

Jiasen Sun; Rui Yang; Xiang Ji; Jie Wu

Evaluation of decision-making units based on the weight-optimized DEA model

*Kybernetika*, Vol. 53 (2017), No. 2, 244–262

Persistent URL: <http://dml.cz/dmlcz/146804>

## Terms of use:

© Institute of Information Theory and Automation AS CR, 2017

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://dml.cz>

## EVALUATION OF DECISION-MAKING UNITS BASED ON THE WEIGHT-OPTIMIZED DEA MODEL

JIASEN SUN, RUI YANG, XIANG JI AND JIE WU

Data envelopment analysis (DEA) is a methodology for measuring best relative efficiencies of a group of peer decision-making units (DMUs) that take multiple inputs to produce multiple outputs. However, the traditional DEA model only aims to maximize the efficiency of the DMU under evaluation. This usually leads to very small weights (even zero weights) being assigned to some inputs or outputs. Correspondingly, these inputs or outputs have little or even no contribution to efficiency, which is unfair and irrational. The purpose of this paper is to address this problem. Two new weight-optimized models are proposed based upon the perspective of cross evaluation. Using the results of an Advanced Manufacturing Technology (AMT) example, it is found that all AMTs are fully sorted. The decision maker can easily choose the best AMT. In addition, unreasonable weights of AMTs are effectively avoided.

*Keywords:* data envelopment analysis (DEA), efficiency, weight-optimized model, cross evaluation

*Classification:* 90B50

### 1. INTRODUCTION

Data envelopment analysis (DEA) is a famous non-parametric mathematical programming method that is used extensively to evaluate relative effectiveness of decision making units (DMUs) in macroeconomics [13], and is particularly adept at estimating efficiency of multiple input and multiple output production activities [51]. The DEA was formally proposed by Charnes et al. [6]. The multiplier model proposed in that famous paper, together with its dual envelopment model, is called the CCR model. Banker et al. [3] later generalized the CCR model by taking variable returns to scale into consideration; their model is called the BCC model. Compared to other efficiency evaluation methods, such as the Stochastic Frontier Analysis (SFA) or Free Disposal Hull (FDH), the DEA has a significant advantage that it does not need any a priori assumptions about weights, production functions, and probability distributions [19]. Accordingly, the DEA is more frequently used in various fields: supply chain management [28], performance evaluation [37], resource allocation [23], procurement management [24], strategic management [7] etc. The conventional DEA model, such as the CCR model or BCC model, is a self-evaluated mode [26]. The self-evaluated DEA model lets each DMU use most favorable

weights to evaluate its efficiency. This may lead to the situation when more than one DMU is evaluated as efficient, and such DEA-efficient DMUs cannot be further distinguished [42]. Therefore, the lack of effective distinction is one of the main drawbacks of the DEA [43]. There is also another significant shortcoming of the fact that the DEA allows each DMU to be evaluated using its most favorable weights. This leads to that the weights obtained by the DEA are usually inconsistent with the practical production process [44].

In order to improve the capability of the DEA in discriminating efficient DMUs, Sexton et al. [36] proposed the cross-efficiency DEA method by integrating the concept of peer evaluation into the classical DEA framework. According to the cross-efficiency DEA, each DMU has  $n-1$  peer-evaluated efficiency scores obtained by using other DMUs' most favorable weights, besides the self-evaluated efficiency score obtained using its own most favorable weight. Then, by making an aggregation of all these efficiency scores, the cross-efficiency score for each DMU is obtained. Compared to the self-evaluated DEA, the cross-efficiency DEA has at least the following three advantages. First, the cross-efficiency DEA can produce a total ranking of all DMUs in most scenarios [18]. Second, the cross-efficiency DEA can eliminate unreasonable weight schemes without any a priori assumptions on weight restrictions [44]. Third, the cross-efficiency DEA is more effective in the distinction of good and poor performers among DMUs [38]. Due to these advantages, the cross-efficiency DEA is widely used for measuring of nursing houses' performance [36], ranking and selecting R&D projects [21], judging suitable computer control machines [40], performance evaluation and ranking in Olympic Games [47], supplier selection in public procurement [20], allocation of emission permits in paper mills [39], measuring airlines' energy efficiency [15] etc.

Although the cross-efficiency DEA has several advantages, and is widely used in various fields, it also has two main drawbacks. One is that the cross-efficiency method has the non-uniqueness problem of optimal weights [36]. In details, since the optimal weights derived by the conventional DEA models are generally not unique, the cross-efficiency scores are somewhat arbitrarily generated [16]. As suggested by Sexton et al. [36], this problem can be solved by integrating secondary goals into the cross-efficiency DEA framework. Based on this idea, numerous secondary goal cross-efficiency DEA models were developed. For instance, Wu et al. [50] proposed a secondary goal that optimizes the ranking position of the DMU under evaluation. Lim [29] suggested the minimization (or maximization) of the best (or worst) cross-efficiency scores of peer DMUs as the secondary goal. Maddahi et al. [32] used the optimization of proportional weights as the secondary goal. Among secondary goal cross-efficiency DEA models, the most commonly used ones are the benevolent and aggressive models developed by Doyle and Green [17]. The main idea of the benevolent (respectively, aggressive) model is to select for each DMU a set of optimal weights that makes all the other DMUs' cross-efficiency scores as large (respectively, small) as possible, keeping its own score optimal. Doyle and Green's [17] model was then extended by Liang et al. [26] with a variety of secondary objective functions being integrated into the cross-efficiency DEA framework. Apart from using secondary goal cross-efficiency DEA models to deal with the non-uniqueness problem, there are also several other methods. For example, Cook and Zhu [11] developed a units-invariant multiplicative DEA model to compute cross-efficiency

scores. The main advantage of the model is that it can generate cross-efficiency scores directly, without the need to identify each DMU's unique set of optimal weights.

