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CONSENSUS-BASED IMPACT-TIME-CONTROL GUIDANCE LAW FOR COOPERATIVE ATTACK OF MULTIPLE MISSILES

Qing Zhu, Xiaoli Wang and Qianyu Lin

In this paper, a new guidance problem with the impact time constraint for cooperative attack of multiple missiles is investigated, which can be applied to salvo attack of anti-ship missiles. It can be used to guide multiple missiles to hit a stationary target simultaneously at a desirable impact time. The considered impact time control problem can be transformed into a range tracking problem. Then the range tracking problem can be viewed a consensus problem of multi-missile systems. As the application of the distributed consensus controller of multi-agent systems, three distributed protocols are given to solve the cooperative attack problem for the advantages such as reducing cost, improving system efficiency, increasing flexibility and reliability of distributed control. Simulation results demonstrate the performance and feasibility of the given protocols.

Keywords: multiple missiles, impact-time-control, missile guidance, distributed control

Classification: 93C10, 93C85, 93C95

1. INTRODUCTION

As we know, Proportional Navigation Guidance (PNG) law is optimal for a constant-velocity missile to attack the target, which can minimize the control efforts. If there is no guidance system lag, the guidance command decreases to zero when the missile approaches the target. For decades, since the PNG does not control the impact time explicitly ([10, 11, 12]), many advanced guidance laws have been devised to improve the guidance performance of the PNG law and achieve some specific objectives. Generally the simultaneous attack requires an impact time which is not the same as the impact time of PNG. In [10], the author designed an impact time control law.It is required to carry out a salvo attack for anti-ship missiles against CIWS (close-in weapon system), which is one of the self-defense measures of almost all modern warfare ships against anti-ship missiles. CIWS usually consists of radars, fire-control-systems, and multiple rapid-fire guns. It has a naval shipboard weapon system which can detect and destroy incoming anti-ship missiles and enemy aircraft at short range. It is a great obstacle for a single anti-ship missile to complete the mission. Salvo attack of anti-ship missiles is devised as one of countermeasures in order to survive the threat of CIWS, in which

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several missiles can hit the target simultaneously. Clearly, it is difficult to defend a group of attackers bursting into sight at the same time. Therefore simultaneously attacking by multiple missiles is a cost-effective and efficient strategy.

The salvo attack problem only required multiple missiles to attack the target simultaneously ([11, 12, 22]). In this case, the target can be destroyed by the remaining ones though several missiles are intercepted. A group of missiles simultaneously attacking against a single common target can be achieved by two ways. The first one is individual homing, in which a common impact time is commanded for all missiles in advance, and then each missile home on the target on time independently ([22]). The second one is cooperative homing, in which the missiles communicate with each other to synchronize the arrival times ([11]). In [11], the authors considered the formulation of the homing problem of multiple missiles against a single target, subject to constraints on the impact time. Cooperative proportional navigation (CPN), which has the same structure as conventional PN except that it has a time-varying navigation gain, was obtained. In [23], the authors investigated the problem of robust cooperative formation tracking control of multiple missile. In [18], the authors studied the cooperative control problem for multi-missile systems and proposed a two-stage control strategy, aiming at simultaneous attack from a group of missiles at a static target. In [16], the authors investigated the consensus problem of multi-missile systems in directed networks with arbitrary finite-time varying communication delays.

In this paper, we study cooperative attack strategies subject to constraints on the impact time of multi-missile systems (MMS). As in [22], we also consider an alternative way to cope with the drawback of the midcourse salvo strategy. The missiles from different platforms or from a single platform approach the expected target position, under the control of a pre-programmed mid-course salvo strategy. However, unpredicted target movement during the mid-course guidance of the missiles makes it difficult to achieve simultaneous attack on the target. In this paper, several **distributed** feedback guidance laws are researched to hit the target at the designated impact time, while the approach given in [22] is individual homing approach. Distributed coordinated multi-missile systems has many advantages such as reducing cost, improving system efficiency, increasing flexibility and reliability. Moreover, multi-missile systems with new cooperative fighting manners can be progressed by following the steps of multi-agent systems. In recent years, there are a lot of papers about the coordination of a group of agents, due to a broad application of multi-agent systems (MAS) including consensus, swarming, flocking and formation (e. g. [2, 4, 16, 17, 19, 20, 21]), among which consensus is one of the important problem in the study of multi-agent systems ([1, 3, 6, 9, 14]). Distributed controller has been widely developed in multi-agent systems. Suitable neighbor-based rules for each agent are adopted based on the average of its own information and that of its neighbors to achieve consensus. In this paper, as the application of the distributed consensus controller of multi-agent systems, several distributed protocols are given to solve the cooperative attack problem.

