mathcal{S}, \tag{18}$$

and similarly the set of time related indices n that identify the period after the failure is denoted by $\mathcal{N}'' \subset \mathcal{N}$. For these indices we replace h_m as follows

$$h_m = h'_{m,s} + h''_{m,s} \qquad \text{for} \quad n \in \mathcal{N}'', \quad m = 1, \ldots, n_{\mathrm{CC}}, \quad s \in \mathcal{S}. \tag{19}$$

Further, note that due to the scenario-based model the temperature variables $T^n_{i,j}$ of the heat transfer model in all equations and constraints presented above are also related to a particular scenario $s \in \mathcal{S}$.

We also assume that the first stage related casting speeds $v_{\mathrm{z},n}(\boldsymbol{\xi}_s)$ and $h'_{m,s}$ variables must satisfy the explicit nonanticipativity constraints. Therefore

$$v_{\mathrm{z},n}(\boldsymbol{\xi}_s) = \sum_{r \in \mathcal{S}} p_s v_{\mathrm{z},n}(\boldsymbol{\xi}_r), \quad \forall s \in \mathcal{S}, \quad n \in \mathcal{N}'. \tag{20}$$

and similarly

$$h'_{m,s} = \sum_{r \in \mathcal{S}} p_r h'_{m,r}, \quad \forall s \in \mathcal{S}. \tag{21}$$

To keep the description compact and general, we will further write about the first stage decision vector $\boldsymbol{x}'_s$ containing first stage components $v_{\mathrm{z},n}, n \in \mathcal{N}'$ and $h'_{m,s}$. Similarly, the second stage related components $v_{\mathrm{z},n}$, $n \in \mathcal{N}''$ and $h''_{m,s}$ will be contained in the second stage decision vector $\boldsymbol{x}''_s$. Both vectors $\boldsymbol{x}'_s$ and $\boldsymbol{x}''_s$ form the composed vector $\boldsymbol{x}_s$.

4.3. Progressive hedging algorithm

The progressive hedging algorithm (PHA) is an optimization method suitable for the solution of multi-stage scenario-based stochastic problems. The PHA, originally proposed by Rockafellar and Wets [25, 34] in 1980s, is a decomposition method which separates the scenario-based problem into smaller independent sub-problems, each related to an individual scenario. The method is therefore particularly applicable in solution of large-scale problems and fits well for parallel computing. The PHA is based on the augmented Lagrangian method with relaxed non-anticipativity constraints. In general, non-anticipativity constraints can be formulated explicitly or implicitly to define an extensive form. An implicit formulation is usually easier for the solution as it introduces shared variables instead of scenario-related variables as in case of the explicit formulation. However, even the implicit formulation can lead to stochastic problems which are difficult for the solution due to their large dimensions as reported in [32].

Let us assume a scenario-based stochastic problem (15) with L scenarios $s \in \mathcal{S}$, each with the probability p_s. Then each particular scenario s forms the sub-problem

$$\begin{cases} \text{minimize} & f(\boldsymbol{x}, \boldsymbol{\xi}_s) \\ \text{subject to} & \boldsymbol{x} \in C_s = \left\{\boldsymbol{x} \in X_s \subset \mathbb{R}^N : \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\xi}_s) \leq \boldsymbol{0}, \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{\xi}_s) = \boldsymbol{0}\right\}. \end{cases} \tag{22}$$

The optimal solution to the problem (15) is denoted as $\boldsymbol{x}^*$. The progressive hedging algorithm is the iterative algorithm which requires solutions $\boldsymbol{x}_s$ of the sub-problems (22) and generates a sequence of solutions $\hat{\boldsymbol{x}}_k$ converging to the solution $\boldsymbol{x}^*$ of the original scenario-based problem (15). In terms of the steel production problem considered in the paper, the objective function f aims at the maximization of the production rate v_{z} (cf. to Eq. (16)). The vector function $\boldsymbol{g}$ of inequality constraints includes inequality conditions (e. g. Eqs. (12), (13), (17)) while the vector function $\boldsymbol{h}$ of equality constraints incorporates the heat transfer model (Eqs. (8) and (9)) and other equality conditions (e. g. Eqs. (18) and (19)).

List of symbols for the PHA

D	the error
ε	the termination parameter
L	the number of scenarios $L = \|\mathcal{S}\|$
ϱ	the penalty parameter
$\boldsymbol{x}'_{k,s}$	the first-stage decision for iteration k and scenario s
$\boldsymbol{x}''_{k,s}$	the second-stage decision for iteration k and scenario s
$\hat{\boldsymbol{x}}'_{k}$	the first-stage "average" decision for iteration k
$\hat{\boldsymbol{x}}''_{k,s}$	the second-stage iterated decision for iteration k and scenario s
$\boldsymbol{w}'_{k,s}$	the first-stage weight vector for iteration k and scenario s

Two-stage Progressive Hedging Algorithm

Initialization. Choose the penalty parameter $\varrho > 0$ and the termination parameter $\varepsilon > 0$. Set $k = 1$, $\boldsymbol{w}'_{0,s} = \boldsymbol{0}$, $\hat{\boldsymbol{x}}'_{0} = \boldsymbol{0}$ and $\hat{\boldsymbol{x}}''_{0,s} = \boldsymbol{0}$ for all $s \in \mathcal{S}$.