The other main drawback of the cross-efficiency DEA is that derived cross-efficiency scores are in general not Pareto optimal [27]. To deal with this problem, Liang et al. [27] extended the classical cross-efficiency DEA model into a game-efficiency DEA model. They showed that the cross-efficiency scores derived by their game-efficiency DEA model constitute a Nash equilibrium for all DMUs. Since no DMU can gain anything by deviating from the Nash equilibrium alone, the evaluation results derived by their method are acceptable. There are also some attempts to overcome this drawback by selecting a set of common weights to compute cross-efficiency scores. For example, by integrating a cooperative game into the DEA framework, Wu et al. [48] obtained a set of common weights associated with the Shapley value vector.

Based on the literature review presented above, most cross efficiency models only focus on how to increase or decrease efficiency of other DMUs. In other words, these studies only care about DMUs' efficiency, ignoring their weight assignment [22]. This may generate assignment results with extreme weights. Specifically, the DEA often assigns weights to a few inputs and outputs in the process of evaluation, ignoring the remaining inputs and outputs by assigning them very small (or even zero) weights [33]. With this scenario, in the configurations of weighted inputs and outputs, some input or output variables with zero weights do not contribute to the efficiency of evaluated DMUs. The input or output variables with large weights may play decisive roles, which is inconsistent with the production process or prior knowledge [35]. In actual production processes, each variable (either input or output) is of critical importance, and none of them can be ignored. Accordingly, in the cross efficiency model, any weighted input or output value can be neither too large nor too small. Otherwise, the main credit for efficiency is assigned to an input or output variable whose weight is too large, and there is no credit assigned to others [49].

The purpose of this paper is to present a weight-optimized DEA model to solve the irrational weight problem of DEA models. The weight-optimized model is also further extended to a cross-efficiency model. Compared to existing DEA models, there are three major advantages of the proposed models. First, the proposed cross efficiency model can reduce differences among weighted inputs and outputs while guaranteeing the maximum self-assessment efficiency of DMUs under evaluation. Second, zero weights of inputs and outputs can be effectively avoided. That is to say, each input or output in the proposed models can be used as much as possible during the evaluation process. Third, taking into account issues similar to those considered in our study, Ramn et al. [33, 34] proposed a multiplier bound approach for assessment of efficiency without slacks. Different from their study, our models choose profiles with similar weighted inputs and outputs, not weight. In addition, their model is a nonlinear programming problem, which makes calculations more difficult. Since our models are built upon the classical cross-efficiency DEA framework and the concept of an ideal DMU, we consider only the CRS condition in this research. Specifically, there are two reasons making it unreasonable to consider the case of the VRS in our research scenario. First, integrating the concept of the VRS into the classical cross-efficiency DEA framework may yield negative cross-efficiency scores [12, 31]. To the best of our knowledge, only two types of methods were proposed to deal

with this problem. One is to add an exogenous constraint restricting cross-efficiency scores from being negative [47], the other is to make suitable data transformation [30]. Unfortunately, neither of these two methods is appropriate in our research scenario. Second, considering the VRS with an ideal DMU is difficult [42]. In fact, when using an ideal DMU in the cross-efficiency DEA, the number of extreme efficient DMUs (for the definition of an extreme efficient DMU, please see [4]) degenerates to one [46]. And it is unreasonable to build a VRS frontier with only one extreme efficient DMU.

The rest of this paper is organized as follows. Section 2 presents a brief introduction of the DEA and cross-efficiency evaluation method. Section 3 provides a detailed process and analysis of the new proposed models. A case study is presented in Section 4, and conclusion is made in Section 5.

## 2. THE TRADITIONAL DEA MODEL AND CROSS-EFFICIENCY EVALUATION METHOD

Data envelopment analysis (DEA), first developed by Charnes et al. [6], is a non-parametric programming tool for assessing efficiency of a set of homogenous decision making units (DMUs). It was proved to be an effective approach for assessing and ranking DMUs, and was extensively applied in fields of engineering and management.

Using the traditional notation, assume that there is a set of  $n$  DMUs. Each  $DMU_j$  ( $j = 1, 2, \dots, n$ ) produces  $s$  different outputs using  $m$  different inputs, which are denoted as  $y_{rj} > 0$  ( $r = 1, 2, \dots, s$ ) and  $x_{ij} > 0$  ( $i = 1, 2, \dots, m$ ), respectively. If the multipliers or prices  $\bar{u}_r, \bar{v}_r$  are associated with  $y_{rj}$  and  $x_{ij}$ , respectively, then the efficiency of  $DMU_j$  can be expressed as the ratio of the weighted outputs to weighted inputs (e.g.  $\sum_r \bar{u}_r y_{rj} / \sum_i \bar{v}_r x_{ij}$ ) according to conventional benefit/cost theory. This benefit/cost ratio is derived from the standard engineering ratio of productivity.

In the case of a known multiplier, Charnes et al. [6] proposed to derive appropriate multipliers for a given DMU by solving a particular programming problem shown in (1). Specifically, for any evaluated  $DMU_d$  ( $d = 1, 2, \dots, n$ ), the efficiency score  $E_{dd}$  can be calculated using the following CCR model.

$$\begin{aligned}
 & \max \sum_{r=1}^s \mu_{rd} y_{rd} = E_{dd} \\
 & \text{s.t.} \sum_{i=1}^m \omega_{id} x_{ij} - \sum_{r=1}^s \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \dots, n, \\
 & \sum_{i=1}^m \omega_{id} x_{id} = 1, \\
 & \omega_{id} \geq \varepsilon, \quad i = 1, 2, \dots, m, \\
 & \mu_{rd} \geq \varepsilon, \quad r = 1, 2, \dots, s.
 \end{aligned} \tag{1}$$

Model (1) is referred to as the CCR (Charnes, Cooper and Rhodes) model, and works for constant returns to scale (CRS). In model (1),  $DMU_d$  is under evaluation, and  $\omega_{id}$  and  $\mu_{rd}$  are the weights assigned to the inputs and outputs, respectively. By duality,

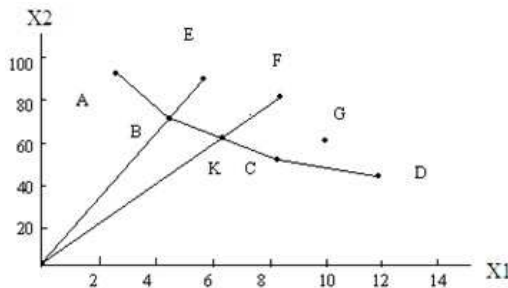
model (1) is equivalent to the linear programming model (2).