In this paper, we investigate an impact-time-control cooperative guidance law for multi-missile systems. Firstly, the time-to-go for each missile is estimated based on PNG and then the desired impact time is determined for each missile. After that, we transform the impact time control problem into a range tracking problem. Secondly, using the tool of feedback linearization, we can transform a nonlinear model for homing guidance into a linear model. Thirdly, a range tracking problem for a linear system can be viewed as a consensus problem of multi-missile systems. Fourthly, several distributed control laws are designed by using consensus protocol of multi-agent systems. Lastly, in order to prove the validity of the algorithm, we get the simulation results. And they show that all missiles can attack the target at the designated impact time very accurately.

The paper is organized as follows. The problem formulation is given in Section 2. In Section 3, we give several distributed protocols of multi-missile-systems, containing consensus with bounded control input and consensus without relative state derivative measurements. Then a numerical simulation results are given in Section 4 to show the effectiveness of our guidance law. The concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

Suppose that n missiles participate in a salvo attack against a single target.Although each missile has a different missile-to-target range and an initial heading angle, their common aim is to reach the target simultaneously. The designated impact time T_d for salvo attack is determined as $T_d \geq \max\{\hat{T}_i,\ i = 1, \ldots, n\}$, where n is the number of missiles involved in the salvo attack and $\hat{T}_i$ is the estimated impact time of the ith missile produced by PNG, $\hat{T}_i$ can be computed using the method in [10], the formula is as follows:

$$\hat{T}_i \approx R_{igo}(1 + (\sigma_i - q_i)^2/10)/V_i, \quad i = 1, \ldots, n, \tag{1}$$

where V_i is the ith missile speed, σ_i, q_i, R_{igo} denote velocity vector angle, the LOS angle in the inertial reference frame, and the range of the ith missile and the target.

Consider the homing guidance geometry of one of the missiles against the single target.

As considered in [10, 11], the target is modeled as being stationary and the speed of the missile V_i $(i = 1, \ldots, n)$ is constant.

In Figure 1, X_i, Y_i and η_i denote missile positions and the heading angle in the inertial reference frame. The subscript $i0$ and if $(i = 1, \ldots, n)$ represent the initial and the final time of ith missile, respectively. Assuming that the terminal time T_{if} $(i = 1, \ldots, n)$ is designated as T_d. The guidance problem can be formulated as a tracking problem for a time-varying nonlinear system.

The guidance relationship of the missile and the target is given as follows:

$$\begin{cases} \dot{r}_i = V_t \cos(\eta_t) - V_i \cos(\eta_i) \\ r_i \dot{q}_i = V_i \sin(\eta_i) - V_t \sin(\eta_t) \\ q_i = \sigma_i + \eta_i \\ \dot{\sigma}_i = \frac{a_i}{V_i} \end{cases}, \ i = 1, \ldots, n, \tag{2}$$

where V_t is the target velocity, and $r_i, \eta_i, \eta_t, \sigma_i, a_i$ denote the range of the ith missile and target, the altitude angle of missile, the altitude angle of target, polar angle of missile, and acceleration command of missile. Then a second-order system can be obtained from (2) as follows

$$\begin{cases} \dot{r}_i = V_t \cos(\eta_t) - V_i \cos(\eta_i) \\ \dot{\eta}_i = \frac{V_i \sin(\eta_i) - V_t \sin(\eta_t)}{r_i} - \frac{a_i}{V_i}, \ i = 1, \ldots, n, \end{cases} \tag{3}$$

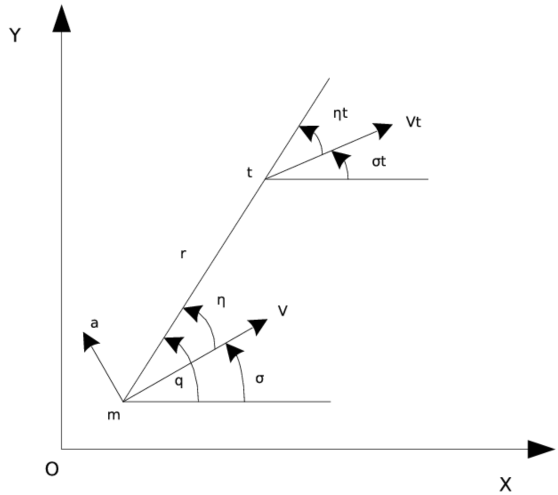

Fig. 1. LOS figure: the relative motion of missile and target.

where a_i is the input and r_i is the output. In order to make the missile attack the target at the designated time, R_i is designed as follows

$$R_i = V_i(T_d - t),\ i = 1, \ldots, n.$$

The key problem is to design a distributed controller making

$$\lim_{t \to \infty} r_i - R_i = 0,\ i = 1, .., n.$$

Since R_i tends to zero at appointed time, the n missiles arrive at target at designated time T_d. Then the guidance law is completed.