Main part.

1. For all $s \in S$ solve the scenario-based sub-problem

$$\left\{\begin{array}{ll} \text{minimize} & f(\boldsymbol{x}', \boldsymbol{x}'', \boldsymbol{\xi}_s) + \boldsymbol{w}'^{\mathsf{T}}_{k-1,s} \cdot \boldsymbol{x}' + \frac{1}{2}\varrho \left\|\boldsymbol{x}' - \hat{\boldsymbol{x}}'_{k-1,s}\right\|^2 \\ \text{subject to} & \boldsymbol{x}', \boldsymbol{x}'' \in C_s \end{array}\right.$$

and let $\boldsymbol{x}'_{k,s}$ and $\boldsymbol{x}''_{k,s}$ be its first-stage and second-stage solution, respectively.

2. Calculate the updated first-stage and second-stage solutions $\hat{\boldsymbol{x}}'_{k}$ and $\hat{\boldsymbol{x}}''_{k,s}$, respectively, for all $s \in \mathcal{S}$:

$$\hat{\boldsymbol{x}}'_{k} = \sum_{s \in S} p_s \boldsymbol{x}'_{k,s} \qquad \text{and} \qquad \hat{\boldsymbol{x}}''_{k,s} = \boldsymbol{x}''_{k,s}.$$

In the case that the termination condition

$$D = \left(L \left\|\hat{\boldsymbol{x}}'_{k-1} - \hat{\boldsymbol{x}}'_{k}\right\|^2 + \sum_{s \in S} \left\|\hat{\boldsymbol{x}}''_{k-1,s} - \hat{\boldsymbol{x}}''_{k,s}\right\|^2 + \sum_{s \in S} p_s \left\|\boldsymbol{x}'_{k,s} - \hat{\boldsymbol{x}}'_{k}\right\|^2 \right)^{\frac{1}{2}} \leq \varepsilon$$

is fulfilled, then stop, $\hat{\boldsymbol{x}}'_{k}$ and $\hat{\boldsymbol{x}}''_{k,s}$ are the the first-stage and the second-stage solution, respectively, to the scenario-based optimization problem (15). Otherwise, for all $s \in S$ update the first-stage weights according to

$$\boldsymbol{w}'_{k,s} = \boldsymbol{w}'_{k-1,s} + \varrho \left(\boldsymbol{x}'_{k,s} - \hat{\boldsymbol{x}}'_{k}\right),$$

set $k = k + 1$ and return to the step 1 of the main part of algorithm.

As can be seen from the detailed description of the PHA, the second-stage decision $\hat{\boldsymbol{x}}''_{k,s}$ is dependent on a particular scenario while the first-stage "average" decision $\hat{\boldsymbol{x}}'_k$ is not. The reason is that the first-stage decision is the *here-and-now* decision [3] and it is made at the beginning of the process, i. e. before the decision maker observes a particular outcome of random failure in our case. The first-stage decision is therefore independent on what will happen in the future and represents the initial setup of control variables. This property is referred to as the non-anticipativity [34]. On the other hand, the second-stage decision is the *wait-and-see* decision and it is made after the decision maker has already observed the failure and particular values of the random vector are fully known at that moment.

4.4. Parallel implementation of the progressive hedging algorithm

As already mentioned, the progressive hedging algorithm allows for the direct parallelism. The parallel processing can be accomplished due to the decomposition of the original optimization problem into smaller optimization sub-problems which are independent to each other. The General Algebraic Modelling System (GAMS) was utilized as the optimization solver for the scenario-based sub-problems. The Message Passing Interface (MPI) was used to run the GAMS instances in parallel, each solving an individual sub-problem. The progressive hedging algorithm itself was implemented as the standalone principal application in C++. For detailed information on the implementation, we refer readers to [15, 16].

Principal PHA application. The principal application aggregates the PHA, MPI, and GAMS. The application also

- controls the flow of the PHA,
- creates the scenario-based sub-problems in the form of input files for the GAMS,
- runs the GAMS instances in parallel by means of MPI,
- loads the solutions to sub-problems from the output files produced by the GAMS,
- computes the "average" first-stage and second-stage decisions and
- evaluates the termination condition.

The block diagram and the flow chart of the algorithm are shown in Figure 5 on the left side and on the right side, respectively.

Message Passing Interface. The MPI is an API Windows library which serves the platform for high performance parallel computing. In particular, the LAM/MPI environment was employed for parallel launch of GAMS instances. Each instance of the GAMS is used to solve one of the scenario-based sub-problems. The maximum number of parallel GAMS instances is dependent on the particular hardware. In case the number of scenarios exceeds the maximum number of parallel processes, parallel runs are repeated until all the scenario-based sub-problems are solved.