$$\begin{aligned}
 &\min \theta - \varepsilon \left( \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^- \right) \\
 &s.t. \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta x_{id}, \quad i = 1, 2, \dots, m, \\
 &\sum_{j=1}^n \lambda_j y_{rj} + s_r^- \geq y_{rd}, \quad r = 1, 2, \dots, s, \\
 &\lambda_j \geq 0, \quad j = 1, 2, \dots, n.
 \end{aligned} \tag{2}$$

Figure 1 provides a geometric interpretation of the CRS model (1). Here both Figure 1 and Table 1 are sourced from Cook and Seiford [9]. This figure provides an illustration of seven DMUs with two inputs [9], and a single common output for all DMUs. By solving model (1) or (2), four DMUs (A, B, C and D) are efficient, i.e.  $\theta_A = \theta_B = \theta_C = \theta_D = 1$ . For DMU E, the efficiency is 0.833, and its projected score in the frontier is  $\theta_E^* x_E$  (DMU B). Therefore, DMU B is considered as a benchmark for DMU E. For DMU G, its projected point in the frontier is point K, then DMUs B and C can provide a benchmark for DMU G.

	A	B	C	D	E	F	G
X1	3	5	8	12	6	8	10
X2	90	70	55	50	84	80	60

**Tab. 1.** Input data of seven DMUs.



**Fig. 1.** Two input illustration of the DEA.

From this example, it can be seen that the traditional DEA model only classifies DMUs into two groups, namely, efficient and inefficient DMUs. Efficient DMUs (the efficiency is one) cannot be distinguished any further. To increase the power of discrimination of efficient DMUs, the cross-efficiency evaluation method was developed as a DEA extension technique [36].

For each  $DMU_d$  ( $d = 1, 2, \dots, n$ ), we can obtain a group of optimal weights  $\omega_{1d}^*, \dots, \omega_{md}^*, \mu_{1d}^*, \dots, \mu_{sd}^*$  by solving the above model (2), and the cross-efficiency of each  $DMU_j$  using the weights of  $DMU_d$ , namely  $E_{dj}$ , can be calculated as follows:

$$E_{dj} = \frac{\sum_{r=1}^s \mu_{rd}^* y_{rj}}{\sum_{i=1}^m \omega_{id}^* x_{ij}}, \quad d, j = 1, 2, \dots, n. \tag{3}$$

For each  $DMU_j$  ( $j = 1, 2, \dots, n$ ), the average of all  $E_{dj}$  ( $d = 1, 2, \dots, n$ ), namely,  $\bar{E}_j = \frac{1}{n} \sum_{d=1}^n E_{dj}$  ( $j = 1, 2, \dots, n$ ), can be treated as a new efficiency measure, namely, the cross-efficiency score of  $DMU_j$ .

### 3. THE PROPOSED DEA MODEL

#### 3.1. The weight-optimized DEA model based on the ideal DMU

The traditional DEA model above allows each DMU to measure efficiency using its favorable weights in order to obtain maximum efficiency. This may lead to a major problem. Very small (or even zero) weights are assigned to some inputs or outputs, and very large weights to other inputs or outputs. To address this problem, we propose the weight-optimized DEA model based on the optimal solution. The proposed model includes two steps. In the first step, a weight restriction of the virtual positive ideal DMU is obtained. In the second step, the weight restriction is incorporated into the DEA model for each DMU. The positive ideal DMU is defined as follows.

**Definition 3.1.** The positive ideal DMU is defined as  $DMU_I = (x_{i,\min}, y_{r,\max})$ , where  $x_{i,\min} = \min\{x_{ij} | j = 1, 2, \dots, n\}$  ( $i = 1, 2, \dots, m$ ), and  $y_{r,\max} = \max\{y_{rj} | j = 1, 2, \dots, n\}$  ( $r = 1, 2, \dots, s$ ). The smallest data in each row of the input matrix is selected to be the input of the virtual positive ideal DMU, and the largest data in each row of the output matrix is selected to be the output of the virtual positive ideal DMU.

Based on this definition, the weight restriction of the positive ideal DMU is obtained by the following model.

$$\begin{aligned}
& \min \beta \\
& \text{s.t. } \sum_{i=1}^m \omega_i x_{ij} - \sum_{r=1}^s \mu_r y_{rj} \geq 0, \quad j = 1, 2, \dots, n, \\
& \quad \sum_{i=1}^m \omega_i x_{i,\min} = 1, \\
& \quad \sum_{r=1}^s \mu_r y_{r,\max} = 1, \\
& \quad |\mu_r y_{r,\max} - 1/s| \leq \beta, \quad r = 1, 2, \dots, s, \\
& \quad |\omega_i x_{i,\min} - 1/m| \leq \beta, \quad i = 1, 2, \dots, m, \\
& \quad \omega_i \geq \varepsilon, \quad i = 1, 2, \dots, m, \\
& \quad \mu_r \geq \varepsilon, \quad r = 1, 2, \dots, s.
\end{aligned} \tag{4}$$

In model (4), each weighted input or output component is considered as an individual. What we are concerned with is whether each weighted input or output component makes a fair contribution to the efficiency of the evaluated  $DMU_d$ . Accordingly, two ideal scores ( $1/s$  and  $1/m$ ) are introduced for each weighted input and output component. The main idea of model (4) is to minimize the absolute distance between the weighted input (output) component and ideal score. The absolute distance is defined as  $\beta_d$ . The smaller  $\beta_d$ , the less the difference between weighted inputs (or outputs). This means that all weighted input and output components can make fair contributions to  $DMU_d$  in the process of efficiency evaluation.