In this paper, the n missiles are regarded as the nodes, and the relationships between n missiles can be conveniently described by a directed graph. In the following, we first introduce some basic concepts in graph theory ([5]). A directed graph (or digraph) is denoted as $\mathcal{G} = (\mathcal{O}, \mathcal{E})$, where $\mathcal{O} = \{1, 2, \ldots, n\}$ is the set of nodes and $\mathcal{E}$ is the set of edges, each element of the directed graph is an ordered pair of distinct nodes in $\mathcal{O}$. $(i, j) \in \mathcal{E}$ denotes an edge leaving from node i and entering into node j if node i can get information from node j. In this case node j is said to be a neighbor of node i. Undirected graph is a special case of directed graph, in which $(i, j) \in \mathcal{E}$ if and only if $(j, i) \in \mathcal{E}$. A path in digraph $\mathcal{G}$ is an alternating sequence $i_1 e_1 i_2 e_2 \ldots e_{k-1} i_k$ of nodes i_m and edges $e_m = (i_m, i_{m+1}) \in \mathcal{E}$ for $m = 1, 2, \ldots, k-1$. The directed graph has a directed spanning tree if and only if the graph has a directed path to all other nodes. In undirected graph, the existence of an undirected spanning tree is equivalent to being connected. The weighted adjacency matrix of $\mathcal{G}$ is denoted as $A = (a_{ij})_{n \times n} \in R^{n \times n}$, where $a_{ii} = 0$ and $a_{ij} \geq 0$ ($a_{ij} > 0$ if there is an edge from agent i to agent j). Its degree matrix $\Delta = diag\{\beta_1, \ldots, \beta_n\} \in R^{n \times n}$ is a diagonal matrix, where diagonal elements

$\beta_i = \sum_{j=1}^{n} a_{ij}$ for $i = 1, \ldots, n$. Then the Laplacian of the weighted graph is defined as $L = \Delta - A$.

The following lemma is about the Laplacian matrix L ([15]).

Lemma 2.1. The Laplacian matrix L of a directed graph has a simple zero eigenvalue with an associated eigenvector 1_n and all of the other eigenvalues are in the open right half plane if and only if the directed graph has a directed spanning tree.

3. DISTRIBUTED CONTROL PROTOCOL

As considered in [10, 11], the target is modeled as being stationary and the speed of ith missile V_i $(i = 1, \ldots, n)$ is constant. Considering the system (3), let

$$\begin{cases} z_{i1} = R_i - r_i \\ z_{i2} = \dot{z}_{i1}, \quad i = 1, \ldots, n. \end{cases} \tag{4}$$

Since V_i $(i = 1, \ldots, n)$ are constant, the new state equation is

$$\begin{cases} \dot{z}_{i1} = z_{i2} \\ \dot{z}_{i2} = -V_i \sin\eta_i (\frac{V_i \sin\eta_i}{r_i} - \frac{a_i}{V_i}), \quad i = 1, \ldots, n. \end{cases} \tag{5}$$

Using the method of feedback linearization, let

$$a_i = \frac{1}{\sin\eta_i}(v_i - \frac{V_i^2 \sin^2\eta_i}{r_i}), \quad i = 1, \ldots, n, \tag{6}$$

where v_i $(i = 1, \ldots, n)$ is an equivalent input to be designed. Then we have

$$\begin{cases} \dot{z}_{i1} = z_{i2} \\ \dot{z}_{i2} = v_i, \quad i = 1, \ldots, n. \end{cases} \tag{7}$$

In the following part, we will give the description of consensus ([3, 9, 15]).

Definition 3.1. Consider the MMS (5), if we have $\lim_{t\to\infty} \|z_{i1}(t) - z_{j1}(t)\| = 0$ and $\lim_{t\to\infty} \|z_{i2}(t) - z_{j2}(t)\| = 0$ for any initial state $z_{i1}(0)$, $z_{i2}(0)$, $i = 1, \ldots, n$. Then consensus is achieved.

It is easy and correct to consider the cooperative guidance of multiple missiles (5) as the consensus problem of multiple missiles (7). In the following subsections, we will give several distributed control protocols for MMS (5).

3.1. Most general consensus protocol

We will give our first algorithm based on the consensus of multi-agent systems in [15]. Consider a MMS of n missiles with dynamics (5) steered by the following protocol:

$$v_i = \sum_{j=1}^{n} a_{ij}[(z_{j1} - z_{i1}) + (z_{j2} - z_{i2})], \quad i = 1, \ldots, n. \tag{8}$$

If the directed graph has a directed spanning tree, the MMS will achieve consensus.