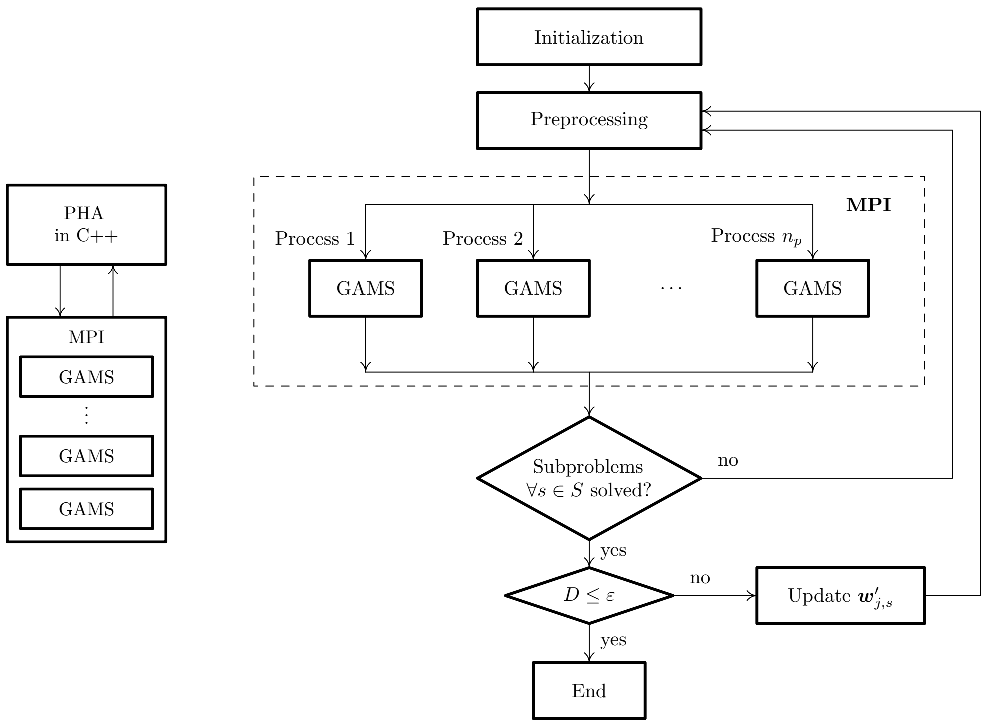

Fig. 5. The block diagram (left) and the flow chart (right) of the parallel implementation of the progressive hedging algorithm.

GAMS. The General Algebraic Modelling System (GAMS) is one of modelling systems applicable to the solution of optimization problems. Various solvers designed for linear, non-linear, integer of mixed integer optimization problems are available. The use of distinct non-linear solvers was investigated. In conclusion, the non-linear solver CONOPT [9] having the best performance was used for the solution of scenario-based sub-problems of the progressive hedging algorithm.

5. NUMERICAL RESULTS AND DISCUSSION

The parallel implementation of the progressive hedging algorithm described in Section 4 was used to solve the two-stage stochastic problem of the steel production described in Section 4.2. The steel-maker is interested in the following questions: "How should the casting machine be set up in order to maximize its productivity but with regard to a failure in the secondary cooling zone which can occur with a given probability?" And in case the failure does occur: "How should the first-stage setup of the machine be modified to preserve optimal casting conditions?"

5.1. Input parameters

As already mentioned in the introduction, we consider the casting machine with four cooling loops, and therefore $n_{\mathrm{CC}} = 4$. Further, we consider that the pump fail can occur in the second cooling loop ($i = 2$) of the secondary cooling; two scenarios s_0 and s_2 are therefore taken into account. The selection of the second cooling loop for a random fail of the pump is justifiable from the metallurgical point of view as follows; among all the loops the second cooling loop is crucial for the steel quality since the liquid pool (liquid phase) ends here (see Figure 1) and any fluctuations in the temperature and in the heat withdrawal can cause the formation of separated liquid bulks surrounded by the solid steel leading to serious defects such as interior cracks or non-homogeneity [4].

A low carbon steel grade with the solidus and liquidus temperatures of 1490 °C and 1515 °C, respectively, was taken into account. The half strand with the height of 125 mm and with the length of 20 m was considered. The spatial domain consisted of 400 control volumes and the time domain of 5 minutes was divided into 300 steps with the time step of 1 s. The scenario-based sub-problem therefore includes about 120 000 variables. The thermal conductivity of steel was 35 W/m K and the ambient air temperature was 20 °C. The heat flux withdrawn from the mould of 3 kW/m^2 was assumed. The constraints for the heat transfer coefficients, see Eq. (11), were $h_1 = 500\,\mathrm{W/m^2K}$, $h_2 = 400\,\mathrm{W/m^2K}$, $h_3 = 400\ \mathrm{W/m^2K}$ and $h_4 = 300\,\mathrm{W/m^2K}$. The constraints for the surface temperatures at the control points, see Eq. (12), are presented in Table 1.

	Control point				
Constraint	Q_1	Q_2	Q_3	Q_4	Q_5
$T_{j,\min}$	1300 °C	1050 °C	900 °C	700 °C	700 °C
$T_{j,\max}$	1480 °C	1250 °C	1000 °C	900 °C	800 °C

Tab. 1. Temperature constraints for the control points.