**Theorem 3.2.** Model (4) is equivalent to the following model:

$$\begin{aligned}
& \min \beta \\
& \text{s.t. } \sum_{i=1}^m \omega_i x_{i,\min} = 1, \\
& \quad \sum_{r=1}^s \mu_r y_{r,\max} = 1, \\
& \quad |\mu_r y_{r,\max} - 1/s| \leq \beta, \quad r = 1, 2, \dots, s, \\
& \quad |\omega_i x_{i,\min} - 1/m| \leq \beta, \quad i = 1, 2, \dots, m, \\
& \quad \omega_i \geq \varepsilon, \quad i = 1, 2, \dots, m, \\
& \quad \mu_r \geq \varepsilon, \quad r = 1, 2, \dots, s.
\end{aligned} \tag{5}$$

**Proof.** Assume that  $(\omega_i^*, \mu_r^*)$  is the optimal solution of model (5). From the definition of the positive ideal DMU, we have  $y_{r,\max} \geq y_{rj}$  and  $x_{i,\min} \leq x_{ij}$ . From the first and second constraints of model (5), we have  $\mu_r^* y_{r,\max} \geq 1 \geq \mu_r^* y_{rj}$  and  $\omega_i^* x_{i,\min} \leq 1 \leq \omega_i^* x_{ij}$ . Then, the constraint  $\sum_{i=1}^m \omega_i x_{ij} - \sum_{r=1}^s \mu_r y_{rj} \geq 0$ , ( $j = 1, 2, \dots, n$ ), can be obtained. Therefore, models (4) and (5) are equivalent.  $\square$



Model (4) is nonlinear, and hence is difficult to solve. It can be transformed into the following linear model:

$$\begin{aligned}
 & \min \beta \\
 & \text{s.t. } \sum_{i=1}^m \omega_i x_{ij} - \sum_{r=1}^s \mu_r y_{rj} \geq 0, \quad j = 1, 2, \dots, n, \\
 & \quad \sum_{i=1}^m \omega_i x_{i,\min} = 1, \\
 & \quad \sum_{r=1}^s \mu_r y_{r,\max} = 1, \\
 & \quad \mu_r y_{r,\max} - 1/s \leq \beta, \quad r = 1, 2, \dots, s, \\
 & \quad -\mu_r y_{r,\max} + 1/s \leq \beta, \quad r = 1, 2, \dots, s, \\
 & \quad \omega_i x_{i,\min} - 1/m \leq \beta, \quad i = 1, 2, \dots, m, \\
 & \quad -\omega_i x_{i,\min} + 1/m \leq \beta, \quad i = 1, 2, \dots, m, \\
 & \quad \omega_i \geq \varepsilon, \quad i = 1, 2, \dots, m, \\
 & \quad \mu_r \geq \varepsilon, \quad r = 1, 2, \dots, s.
 \end{aligned} \tag{6}$$

Let  $DMU_d$  ( $d = 1, 2, \dots, n$ ) be the DMU that needs to be evaluated. Then, its efficiency score can be determined by the following programming problem:

$$\begin{aligned}
 & \max \theta_d = \sum_{r=1}^s \mu_{rd} y_{rd} \\
 & \text{s.t. } \sum_{i=1}^m \omega_{id} x_{ij} - \sum_{r=1}^s \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \dots, n, \\
 & \quad \sum_{i=1}^m \omega_{id} x_{id} = 1, \\
 & \quad \left| \mu_{rd} y_{rd} - \sum_{r=1}^s \mu_{rd} y_{rd} / s \right| \leq \beta^*, \quad r = 1, 2, \dots, s, \\
 & \quad \left| \omega_{id} x_{id} - 1/m \right| \leq \beta^*, \quad i = 1, 2, \dots, m, \\
 & \quad \omega_i \geq \varepsilon, \quad i = 1, 2, \dots, m, \\
 & \quad \mu_r \geq \varepsilon, \quad r = 1, 2, \dots, s.
 \end{aligned} \tag{7}$$

In model (7),  $\beta^*$  is obtained from model (4). Compared to the traditional DEA model, model (7) incorporates the third and fourth constraints into the formulation. As a consequence, with this model, each  $DMU_d$  is assessed with the constraint that weighted inputs or outputs cannot be much different. Model (7) is also nonlinear, but it is not difficult to transform it into a linear model.

### 3.2. Cross efficiency evaluation method based on the weight-optimized DEA model

In model (7), each DMU is evaluated in the self-evaluation mode, and chooses its favorable weights. However, Doyle and Green [17] pointed out that the peer-evaluation mode is also important and common, and proposed the cross-efficiency evaluation model. The main idea of the cross-efficiency evaluation is to use the DEA in the peer evaluation mode instead of the self-evaluation mode. There are at least three advantages of cross-efficiency evaluation, such as ranking DMUs in a unique order [36], eliminating unrealistic weight schemes [36], and effectively differentiating between good and poor performers [5]. These advantages make the method widely applied for evaluating DMUs.

The purpose of this section is to extend the weight-optimized DEA model to the cross-efficiency evaluation model. The proposed model can not only reduce differences in weighted inputs and outputs during the evaluation process, but also rank DMUs in the peer-evaluation mode. The proposed cross-efficiency evaluation model can be written as follows:

$$\begin{aligned}
 & \max \sum_{j=1}^n \sum_{r=1}^s \mu_{rj} y_{rj} - \sum_{j=1}^n \sum_{i=1}^m \omega_{ij} x_{ij} \\
 & s.t. \sum_{i=1}^m \omega_{id} x_{ij} - \sum_{r=1}^s \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \dots, n, \\
 & \theta_d^* \sum_{i=1}^m \omega_{id} x_{id} - \sum_{r=1}^s \mu_{rd} y_{rd} = 0, \\
 & |\mu_{rd} y_{rd} - \sum_{r=1}^s \mu_{rd} y_{rd} / s| \leq \beta^*, \quad r = 1, 2, \dots, s, \\
 & |\omega_{id} x_{id} - 1/m| \leq \beta^*, \quad i = 1, 2, \dots, m, \\
 & \omega_i \geq \varepsilon, \quad i = 1, 2, \dots, m, \\
 & \mu_r \geq \varepsilon, \quad r = 1, 2, \dots, s,
 \end{aligned} \tag{8}$$

where  $\theta_d^*$  obtained from model (7) is the self-evaluation efficiency of  $DMU_d$ ,  $\beta^*$  is the weight restriction obtained by model (4), and  $\omega_{1d}, \dots, \omega_{md}, \mu_{1d}, \dots, \mu_{sd}$  are the variables for which the problem needs to be solved. The model (8) needs to be solved  $n$  times, i. e. for each DMU.