Recalling (6), the control law for (5) is

$$\begin{aligned} a_i &= \frac{1}{\sin \eta_i} \sum_{j=1}^{n} a_{ij}(V_j(T_d - t) - r_j - V_i(T_d - t) + r_i) \\ &+ \frac{1}{\sin \eta_i} \sum_{j=1}^{n} a_{ij}(V_j \cos \eta_j - V_i \cos \eta_i) \\ &- \frac{1}{\sin \eta_i} \frac{V_i^2 \sin^2 \eta_i}{r_i}, \quad i = 1, \dots, n. \end{aligned}$$

Consider the above control law, it will be singular when $r_i = 0$ or $\eta_i = 0$. Then we have the following theorem.

Theorem 3.2. (1) When $\eta_i = 0$, we can replace it with a small positive number ϵ:

$$a_i = \frac{1}{\sin \epsilon} \sum_{j=1}^{n} a_{ij}(V_j(T_d - t) - r_j - V_i(T_d - t) + r_i) \tag{9}$$

$$+ \frac{1}{\sin \epsilon} \sum_{j=1}^{n} a_{ij}(V_j \cos \eta_j - V_i \cos \eta_i), \quad i = 1, \dots, n.$$

(2) When $r_i = 0$, it means missile attacking the target. But $r_i(t) \to 0$ for large t, the control will divergent, so when $|R_i - r_i| < 30$, we switch the guidance law to PNG

$$a_i = N V_i \dot{q}_i, \quad i = 1, \dots, n. \tag{10}$$

So **the guidance law for MMS (5)** is

$$a_i = \begin{cases} \frac{1}{\sin \eta_i} \sum_{j=1}^{n} a_{ij}(V_j(T_d - t) - r_j - V_i(T_d - t) + r_i) \\ + \frac{1}{\sin \eta_i} (\sum_{j=1}^{n} a_{ij}(V_j \cos \eta_j - V_i \cos \eta_i) - \frac{V_i^2 \sin^2 \eta_i}{r_i}) \\ \qquad \text{if } |R_i - r_i| \geq 30 \ \& \ \eta_i \neq 0 \\ \frac{1}{\sin \epsilon} (\sum_{j=1}^{n} a_{ij}(V_j(T_d - t) - r_j - V_i(T_d - t) + r_i)) \\ + \frac{1}{\sin \epsilon} \sum_{j=1}^{n} a_{ij}(V_j \cos \eta_j - V_i \cos \eta_i) \qquad \text{if } |R_i - r_i| \geq 30 \ \& \ \eta_i = 0 \\ N V_i \dot{q}_i \qquad \text{if } |R_i - r_i| < 30. \end{cases} \tag{11}$$

3.2. Consensus with bounded control inputs

In the following, we take into account a bounded control for (7) with bounded control inputs as

$$v_i = \sum_{j=1}^{n} a_{ij}[\tanh(z_{j1} - z_{i1}) + \tanh(z_{j2} - z_{i2})]. \tag{12}$$

Note that with (12), v_i is bounded because $\tanh(\cdot)$ is bounded with $\|v_i\|_\infty \leq \sum_{j=1}^{n} 2a_{ij}$ independent of initial conditions of the information states and their derivatives.

Theorem 3.3. Consider a MMS of n missiles with dynamics (5) steered by the protocol (15). If the undirected graph is connected, the MMS will achieve consensus.

Proof. Note that with (12), (7) can be written as

$$\begin{cases} \dot{z}_{i1} = z_{i2} \\ \dot{z}_{i2} = \sum_{j=1}^{n} a_{ij}[\tanh(z_{j1} - z_{i1}) + \tanh(z_{j2} - z_{i2})]. \end{cases} \tag{13}$$

Consider the following Lyapunov function for (13) as

$$V = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \log[\cosh(z_{i1} - z_{j1})] + \frac{1}{2} \sum_{i=1}^{n} z_{i2}^2, \tag{14}$$

where $\cosh(\cdot)$, $\log(\cdot)$ are defined componentwise. Note that V is positive definite and radially unbounded with respect to $z_{i1} - z_{j1}$, $\forall i \neq j$, z_{i2}, i,j=1,. . . ,n. Differentiating V gives

$$\begin{aligned} \dot{V} &= \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} (z_{i2} - z_{j2})^T \tanh(z_{i1} - z_{j1}) \\ &\quad + \sum_{i=1}^{n} z_{i2} (\sum_{j=1}^{n} a_{ij} [\tanh(z_{j1} - z_{i1}) \\ &\quad + \tanh(z_{j2} - z_{i2})]) \\ &= \sum_{i=1}^{n} z_{i2} \sum_{j=1}^{n} a_{ij} \tanh(z_{j2} - z_{i2}) \\ &= -\frac{1}{2} \sum_{i=1}^{n} (z_{j2} - z_{i2}) \sum_{j=1}^{n} a_{ij} \tanh(z_{j2} - z_{i2}) \\ &\leq 0. \end{aligned}$$