5.2. Set-up of scenarios

The problem with two scenarios s_0 and s_2 is considered. The scenario s_0 is used for the failure-free situation in the casting process and its probability was $p_0 = 0.95$. The scenario s_2 models the failure of the pump in the second cooling loop of the secondary cooling zone and its probability is $p_2 = 0.05$. Without the loss of generality of the presented stochastic model, we consider the simplified case with a possible fail in one particular cooling loop. It should be noted here that the above presented stochastic problem is formulated for the two-stage structure with a general number of scenarios and additional scenarios can be added straightforwardly to the model along with constraints. The described simplification will lead to the stochastic steel production problem with two scenarios. A problem including two scenarios may seem to be quite non-comprehensive. However, it should be noted that an attempt to solve the complete (non-decomposed) steel production programme including the two scenarios was made with no success as the problem exceeded the computational capability of the CONOPT solver. On the other hand, the PHA separates the problem into scenario-related sub-problems which are simpler and solvable by the CONOPT solver. The decomposition principle is therefore a crucial point in the solution of the steel production problem under uncertainty by means of the PHA even for small number of scenarios.

Though the simplified problem with two scenarios was considered, such solution is still quite realistic and applicable in practice. The reason is that the casting problem with a possible failure generally does not require a large number of scenarios for the description of various combinations of failures in cooling loops. The possibility of multiple breakdowns in distinct cooling loops is very low as individual cooling loops are operated by independent pumps and control systems. As explained above, a failure in the second cooling loop was selected since the second cooling loop controls the end of solidification zone in the strand having an important influence on the quality. The second cooling loop can be therefore considered as the most important loop in the secondary cooling zone. As for computational requirements of the proposed PHA implementation, the method is definitely not applicable for real-time applications. However, the PHA implementation can reasonably be used for a pre-calculation of various failure situations in advance and their solutions can be then concatenated into a solution manual applicable by operators of the casting machine in failure cases.

For the failure-free scenario s_0, the void second-stage decision $h''_{m,s_0} = 0$ for $m = 1, 2, 3, 4$ is obviously required (see Eq. (17)) since the operators of the casting machine modify its setting only in case of the failure. As for the scenario s_2, the failure situation within the second cooling loop at the time $t_\mathrm{f} = 60\,\mathrm{s}$ occurs, and thus $h'_{2,s_2} + h''_{2,s_2} = 0$ (see Eq. (19)).

5.3. Discussion on parameters

The problem was solved with the use of a computer running 64-bit Ubuntu operating system equipped with Intel Quad CPU having four cores and 8 GB of the RAM memory. A crucial point in case of the practical application of the PHA is the choice of the penalty parameter ϱ since the penalty parameter can significantly influence the feasibility of iteratively generated solutions and the convergence of the PHA. An improper value of the penalty parameter can considerably increase the number of iterations required by the PHA, or it may even lead to divergence (oscillatory behaviour) of the algorithm. The penalty parameter can be considered a constant value or it can vary through iterations [25]. Unfortunately, there is no general procedure for the determination of a suitable value of ϱ. Due to this reason a trial-and-error approach is often used by users of the PHA. In the paper we determined the penalty parameter ϱ to 10^{-3}. The termination parameter D was set to 10^{-9}.

5.4. Results

Efficiency of the parallel MPI implementation. The parallel MPI implementation of the PHA was used to solve the presented two-stage stochastic steel production problem. The PHA required 74 iterations performed in about 14 hours and 30 min of the computational time. After 74 iterations the algorithm converged to an optimal solution. The solution of a scenario-related sub-problem (in a particular iteration of the PHA)

			Casting parameters				
Stage	Scenario	Decision	h_1	h_2	h_3	h_4	v_z
First	s_0, s_2	$\boldsymbol{x}'$	$476\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$249\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$309\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$87\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$2.2156\,\frac{\mathrm{m}}{\mathrm{min}}$
Second	s_0	$\boldsymbol{x}''_1$	$0\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$0\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$0\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$0\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$2.2156\,\frac{\mathrm{m}}{\mathrm{min}}$
Second	s_2	$\boldsymbol{x}''_2$	$24\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$-249\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$-240\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$-15\,\frac{\mathrm{W}}{\mathrm{m^2K}}$	$1.8366\,\frac{\mathrm{m}}{\mathrm{min}}$

Tab. 2. The first-stage and second-stage decisions to the steel production problem determined with the use of the PHA.

took about 12 min in average and two sub-problems (two scenarios) were concurrently solved via MPI at the same time. On the other hand, the computational time required by the PHA to load results, compute the aggregated solution, update the weights, evaluate the termination condition and to assemble the GAMS files for the next iteration was virtually negligible and less than 1 s. From that point of view the parallel implementation enabled the reduction of the computational time to about a half in the comparison to a sequential (non-parallel) processing.

Solution to the two-stage steel production problem. The scenario-dependent optimal decisions are presented in Table 2. The first-stage "here-and-now" decision $\boldsymbol{x}'$ is scenario-independent due to the non-anticipativity as mentioned in Section 4.3. The value of the objective function (i. e. the casting speed v_z) corresponding to the first-stage decision is equal to 2.216 m/min.

In case the failure does not occur in the secondary cooling zone (scenario s_0), the second-stage "wait-and-see" decision $\boldsymbol{x}''_1$ prescribes no change in the casting parameters. This means that the casting parameters in the second stage are $\boldsymbol{x}' + \boldsymbol{x}''_1 = \boldsymbol{x}'$ with the casting speed unchanged. On the other hand, if the failure in the second loop of the secondary cooling zone does occur, the second stage decision $\boldsymbol{x}''_2$ is applied by the operator of the casting machine according to Table 2. In that case the casting parameters are updated to $\boldsymbol{x}' + \boldsymbol{x}''_2$. Observe that the heat transfer coefficient for the second cooling loop h_2 is equal to $0\,\frac{\mathrm{W}}{\mathrm{m^2K}}$ as it is used to simulate the failure. Since there is almost no heat withdrawal from the strand in the second loop, the casting speed has to be reduced to 1.8366 m/min in order to fulfil the temperature constraints and the range of the metallurgical length given by Eqs. (12) and (13), respectively.