The DEA method mainly has two orientation modes, which are the input and output orientation (10). In the envelope model, the input orientation mode is to appropriately adjust inputs under fixed outputs. In the multiplier model, the input orientation mode is expressed as maximizing the ratio of the DMU's sum of weighted outputs to its sum of weighted inputs [8, 14]. According to the definition, it is clear that the proposed weight-optimized DEA models are input orientation modes.

If we denote the optimal solution of model (7) by  $(\omega_{1d}^*, \dots, \omega_{md}^*, \mu_{1d}^*, \dots, \mu_{sd}^*)$  for the corresponding  $DMU_d$ , then the cross-efficiency of a given  $DMU_j$  with the profile of

weights provided by  $DMU_d$  can be obtained as follows:

$$E_{dj} = \frac{\sum_{r=1}^s \mu_{rd}^* y_{rj}}{\sum_{i=1}^m \omega_{id}^* x_{ij}}. \tag{9}$$

Therefore, the cross-efficiency score of  $DMU_j$  is the average of these cross-efficiencies

$$\bar{E}_j = \frac{1}{n} \sum_{d=1}^j E_{dj}^*, \quad j = 1, 2, \dots, n. \tag{10}$$

This measures the average efficiency according to all DMUs.

#### 4. ILLUSTRATIONS

To illustrate the proposed methods, we consider an Advanced Manufacturing Technology (AMT) example with data presented in Table 2 (Table 2-5 are all contained in the appendix). The data is from Khouja [25]. In this example, there are 27 industrial robots that need to be evaluated and selected. The inputs are the cost (in \$10,000) and repeatability (in millimeters). The outputs include the load capacity (in kilograms) and velocity (in meters per second).

We evaluate and rank the DMUs using models (1), (7) and (8). In order to compare with other models, the slacks-based measure (SBM) and super DEA models, the two most commonly used models, are also employed to evaluate these DMUs. The SBM model was proposed by Tone [41], and the super DEA model was proposed by Andersen and Petersen [2]. The results of the SBM and super DEA models are listed in the third and fourth columns. By comparing the results, several findings can be identified. First, the CCR efficiency scores show that nine DMUs are identified to be efficient, which cannot be discriminated any further. Second, comparing the results of the CCR and SBM models, it is noted that the efficiency scores obtained by the SBM model are smaller than those obtained by the CCR model, but there are all nine efficient DMUs. These two findings indicate that the traditional CCR and SBM models fail to completely discriminate all DMUs. Third, from the data of the fourth column, it is noted that the super efficiency of all DMUs can be completely ranked. However, according to Table 5, we find that the super model still has unreasonable extreme weight results. Fourth, the fourth and sixth columns list the efficiency scores of models (7) and (8), respectively, in which only one DMU is efficient. Other DMUs are all inefficient. This indicates that all DMUs can be fully discriminated. Fifth, comparing to the traditional models, ranking results obtained from the proposed models are significantly different. The reason is that each input or output is given a non-zero weight, so that it can be used as much as possible in efficiency evaluation. Finally, the difference in results between model (7) and (8) is that ranks of some DMUs are changed. This difference results from the evaluation mode. Model (7) is the self-evaluation mode, and model (8) is the peer-evaluation mode.

Tables 4 and 5 provide the results for weights of inputs and outputs using different models. In Table 4, the weights are obtained using the traditional CCR and super DEA

models. It is noted that there are many zero weights. For example, DMU2 has a very small weight under the CCR or super DEA model, but its CCR or super DEA efficiency is 0.9038. This shows that this DMU allocate unreasonable weights to itself in order to pursue maximum efficiency. Similar situations can also be seen for DMU5, DMU6, and DMU22. In Table 5, we see that there are no zero weights. This guarantees that each input or output makes an important contribution to the process of efficiency evaluation.

From Table 3, we can find that there are several special DMUs, such as DMU4 and DMU20. These two DMUs are efficient in the traditional CCR model, but they are almost the worst units in model (8). This is because the self-evaluation CCR model allows each DMU to rate its efficiency using the most favorable weights, generating unrealistic weights. For example, under the self-evaluation CCR model, the weights of DMU20 are 6.1477, 0.0082, 0.0235 and 1.2059. Then, the results for the weighted inputs and outputs of DMU20 are 0.9836, 0.0164, 0.03525 and 0.9647. This indicates that only input 1 and output 2 play major roles in the efficiency, while input 2 and output 1 made little contribution to the efficiency. The results for the weighted inputs and outputs of DMU20 are 0.5, 0.5, 0.0945 and 0.09452, using the weights obtained by model (8). The results show that the contribution of each input (or output) variable is equivalent. Similar findings can also be obtained by DMU4. These findings indicate that the optimized DEA models proposed in this paper ensure that each weighted input and output component is given power as much as possible to make equally important contributions to the efficiency. Thus, weighted inputs and outputs are more balanced.

## 5. CONCLUSIONS

As an effective method of evaluation and ranking of DMUs, the DEA and its extended cross-efficiency evaluation are applied in a wide variety of fields. However, the unreasonable weight problem reduces the usefulness of the cross-efficiency evaluation method. Due to the weight flexibility problem of the traditional DEA, zero weights are usually assigned to some inputs or outputs. This may lead to that the efficiency scores of some DMUs do not use all inputs or outputs. The purpose of this paper is to solve this problem. Two new models are proposed. From the results of an AMT example, the models show superiority in effectively avoiding unreasonable weights, and also offer a unique ranking of all AMTs. Therefore, the models proposed in this paper can be seen as improvements and extensions of traditional DEA models, making them meaningful contributions to the DEA research.