Let $S = \{(z_{i1} - z_{j1},\ z_{i2}) | \dot{V} = 0\}$. Note that $\dot{V} \equiv 0$ implies that $z_{i2} \equiv z_{j2}$, $\forall i \neq j$, which in turn implies that $\dot{z}_{i2} \equiv \dot{z}_{j2}$, $\forall i \neq j$. Since the undirected graph is connected and $\tanh(z_{i1} - z_{j1}) = -\tanh(z_{j1} - z_{i1})$, $\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \tanh(z_{j1} - z_{i1}) = 0$, i. e., $\sum_{i=1}^{n} \dot{z}_{i2} = 0$. Then $\dot{z}_{i2} = 0$, $i = 1, \ldots, n$. From $\dot{z}_{i2} = \sum_{j=1}^{n} a_{ij} \tanh(z_{j1} - z_{i1}) = 0$, we can have

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} z_{i1} \tanh(z_{j1} - z_{i1}) = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} (\tanh(z_{j1} - z_{i1}))^2 = 0.$$

Therefore $z_{i1} \equiv z_{j1}$, $\forall i \neq j$. It follows that $\lim_{t\to\infty} z_{i1}(t) = z_{j1}(t)$, $\lim_{t\to\infty} z_{i2}(t) = z_{j2}(t)$, $\forall i \neq j$, $i, j = 1, \ldots, n$, by Laselle Invariance Theorem. Thus the consensus is achieved. □

Similar to the discussion in Theorem 3.2, **the guidance law for MMS (5)** is:

$$a_i = \begin{cases} \frac{1}{\sin\eta_i}\sum_{j=1}^{n} a_{ij}\tanh(V_j(T_d-t)-r_j) \\ -\frac{1}{\sin\eta_i}\sum_{j=1}^{n} a_{ij}\tanh(V_i(T_d-t)+r_i) \\ +\frac{1}{\sin\eta_i}\sum_{j=1}^{n} a_{ij}(\tanh(V_j\cos\eta_j)-\tanh(V_i\cos\eta_i)) \\ -\frac{1}{\sin\eta_i}\frac{V_i^2\sin^2\eta_i}{r_i} & \text{if } |R_i-r_i|\geq 30 \ \&\ \eta_i\neq 0 \\ \frac{1}{\sin\epsilon}\sum_{j=1}^{n} a_{ij}\tanh(V_j(T_d-t)-r_j) \\ -\frac{1}{\sin\epsilon}\sum_{j=1}^{n} a_{ij}\tanh(V_i(T_d-t)+r_i) \\ +\frac{1}{\sin\epsilon}\sum_{j=1}^{n} a_{ij}(\tanh(V_j\cos\eta_j)-\tanh(V_i\cos\eta_i)) \\ & \text{if } |R_i-r_i|\geq 30 \ \&\ \eta_i=0 \\ NV_i\dot{q}_i & \text{if } |R_i-r_i|<30. \end{cases} \tag{15}$$

The following corollary is mainly based on Theorem 3.2 by which $\lim_{t\to\infty} z_{i2}(t) = 0,\ i=1,\dots,n$.

Corollary 3.4. Consider a MMS of n missiles with dynamics (7) steered by the protocol

$$v_i = \sum_{j=1}^{n} a_{ij}[\tanh(z_{j1}-z_{i1})-\tanh(z_{i2})], \quad i=1,\dots,n. \tag{16}$$

If the undirected graph is connected, the MMS will achieve consensus with $\lim_{t\to\infty} z_{j1}(t) = z_{i1}(t),\ \forall j\neq i,\ \lim_{t\to\infty} z_{i2}(t)=0,\ i,j=1,\dots,n$. Thus **the guidance law for MMS (5)** is:

$$a_i = \begin{cases} \frac{1}{\sin\eta_i}\sum_{j=1}^{n} a_{ij}\tanh(V_j(T_d-t)-r_j) \\ -\frac{1}{\sin\eta_i}\sum_{j=1}^{n} a_{ij}\tanh(V_i(T_d-t)+r_i) \\ -\frac{1}{\sin\eta_i}\tanh(V_i\cos\eta_i-\frac{V_i^2\sin^2\eta_i}{r_i}) & \text{if } |R_i-r_i|\geq 30 \ \&\ \eta_i\neq 0 \\ \frac{1}{\sin\epsilon}\sum_{j=1}^{n} a_{ij}\tanh(V_j(T_d-t)-r_j) \\ -\frac{1}{\sin\epsilon}\sum_{j=1}^{n} a_{ij}\tanh(V_i(T_d-t)+r_i) \\ -\frac{1}{\sin\epsilon}\tanh(V_i\cos\eta_i) & \text{if } |R_i-r_i|\geq 30 \ \&\ \eta_i=0 \\ NV_i\dot{q}_i & \text{if } |R_i-r_i|<30. \end{cases} \tag{17}$$