The resultant temperature distributions of the strand for the failure-free scenario s_0 and for the scenario with the failure s_2 are shown in Figures 6 and 8, respectively. The temperature profiles at the selected longitudinal cross-sections of the strand are pictured in Figures 7 and 9. As can be seen in the figures, the temperature distributions for both the scenarios fulfil quite well the requirement of the gradual decrease of the temperature. In case of the scenario with the failure, there is a small reheating at the surface (see Figure 9) at the distance of about 6 m from the mould. However, such reheating is only local with the peak of about 150 °C and it can therefore be accepted by metallurgists with no negative influence on the surface quality of the strand.

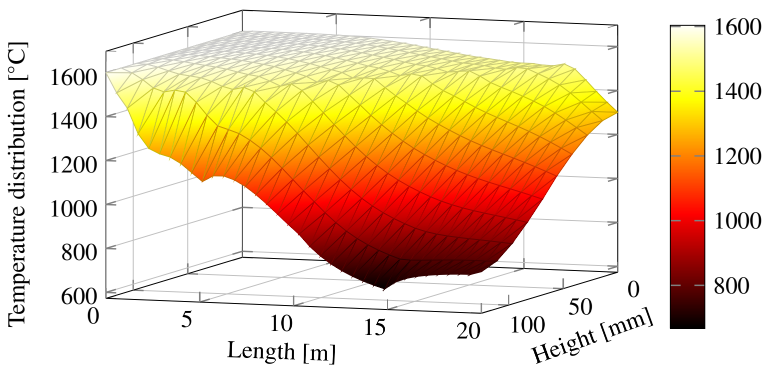

Fig. 6. The resultant temperature distribution of the strand for the failure-free scenario s_0.

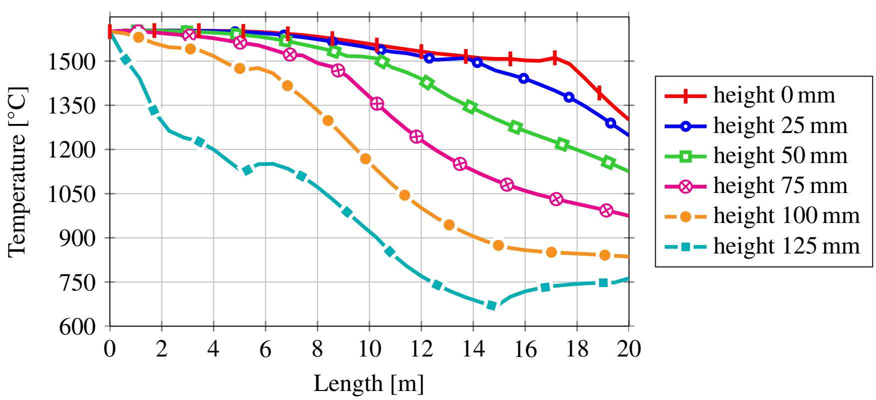

Fig. 7. The temperature profiles in the longitudinal cross-sections of the strand for the failure-free scenario s_0.

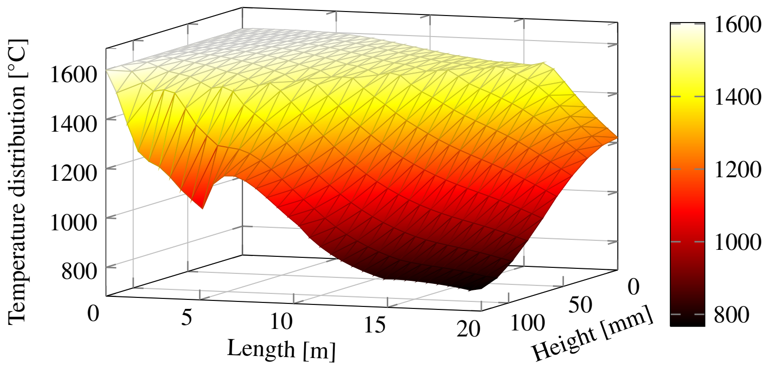

Fig. 8. The resultant temperature distribution of the strand for the scenario s_2 with the failure.