The proposed models have at least four advantages. First, the proposed models do not need any prior information on weight restrictions from application area experts, avoiding the impact of subjective factors on the results of evaluation. Second, the proposed cross efficiency models can not only guarantee self-assessment efficiency of DMUs under evaluation, but also reduce differences in weighted inputs and outputs during the evaluation process. Third, the proposed models can effectively reduce the number of zero weights of inputs and outputs. In other words, all inputs and outputs in this newly proposed model have played contributions in the process of efficiency evaluation can be used as much as possible. Finally, the results of an AMT example show that our weight-optimized DEA models have strong power in discriminating among DMUs, and also show strong applicability, which provides more choices for decision-

makers.

This work can be extended along at least two directions. On one hand, our models do not consider the situation when input and/or output data is stochastic, which may often be seen in real-world applications. Further research may consider this problem, and propose new DEA methods according to the stochastic or fuzzy theory. On the other hand, the model proposed in this paper can also be extended for other application fields, such as supplier selection, investment selection and so on.

6. APPENDIX

Robots	Inputs		Outputs	
	Cost (\$10000)	Repeatability (mm)	Load capacity (Kg)	Velocity (m/s)
1	7.2	0.15	60	1.35
2	4.8	0.05	6	1.1
3	5	1.27	45	1.27
4	7.2	0.025	1.5	0.66
5	9.6	0.25	50	0.05
6	1.07	0.1	1	0.3
7	1.76	0.1	5	1
8	3.2	0.1	15	1
9	6.72	0.2	10	1.11
10	2.4	0.05	6	1
11	2.88	0.5	30	0.9
12	6.9	1	13.6	0.15
13	3.2	0.05	10	1.2
14	4	0.05	30	1.2
15	3.68	1	47	1
16	6.88	1	80	1
17	8	2	15	2
18	6.3	0.2	10	1
19	0.94	0.05	10	0.3
20	0.16	2	1.5	0.8
21	2.81	2	27	1.7
22	3.8	0.05	0.9	1
23	1.25	0.1	2.5	0.5
24	1.37	0.1	2.5	0.5
25	3.63	0.2	10	1
26	5.3	1.27	70	1.25
27	4	2.03	205	0.75

**Tab. 2.** Data for 27 industrial robots [25].

Robot	CCR efficiency	SBM	Super	Model (7)	Rank	Model (8)	Rank
1	1	1.0000	1.0118	0.6400	3	0.6933	2
2	0.9038	0.3010	0.9038	0.3010	16	0.3697	9
3	0.5289	0.4687	0.5288	0.4661	9	0.2416	19
4	1	1.0000	1.1000	0.1171	22	0.1473	24
5	0.5924	0.0233	0.5924	0.0220	27	0.2921	18
6	0.4824	0.1199	0.4824	0.1158	23	0.2020	22
7	1	1.0000	1.3217	0.3851	11	0.5428	7
8	0.7825	0.5741	0.7825	0.5469	6	0.5308	8
9	0.3814	0.2071	0.3814	0.2071	20	0.2336	20
10	1	1.0000	1.0432	0.4301	10	0.5830	5
11	0.6713	0.6394	0.6713	0.6348	4	0.3624	10
12	0.1024	0.0505	0.1024	0.0505	26	0.0574	27
13	1	1.0000	1.0909	0.5625	5	0.6236	4
14	1	1.0000	1.7692	1	1	0.8781	1
15	0.6125	0.5526	0.6125	0.5227	7	0.2968	17
16	0.6035	0.4407	0.6035	0.3349	13	0.3464	11
17	0.4045	0.1232	0.4045	0.1079	24	0.1243	25
18	0.3652	0.2107	0.3652	0.2107	19	0.2277	21
19	1	1.0000	1.0208	0.7508	2	0.6800	3
20	1	1.0000	8.2647	0.1891	21	0.0800	26
21	0.8515	0.6346	0.8515	0.3035	15	0.1859	23
22	0.8289	0.0594	0.8289	0.0594	25	0.3208	14
23	0.6943	0.3290	0.6943	0.2569	17	0.3309	12
24	0.6361	0.2876	0.6361	0.2375	18	0.3143	16
25	0.5533	0.3219	0.5533	0.3219	14	0.3214	13
26	0.5810	0.5353	0.5810	0.4714	8	0.3158	15
27	1	1.0000	3.8804	0.3604	12	0.5599	6

**Tab. 3.** Evaluation results for 27 industrial robots.

Robot	Weight of Input1		Weight of Input2		Weight of Output1		Weight of Output2	
	Model (1)	Super	Model (1)	Super	Model (1)	Super	Model (1)	Super
1	0.1056	0.1078	1.5998	1.4904	0.0166	0.0169	0.0026	1E-06
2	0.0088	0.0088	19.1549	19.1550	1E-06	1E-06	0.8216	0.8216
3	0.1726	0.1726	0.1078	0.1078	0.0033	0.0033	0.2978	0.2978
4	0.0034	1E-06	39.0201	39.9997	0.0021	1E-06	1.5103	1.6666
5	0.0770	0.0770	1.0447	1.0447	0.0118	0.0118	1E-06	1E-06
6	0.8811	0.8811	0.5727	0.5727	1E-06	1E-06	1.6079	1.6079
7	0.3380	0.5406	4.0513	0.4854	0.0021	1E-06	0.9896	1.3217
8	0.2650	0.2650	1.5187	1.5187	0.0164	0.0164	0.5362	0.5362
9	0.1098	0.1098	1.3095	1.3095	0.0048	0.0048	0.3002	0.3002
10	0.2960	0.3147	5.7924	4.8920	0.0105	1E-06	0.9371	1.0432
11	0.1898	0.1898	0.9066	0.9066	0.0113	0.0113	0.3681	0.3681
12	0.0497	0.0497	0.6574	0.6574	0.0075	0.0075	0.0065	0.0065
13	0.0822	0.1136	14.7372	12.7264	0.0008	1E-06	0.8271	0.9091
14	0.0068	1E-06	19.4523	19.9999	0.0208	0.0385	0.3145	0.5128
15	0.2323	0.2323	0.1451	0.1451	0.0045	0.0045	0.4008	0.4008
16	0.0497	0.0497	0.6581	0.6581	0.0075	0.0075	0.0065	0.0065
17	0.1081	0.1081	0.0675	0.0675	0.0021	0.0021	0.1866	0.1866
18	0.1151	0.1151	1.3728	1.3728	0.0051	0.0051	0.3147	0.3147
19	0.7949	0.8342	5.0563	4.3162	0.0606	0.0530	1.3119	1.6347
20	6.1477	6.2500	0.0082	1E-06	0.0235	1E-06	1.2059	10.3308
21	0.2463	0.2463	0.1539	0.1539	0.0048	0.0048	0.4251	0.4251
22	0.0089	0.0089	19.3250	19.3251	1E-06	1E-06	0.8289	0.8289
23	0.7619	0.7619	0.4760	0.4760	0.0148	0.0148	1.3147	1.3147
24	0.6981	0.6981	0.4361	0.4361	0.0135	0.0135	1.2046	1.2046
25	0.2094	0.2094	1.1998	1.1998	0.0130	0.0130	0.4236	0.4236
26	0.0880	0.0880	0.4202	0.4202	0.0053	0.0053	0.1706	0.1706
27	0.2461	0.2500	0.0076	1E-06	0.0048	0.0189	0.0273	1E-06