Proof. Following the proof of Theorem 3.3, consider the Lyapunov function given by (14). Differentiating V, gives

$$\dot{V} = -\sum_{i=1}^{n} z_{i2}\tanh(z_{i2}) \leq 0.$$

Let $S = \{(z_{i1}-z_{j1},\ z_{i2}) | \dot{V}=0\}$. Note that $\dot{V}\equiv 0$ implies that $z_{i2}=0,\ \dot{z}_{i2}=0$. Then $\sum_{j=1}^{n} a_{ij}[\tanh(z_{j1}-z_{i1})]\equiv 0$. Thus, an argument similar to that in the proof of Theorem 3.3 shows that $\lim_{t\to\infty} z_{j1}(t) = z_{i1}(t),\ \forall j\neq i,\ \lim_{t\to\infty} z_{i2}(t)=0,\ i,j=1,\dots,n$. □

3.3. Consensus without relative state derivative measurements

Note that (8) requires measurements of relative information state derivatives between neighboring vehicles. We propose a consensus algorithm without measurements of relative information state derivatives based on a passive approach as:

$$\dot{\hat{x}}_i = -\hat{x}_i + \sum_{j=1}^{n} a_{ij}(z_{i1} - z_{j1}) \tag{18}$$

$$y_i = -\hat{x}_i + \sum_{j=1}^{n} a_{ij}(z_{i1} - z_{j1}) \tag{19}$$

$$v_i = -y_i - \sum_{j=1}^{n} a_{ij}(z_{i1} - z_{j1}). \tag{20}$$

Theorem 3.5. Consider a MMS of n missiles with dynamics (5) achieves consensus if undirected graph is connected.

Proof. Let $z_1 = [z_{11}, \ldots, z_{n1}]^T$, $z_2 = [z_{12}, \ldots, z_{n2}]^T$, $y = [y_1, \ldots, y_n]^T$, $\hat{x} = [\hat{x}_1, \ldots, \hat{x}_n]^T$ and $v = [v_1, \ldots, v_n]$. Algorithm can be written as

$$\dot{\hat{x}} = -\hat{x} + Lz_1 \tag{21}$$

$$y = \dot{\hat{x}} \tag{22}$$

$$v = -y - Lz_1. \tag{23}$$

Thus (7) can be written as

$$\dot{z}_{i1} - \dot{z}_{j1} = z_{i2} - z_{j2} \tag{24}$$

$$z_{i2} - z_{j2} = -\sum_{j=1}^{n} a_{ij}(z_{i1} - z_{j1}) - \dot{\hat{x}}_i + \sum_{k=1}^{n} a_{jk}(z_{j1} - z_{k1}) + \dot{\hat{x}}_j \tag{25}$$

$$\ddot{\hat{x}}_i = -\dot{\hat{x}}_i + \sum_{j=1}^{n} a_{ij}(z_{i2} - z_{j2}). \tag{26}$$

Consider the Lyapunov function as

$$V = \frac{1}{2} z_1^T L^2 z_1 + \frac{1}{2} z_2^T z_2 + \frac{1}{2} \dot{\hat{x}}^T \dot{\hat{x}}.$$

Note that V is positive definite and radially unbounded with respect to $z_{i1} - z_{j1},\ z_{i2} - z_{j2},\ \forall i \neq j,\ \dot{\hat{x}}_i,\ i, j = 1, \ldots, n$. Differentiating V gives

$$\begin{aligned}
\dot{V} &= z_2^T L^2 z_1 + z_2^T Lu + \frac{1}{2} \ddot{\hat{x}}^T \dot{\hat{x}} + \frac{1}{2} \dot{\hat{x}}^T \ddot{\hat{x}} \\
&= z_2^T L^2 z_1 + z_2^T Lu \\
&\quad -\frac{1}{2} \dot{\hat{x}}^T \dot{\hat{x}} + \frac{1}{2} z_2^T L \dot{\hat{x}} - \frac{1}{2} \dot{\hat{x}}^T \dot{\hat{x}} + \frac{1}{2} \dot{\hat{x}}^T L z_2 \\
&= z_2^T L^2 z_1 + z_2^T Lu - \frac{1}{2} \dot{\hat{x}}^T \dot{\hat{x}} + z_2^T L \dot{\hat{x}} \\
&= -\frac{1}{2} \dot{\hat{x}}^T \dot{\hat{x}} \leq 0, \quad i = 1, \ldots, n.
\end{aligned}$$