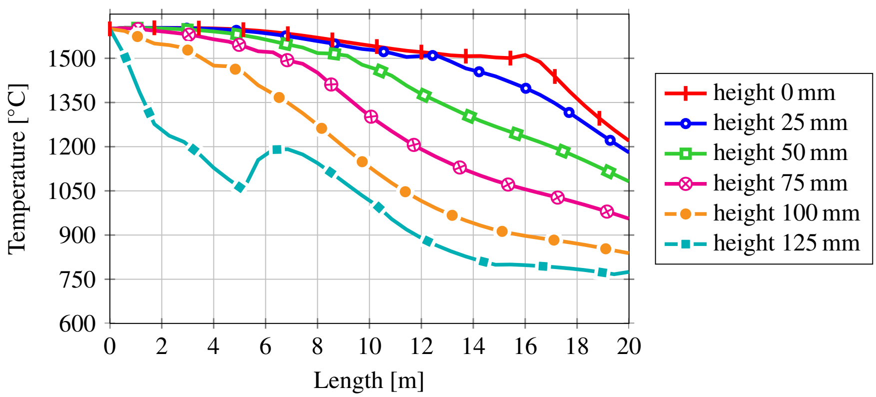

Fig. 9. The temperature profiles in the longitudinal cross-sections of the strand for the scenario s_2 with the failure.

5.5. Evaluation of the results

As mentioned in Section 4 the progressive hedging algorithm was used to determine the solution to the expected objective (EO) problem having the general form

$$\begin{cases} \text{minimize} & \mathbb{E}_{\boldsymbol{\xi}}\big(f(\boldsymbol{x},\boldsymbol{\xi})\big) \\ \text{subject to} & \boldsymbol{x} \in X : \boldsymbol{g}(\boldsymbol{x},\boldsymbol{\xi}) = \mathbf{0}, \boldsymbol{h}(\boldsymbol{x},\boldsymbol{\xi}) \leq \mathbf{0} \text{ almost surely.} \end{cases} \tag{23}$$

In general, it is more difficult to solve the EO problem (23) than the simpler expected value (EV) problem (24)

$$\begin{cases} \text{minimize} & f\big(\boldsymbol{x}, \mathbb{E}(\boldsymbol{\xi})\big) \\ \text{subject to} & \boldsymbol{x} \in X : \boldsymbol{g}\big(\boldsymbol{x}, \mathbb{E}(\boldsymbol{\xi})\big) = \mathbf{0}, \boldsymbol{h}\big(\boldsymbol{x}, \mathbb{E}(\boldsymbol{\xi})\big) \leq \mathbf{0} \end{cases} \tag{24}$$

as the EV formulation reduces the stochastic problem into the deterministic problem because the random variables are replaced by their expected values [3].

Value of stochastic solution (VSS). The VSS is the useful measure for the evaluation of the potential profit which can be obtained if the EO problem (23) is solved instead of the EV problem (24). The VSS is defined as [3]

$$\mathrm{VSS} = \mathrm{EEV} - \mathbb{E}_{\boldsymbol{\xi}}\big(f(\boldsymbol{x}_{\mathrm{EO}}, \boldsymbol{\xi})\big) \tag{25}$$

where $\boldsymbol{x}_{\mathrm{EO}}$ is the optimal solution to the problem (23) and the EEV is called the expected result of using EV solution [3], $\mathrm{EEV} = \mathbb{E}_{\boldsymbol{\xi}}\big(f(\boldsymbol{x}_{\mathrm{EV}}, \boldsymbol{\xi})\big)$ where $\boldsymbol{x}_{\mathrm{EV}}$ is the optimal solution to the problem (24). The EV solution instead of the EO solution can be well applied in cases having small values of the VSS. On the other hand, the higher the value of the VSS the higher the profit that can be acquired using the EO solution instead of the EV solution.

The EV formulation of the steel production problem presented in the paper was solved and the values of the objective function are 2.1885 m/min and 1.8355 m/min for the failure-free scenario s_0 and for the scenario with the failure s_2, respectively. The value of the EEV is 2.1709 m/min and the VSS for the maximization problem then is

$$\mathrm{VSS} = \mathbb{E}_{\boldsymbol{\xi}}\big(f(\boldsymbol{x}_{\mathrm{EO}}, \boldsymbol{\xi})\big) - \mathrm{EEV} = 0.0258\,\mathrm{m/min}. \tag{26}$$

Though the value of the VSS is rather small, the profit in the strand production of 2.6 cm per minute is not negligible when considering the 24-hour casting operation and the price of the steel strand between 1,000 € and 10,000 € per meter. Moreover, in case of the failure the important aim of steelworkers is not only to optimally solve the problem from the point of the maximum productivity. A crucial task is to maintain the casting process in operation and to provide sufficient time for a repair/solution of the failure. In some cases, a failure in the secondary cooling zone could cause a serious breakout situation in which the solid shell at the surface of the strand cracks and the molten steel inside the strand then leaks out stopping the casting machine for several days.

Expected value of perfect information (EVPI). The EVPI, another useful measure, evaluates the profit which can be acquired in case the full information about the future is available [3]. The EVPI therefore states the maximum amount of "money" which is reasonable to pay for information about the future. The EVPI is defined as

$$\mathrm{EVPI} = \mathbb{E}_{\boldsymbol{\xi}}\big(f(\boldsymbol{x}_{\mathrm{EO}}, \boldsymbol{\xi})\big) - z_{\mathrm{WS}} \tag{27}$$

where z_{WS} is the objective value of the wait-and-see (WS) deterministic formulation to the problem, see [3]. The WS formulation of the steel production problem presented in the paper was solved. The values of the objective function are 2.2158 m/min and 1.8396 m/min for the failure-free scenario s_0 and for the scenario with the failure s_2, respectively. The value of z_{WS} is 2.1970 m/min and the EVPI for the maximization problem is then

$$\mathrm{EVPI} = z_{\mathrm{WS}} - \mathbb{E}_{\boldsymbol{\xi}}\big(f(\boldsymbol{x}_{\mathrm{EO}}, \boldsymbol{\xi})\big) = 0.0003\,\mathrm{m/min}. \tag{28}$$

Nonetheless, the information about the future is usually not available at any price in technical optimization problems. The VSS is therefore more informative parameter in the studied steel production problem than the EVPI.