**Tab. 4.** Weight results for 27 industrial robots from the traditional DEA model.

Robot	Weight of Input1	Weight of Input2	Weight of Output1	Weight of Output2
1	0.0694	0.3333	0.0533	0.2370
2	0.1042	1.0000	0.2508	0.1368
3	0.1000	0.0394	0.0518	0.1835
4	0.0694	2.0000	0.3904	0.0887
5	0.0521	0.2000	0.0022	0.2204
6	0.4673	0.5000	0.5792	0.1931
7	0.2841	0.5000	0.3851	0.1926
8	0.1563	0.5000	0.1823	0.2734
9	0.0744	0.2500	0.1036	0.0933
10	0.2083	1.0000	0.3584	0.2151
11	0.1736	0.1000	0.1058	0.3527
12	0.0725	0.0500	0.0186	0.1683
13	0.1563	1.0000	0.2812	0.2344
14	0.1250	1.0000	0.1667	0.4167
15	0.1359	0.0500	0.0556	0.2613
16	0.0727	0.0500	0.0209	0.1674
17	0.0625	0.0250	0.0360	0.0270
18	0.0794	0.2500	0.1053	0.1053
19	0.5319	1.0000	0.3754	1.2513
20	3.1250	0.0250	0.6302	0.1182
21	0.1779	0.0250	0.0562	0.0893
22	0.1316	1.0000	0.3302	0.0297
23	0.4000	0.5000	0.5139	0.2569
24	0.3650	0.5000	0.4750	0.2375
25	0.1377	0.2500	0.1609	0.1609
26	0.0943	0.0394	0.0337	0.1886
27	0.1250	0.0246	0.0088	0.2402

**Tab. 5.** Weight results for 27 industrial robots from the proposed model (8).

#### ACKNOWLEDGEMENT

This work was partially supported by the National Natural Science Foundation of China (Nos. 71471125, 71501139, 71571173 and 71602182), the Natural Science Funds of Jiangsu Province (No. BK20150307), the Research Project of Philosophy and Social Sciences in Universities of Jiangsu Province (2015SJB525), the China Scholarship Council (No. 201506340126) and the Support Funds for Excellent Doctoral Dissertations of USTC (2016-2017).

(Received March 5, 2015)



## REFERENCES

- 
- [1] T.R. Anderson, K. Hollingsworth, and L. Inman: The fixed weighting nature of a cross-evaluation model. *J. Productivity Anal.* *17* (2002), 3, 249–255. DOI:10.1023/a:1015012121760
  - [2] P. Andersen, and N.C. Petersen: A procedure for ranking efficient units in data envelopment analysis. *Management Sci.* *39* (1993), 10, 1261–1264. DOI:10.1287/mnsc.39.10.1261
  - [3] R.D. Banker, A. Charnes, and W.W. Cooper: Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Sci.* *30* (1984), 9, 1078–1092. DOI:10.1287/mnsc.30.9.1078
  - [4] M.L. Bouniol, and J.H. Dulá: Anchor points in DEA. *Europ. J. Oper. Res.* *192* (2009), 2, 668–676. DOI:10.1016/j.ejor.2007.10.034
  - [5] A. Boussoufiane, R.G. Dyson, and E. Thanassoulis: Applied data envelopment analysis. *Europ. J. Oper. Res.* *52* (1991), 1, 1–15. DOI:10.1016/0377-2217(91)90331-o
  - [6] A. Charnes, W.W. Cooper, and E. Rhodes: Measuring the efficiency of decision making units. *Europ. J. Oper. Res.* *2* (1978), 6, 429–444. DOI:10.1016/0377-2217(78)90138-8
  - [7] C.M. Chen, M.A. Delmas, and M.B. Lieberman: Production frontier methodologies and efficiency as a performance measure in strategic management research. *Strategic Management J.* *36* (2015), 1, 19–36. DOI:10.1002/smj.2199
  - [8] W.D. Cook and K. Bala: Performance measurement and classification data in DEA: input-oriented model. *Omega* *35* (2007), 1, 39–52. DOI:10.1016/j.omega.2005.02.002
  - [9] W.D. Cook and L.M. Seiford: Data envelopment analysis (DEA) – Thirty years on. *Europ. J. Oper. Res.* *192* (2009), 1, 1–17. DOI:10.1016/j.ejor.2008.01.032
  - [10] W.D. Cook, K. Tone, and J. Zhu: Data envelopment analysis: Prior to choosing a model. *Omega* *44* (2014), 1–4. DOI:10.1016/j.omega.2013.09.004
  - [11] W.D. Cook and J. Zhu: DEA Cobb-Douglas frontier and cross-efficiency. *J. Oper. Res. Soc.* *65* (2013), 2, 265–268. DOI:10.1057/jors.2013.13
  - [12] W.D. Cook and J. Zhu: DEA Cross Efficiency. In *Data Envelopment Analysis*. Springer US 2015. DOI:10.1007/978-1-4899-7553-9\_2
  - [13] W.W. Cooper, L.M. Seiford, and K. Tone: *Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software*. Springer-Verlag US, 2007. DOI:10.1007/978-0-387-45283-8
  - [14] W.W. Cooper, L.M. Seiford, and J. Zhu: *Data envelopment analysis*. In *Handbook on data envelopment analysis*. Springer US 2004. DOI:10.1007/1-4020-7798-x\_1
  - [15] Q. Cui and Y. Li: Evaluating energy efficiency for airlines: An application of VFB-DEA. *J. Air Transport Management* *44* (2015), 34–41. DOI:10.1016/j.jairtraman.2015.02.008
  - [16] D.K. Despotis: Improving the discriminating power of DEA: focus on globally efficient units. *J. Oper. Res. Soc.* *53* (2002), 3, 314–323. DOI:10.1057/palgrave/jors/2601253
  - [17] J.R. Doyle and R.H. Green: Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. *J. Oper. Res. Soc.* *45* (1994), 5, 567–578. DOI:10.1038/sj/jors/0450509
  - [18] J.R. Doyle and R.H. Green: Cross-evaluation in DEA: improving discrimination among DMUs. *Infor* *33* (1995), 3, 205–222. DOI:10.1080/03155986.1995.11732281
  - [19] A. Emrouznejad, B.R. Parker, and G. Tavares: Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA. *Socio-economic Planning Sci.* *42* (2008), 3, 151–157. DOI:10.1016/j.seps.2007.07.002