Let $S = \{(z_{i1} - z_{j1},\ z_{i2} - z_{j2},\ \dot{\hat{x}})|\dot{V} = 0\}$. Note that $\dot{V} = 0$ implies $\dot{\hat{x}} = 0$, which in turn implies that $y = 0, \ddot{\hat{x}} = 0$. With the same analysis as in Theorem 3.2, we have $\dot{z}_{i1} = \dot{z}_{j1},\ \forall i \neq j$. Using Laselle Invariant Theorem, we obtain $\lim_{t\to\infty} z_{i1}(t) = z_{j1}(t),\ \lim_{t\to\infty} z_{i2}(t) = z_{j2}(t),\ \forall i \neq j,\ i,j = 1,\dots,n$. Then the consensus is achieved. □

Following the same discussion as in Remark, **the guidance law for MMS (5)** is

$$a_i = \begin{cases} \frac{1}{\sin\eta_i}\hat{x}_i + 2\frac{1}{\sin\eta_i}\sum_{j=1}^n a_{ij}(V_j(T_d - t) - r_j) \\ -2\frac{1}{\sin\eta_i}\sum_{j=1}^n a_{ij}(V_i(T_d - t) + r_i) \\ -\frac{1}{\sin\eta_i}\frac{V_i^2\sin^2\eta_i}{r_i} & \text{if } |R_i - r_i| \geq 30\ \&\ \eta_i \neq 0 \\ \frac{1}{\sin\epsilon}\hat{x}_i + 2\frac{1}{\sin\epsilon}\sum_{j=1}^n a_{ij}(V_j(T_d - t) - r_j) \\ -2\frac{1}{\sin\epsilon}\sum_{j=1}^n a_{ij}(V_i(T_d - t) + r_i) & \text{if } |R_i - r_i| \geq 30\ \&\ \eta_i = 0 \\ NV_i\dot{q}_i & \text{if } |R_i - r_i| < 30. \end{cases} \tag{27}$$

In the following corollary, we will give a protocol which is a simpler case than that in Theorem 3.5 making $\lim_{t\to\infty} z_{i2}(t) = 0,\ i = 1,\dots,n$.

Corollary 3.6. Consider a MMS of n missiles with dynamics (7) steered by the protocol

$$\dot{\hat{x}}_i = -\hat{x}_i + z_{i1} \tag{28}$$

$$y_i = -\hat{x}_i + z_{i1} \tag{29}$$

$$v_i = -y_i - \sum_{j=1}^n a_{ij}(z_{i1} - z_{j1}). \tag{30}$$

If the undirected graph is connected, the MMS will achieve consensus, $\lim_{t\to\infty} z_{j1}(t) = z_{i1}(t),\ \forall j \neq i,\ \lim_{t\to\infty} z_{i2}(t) = 0$, i,j=1,. . . ,n. Thus **the guidance law for MMS (5)** is

$$a_i = \begin{cases} \frac{1}{\sin\eta_i}(\hat{x}_i + r_i - V_i(T_d - t)) \\ +\frac{1}{\sin\eta_i}\sum_{j=1}^n a_{ij}(V_j(T_d - t) - r_j) \\ -\frac{1}{\sin\eta_i}\sum_{j=1}^n a_{ij}(V_i(T_d - t) + r_i) \\ -\frac{1}{\sin\eta_i}\frac{V_i^2\sin^2\eta_i}{r_i} & \text{if } |R_i - r_i| \geq 30\ \&\ \eta_i \neq 0 \\ \frac{1}{\sin\epsilon}(\hat{x}_i + r_i - V_i(T_d - t)) \\ +\frac{1}{\sin\epsilon}\sum_{j=1}^n a_{ij}(V_j(T_d - t) - r_j) \\ -\frac{1}{\sin\epsilon}\sum_{j=1}^n a_{ij}(V_i(T_d - t) + r_i) & \text{if } |R_i - r_i| \geq 30\ \&\ \eta_i = 0 \\ NV_i\dot{q}_i & \text{if } |R_i - r_i| < 30. \end{cases}$$

4. SIMULATION

In order to investigate the characteristics of the proposed guidance law, simulations under three cases are performed. The initial conditions are randomly selected as follows:

Missile	Position(km)	Velocity vector angle(∘)
1	(-6,-6)	60
2	(-8,-2)	60
3	(4,-7)	120
T	(0,0)	0

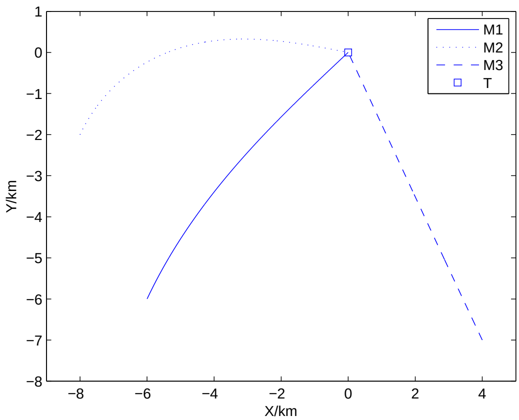

Fig. 2. Trajectories of three missiles.