6. CONCLUSIONS

The parallel implementation of the two-stage progressive hedging algorithm (PHA) is presented. The Message Passing Interface (MPI) was used for the parallel run of the General Algebraic Modelling System (GAMS) which solves the scenario-based sub-problems. The standalone application wrapping the PHA, MPI, and GAMS was created in C++. The applicability of the developed software was demonstrated for the solution of the two-stage PDE-constrained steel production problem considering a random failure. The discretization of the heat transfer model with phase changes for the steel production by means of the control volume method is discussed. The evaluation of the solution quality is presented and the proposed computational approach seems effective and sufficiently robust for similar kinds of problems. The further research will be mainly aimed at the improvement of the heat transfer model of continuous casting as numerous simplifications were made. Due to requirements from the steel industry, the attention has recently also focused on real-time control algorithms based on predictive control and fuzzy logic approach.

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REFERENCES

[1] A. Alquarashi, A. H. Etemadi, and A. Khodaei: Treatment of uncertainty for next generation power systems: State-of-the-art in stochastic optimization. Electr. Power Syst. Res. *141* (2016), 233–245. DOI:10.1016/j.epsr.2016.08.009

[2] M. Barttfeld, N. Alleborn, and F. Durst: Dynamic optimization of multiple-zone air impingement drying process. Comput. Chem. Engrg. *30* (2006), 467–489. DOI:10.1016/j.compchemeng.2005.10.016

[3] J. R. Birge and F. Louveaux: Introduction to Stochastic Programming. Springer, New York 2011.

[4] J. K. Brimacombe and K. Sorimachi: Crack formation in continuous-casting of steel. Metal. Trans. B. Proc. Metal. *8* (1977), 489–505. DOI:10.1007/bf02696937

[5] E. P. Carvalho, J. Martínez, J. M. Martínez, and F. Pisnitchenko: On optimization strategies for parameter estimation in models governed by partial differential equations. Math. Comput. Simul. *114* (2015), 14–24. DOI:10.1016/j.matcom.2010.07.020

[6] M. Carrasco, B. Ivorra, and A. M. Ramos: Stochastic topology design optimization for continuous elastic materials. Comput. Meth. Appl. Mech. Engrg. *289* (2015), 131–154. DOI:10.1016/j.cma.2015.02.003

[7] P. L. Carpentier, M. Gendreau, and F. Bastin: Long-term management of a hydroelectric multireservoir system under uncertainty using the progressive hedging algorithm. Water Resour. Res. *49* (2013), 2812–2827. DOI:10.1002/wrcr.20254

[8] Y. M. Cheng, D. Z. Li, N. Li, Y. Y. Lee, S. K. Au: Solution of some engineering partial differential equations governed by the minimal of a functional by global optimization method. J. Mech. *29* (2013), 507–516. DOI:10.1017/jmech.2013.26

[9] A. Drud: CONOPT – A GRG code for large sparse dynamic nonlinear optimization problems. Math. Program. *31* (1985), 153–191. DOI:10.1007/bf02591747

[10] D. Gade, G. Hackebeil. S. M. Ryan, J.-P. Watson, R. J.-B. Wets, and D. L. Woodruff: Obtaining lower bounds from the progressive hedging algorithm for stochastic mixed-integer programs. Math. Prog. *157* (2016), 47–67. DOI:10.1007/s10107-016-1000-z

[11] R. E. C. Gonçalves, E. C. Finardi, and E. L. da Silva: Applying different decomposition schemes using the progressive hedging algorithm to the operation planning problem of a hydrothermal system. Electr. Power Syst. Res. *83* (2012), 19–27. DOI:10.1016/j.epsr.2011.09.006

[12] S. Gul, B. T. Denton, and J. W. Fowler: A progressive hedging approach for surgery planning under uncertainty. INFORMS J. Comput. *27* (2015), 755–772. DOI:10.1287/ijoc.2015.0658

[13] S. Ikeda and R. Ooka: A new optimization strategy for the operating schedule of energy systems under uncertainty of renewable energy sources and demand changes. Energ. Build. *125* (2016), 75–85. DOI:10.1016/j.enbuild.2016.04.080

[14] T. L. Bergman, A. S. Lavine, F. P. Incropera, and D. P. Dewitt: Fundamentals of Heat and Mass Transfer. Seventh edition. Wiley, New York 2011.

[15] L. Klimeš: Stochastic Programming Algorithms. Master Thesis. Brno University of Technology, 2010.

[16] L. Klimeš and P. Popela: An implementation of progressive hedging algorithm for engineering problem. In: Proc. 16th International Conference on Soft Computing MENDEL, Brno 2010, pp. 459–464.

[17] L. Klimeš, P. Popela, and J. Štětina: Decomposition approach applied to stochastic optimization of continuous steel casting. In: Proc. 17th International Conference on Soft Computing MENDEL, Brno 2011, pp. 314–319.