The simulation results are shown from Figure 1 to Figure 4. They are presented in three cases: (1) a salvo attack with the designed impact time in simple situation; (2) a salvo attack with bounded inputs; (3) a salvo attack without relative state derivative measurement. Figure 1 shows the missiles trajectories based on the consensus guidance law. It illustrates that the three missiles can attack the target, which proves the accuracy of the guidance law. Figure 2 shows the distance variation between the missiles and the target. From Figure 2, we can see that the distance between the three missiles and the target close to zero at the same time. Also in this scenario, it is observed that the dispersion of impact time is about 0.18 seconds, and the longest impact time is 27.64 sec of missile 1, the shortest impact time is 27.46 sec of missile 3. Compare with the designed time 27 sec, we can find it demonstrates that the three missiles can hit the target at the designed time simultaneously.

Figure 3 shows trajectories of the missiles based on the consensus with bounded control input guidance law. The impact time is 27.41 sec of missile 1, 27.39 sec of missile 2, 27.21 sec of missile 3. The dispersion of impact time is about 0.20 sec. Figure 4 shows trajectories based on the consensus guidance law without the relative state derivative

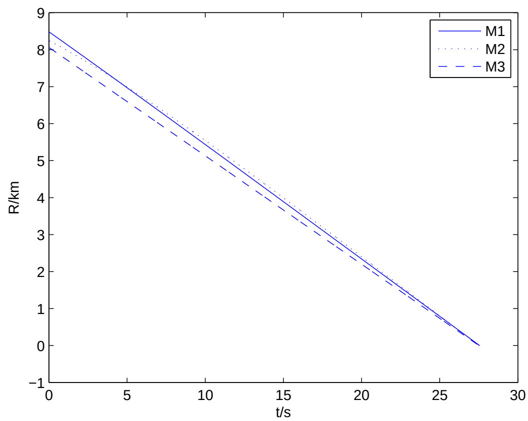

Fig. 3. The relative distance of missile-target.

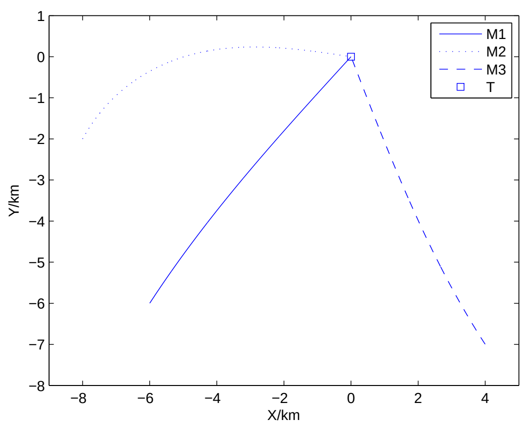

Fig. 4. Trajectories of three missiles.

measures. Their impact time is 27.69 sec of missile 1, 27.68 sec of missile 2, and 27.28 sec of missile 3. The dispersion of impact time is about 0.41 sec. Both of them are similar with Figure 1, though their impact time has a little difference. Figure 3 and Figure 4 illustrate the designed guidance law can make three missiles hit the target at the same time, so the guidance law is effective. All the simulation results prove that the proposed multiple missiles guidance law can drive the missiles hit the target at the designed time.

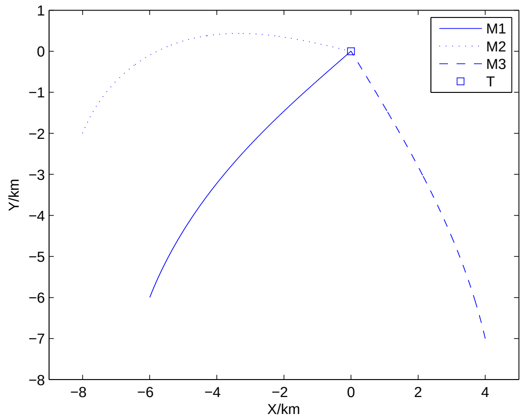

Fig. 5. Trajectories of three missiles.

5. CONCLUSIONS

In this paper, a new cooperative guidance with the impact time constraint was investigated, which can be applied to salvo attack of anti-ship missiles. The missiles estimated the arriving times which can be computed by PNG. We designed the distributed guidance law which can force the range of missile and the target to trace the nominal range. When the time-to-go tends to zero, the nominal range tends to zero too, which means the multiple missiles can arrive the object at the appointed time. In this paper, the simultaneous attack problem can be viewed as a range tracking problem problem for multi-missile systems which can be solved by consensus protocol based on multi-agent systems. Moreover, simulation results showed that all missiles could impact the target at the designated impact time very accurately.

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