[18] L. Klimeš, T. Mauder, and J. Štětina: Stochastic approach and optimal control of continuous steel casting process by using progressive hedging algorithm. In: Proc. 20th International Conference on Materials and Metallurgy METAL, Brno 2011, pp. 146–151.

[19] M. La Marca, D. Armbruster, M. Herty, and C. Ringhofer: Control of continuum models of production systems. IEEE Trans. Automat. Control *55* (2010), 2511–2526. DOI:10.1109/tac.2010.2046925

[20] A. Lamghari and R. Dimitrakopoulos: Progressive hedging applied as a metaheuristic to schedule production in open-pit mines accounting for reserve uncertainty. Eur. J. Oper. Res. *253* (2016), 843–855. DOI:10.1016/j.ejor.2016.03.007

[21] J. Liu and C. Liu: Optimization of mold inverse oscillation control parameters in continuous casting process. Mater. Manuf. Process. *30* (2015), 563-568. DOI:10.1080/10426914.2015.1004696

[22] K. C. Mills, P. Ramirez-Lopez, P. D. Lee, B. Santillana, B. G. Thomas, and R. Morales: Looking into continuous casting mould. Ironmak. Steelmak. *41* (2014), 242–249. DOI:10.1179/0301923313z.000000000255

[23] T. Mauder, F. Kavička, J. Štětina, Z. Franěk, and M. Masarik: A mathematical & stochatic modelling of the concasting of steel slabs. In: Proc. International Conference on Materials and Metallurgy, Hradec nad Moravicí 2009, pp. 41–48.

[24] T. Mauder and J. Novotný: Two mathematical approaches for optimal control of the continuous slab casting process. In: Proc. 16th International Conference on Soft Computing MENDEL, Brno 2010, pp. 41–48.

[25] R. T. Rockafellar and R. J.-B. Wets: Scenarios and policy aggregation in optimization under uncertainty. Math. Oper. Res. *16* (1991), 119–147. DOI:10.1287/moor.16.1.119

[26] A. Ruszczynski and A. Shapiro: Stochastic Programming Models. Handbooks in Operations Research and Management Science, Volume 10: Stochastic Programming, Elsevier, Amsterdam 2003. DOI:10.1016/s0927-0507(03)10001-1

[27] A. Shioura, N. V. Shakhlevich, and V. A. Strusevich: Application of submodular optimization to single machine scheduling with controllable processing times subject to release dates and deadlines. INFORMS J. Comput. *28* (2016), 148–161. DOI:10.1287/ijoc.2015.0660

[28] D. M. Stefanescu: Science and Engineering of Casting Solidification. Second edition. Springer, New York 2009.

[29] J. Štětina, L. Klimeš, and T. Mauder: Minimization of surface defects by increasing the surface temperature during the straightening of a continuously cast slab. Mater. Tehnol. *47* (2013), 311–316.

[30] H. Ugail and M. J. Wilson: Efficient shape parametrisation for automatic design optimisation using a partial differential equation formulation. Comput. Struct. *81* (2003), 2601–2609. DOI:10.1016/s0045-7949(03)00321-3

[31] P. Varaiya and R. J.-B. Wets: Stochastic dynamic optimization approaches and computation. In: Proc. 13th International Symposium on Mathematical Programming, Tokio 1989, pp. 309–331. DOI:10.1007/978-3-642-82450-0_11

[32] F. B. Veliz, J. P. Watson, A. Weintraub, R. J.-B. Wets, and D. L. Woodruff: Stochastic optimization models in forest planning: a progressive hedging solution approach. Ann. Oper. Res. *232* (2015), 259–274. DOI:10.1007/s10479-014-1608-4

[33] B. G. V. Waanders and B. R. Carnes: Optimization under adaptive error control for finite element based simulations. Comput. Mech. *47* (2011), 49–63. DOI:10.1007/s00466-010-0530-0

[34] R. J.-B. Wets: The aggretation principle in scenario analysis and stochastic optimization. In: Algorithms and Model Formulations in Mathematical Programming (S. W. Wallace, ed.), Springer, Berlin 1989. DOI:10.1007/978-3-642-83724-1_4

[35] Z. Yang, H. L. Qui, X. W. Luo, and D. Shen: Simulating schedule optimization problem in steelmaking continuous casting process. Int. J. Simul. Model. *14* (2015), 710–718. DOI:10.2507/ijsimm14(4)co17

[36] J. Yang, Z. P. Ji, S. Liu, and Q. Jia: Multi-objective optimization based on pareto optimum in secondary cooling and EMS of continuous casting. In: Proc. International Conference on Advanced Robotics and Mechatronics (ICARM), Macau 2016, pp. 283–287. DOI:10.1109/icarm.2016.7606933

[37] E. Žampachová, P. Popela, and M. Mrázek: Optimum beam design via stochastic programming. Kybernetika *46* (2010), 571–582.

[38] M. H. F. Zarandi, F. Dorry, and F. S. Moghadam: Steelmaking-continuous casting scheduling problem with interval type 2 fuzzy random due dates. In: Proc. IEEE Conference on Norbert Wiener in the 21st Century (21CW), Boston 2014. DOI:10.1109/norbert.2014.6893896

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