- [20] M. Falagario, F. Sciancalepore, N. Costantino, and R. Pietroforte: Using a DEA-cross efficiency approach in public procurement tenders. *Europ. J. Oper. Res.* *218* (2012), 2, 523–529. DOI:10.1016/j.ejor.2011.10.031
- [21] R. H. Green, J. R. Doyle, and W. D. Cook: Preference voting and project ranking using DEA and cross-evaluation. *Europ. J. Oper. Res.* *90* (1996), 3, 461–472. DOI:10.1016/0377-2217(95)00039-9
- [22] G. R. Jahanshahloo, F. H. Lotfi, Y. Jafari, and R. Maddahi: Selecting symmetric weights as a secondary goal in DEA cross-efficiency evaluation. *Appl. Math. Modelling* *35* (2011), 1, 544–549. DOI:10.1016/j.apm.2010.07.020
- [23] X. Ji, J. Sun, Y. Wang, and Q. Yuan: Allocation of emission permits in large data sets: a robust multi-criteria approach. *J. Cleaner Product.* *142* (2017), 894–906. DOI:10.1016/j.jclepro.2016.02.117
- [24] X. Ji, J. Wu, and Q. Zhu: Eco-design of transportation in sustainable supply chain management: A DEA-like method. *Transport. Res. Part D: Transport and Environment* *48* (2016), 451–459. DOI:10.1016/j.trd.2015.08.007
- [25] M. Khouja: The use of data envelopment analysis for technology selection. *Computers Industr. Engrg.* *28* (1995), 1, 123–132. DOI:10.1016/0360-8352(94)00032-i
- [26] L. Liang, J. Wu, W. D. Cook, and J. Zhu: Alternative secondary goals in DEA cross-efficiency evaluation. *Int. J. Product. Econom.* *113* (2008), 2, 1025–1030. DOI:10.1016/j.ijpe.2007.12.006
- [27] L. Liang, J. Wu, W. D. Cook, and J. Zhu: The DEA game cross-efficiency model and its Nash equilibrium. *Oper. Res.* *56* (2008), 5, 1278–1288. DOI:10.1287/opre.1070.0487
- [28] L. Liang, F. Yang, W. D. Cook, and J. Zhu: DEA models for supply chain efficiency evaluation. *Ann. Oper. Res.* *145* (2006), 1, 35–49. DOI:10.1007/s10479-006-0026-7
- [29] S. Lim: Minimax and maximin formulations of cross-efficiency in DEA. *Computers Industr. Engrg.* *62* (2002), 3, 726–731. DOI:10.1016/j.cie.2011.11.010
- [30] S. Lim and J. Zhu: DEA cross-efficiency evaluation under variable returns to scale. *J. Oper. Res. Soc.* *66* (2014), 3, 476–487. DOI:10.1057/jors.2014.13
- [31] S. Lim and J. Zhu: DEA Cross Efficiency Under Variable Returns to Scale. In *Data Envelopment Analysis*. Springer US 2015.
- [32] R. Maddahi, G. J. Zhu, R. Jahanshahloo, F. Hosseinzadeh Lotfi, and A. Ebrahimnejad: Optimising proportional weights as a secondary goal in DEA cross-efficiency evaluation. *Int. J. Oper. Res.* *19* (2014), 2, 234–245. DOI:10.1504/ijor.2014.058953
- [33] N. Ramón, J. J. Zhu, L. Ruiz, and I. Sirvent: A multiplier bound approach to assess relative efficiency in DEA without slacks. *Europ. J. Oper. Res.* *203* (2010), 1, 261–269. DOI:10.1016/j.ejor.2009.07.009
- [34] N. Ramón, J. L. Ruiz, and I. Sirvent: On the choice of weights profiles in cross-efficiency evaluations. *Europ. J. Oper. Res.* *207* (2010), 3, 1564–1572. DOI:10.1016/j.ejor.2010.07.022
- [35] N. Ramón, J. L. Ruiz, and I. Sirvent: Reducing differences between profiles of weights: A "peer-restricted" cross-efficiency evaluation. *Omega* *39* (2011), 6, 634–641. DOI:10.1016/j.omega.2011.01.004
- [36] T. R. Sexton, R. H. Silkman, and A. J. Hogan: Data envelopment analysis: Critique and extensions. *New Directions for Program Evaluation* *1986* (1986), 32, 73–105. DOI:10.1002/ev.1441

- [37] J. Sun, Y. Miao, J. Wu, L. Cui, and R. Zhong: Improved interval DEA models with common weight. *Kybernetika* 50 (2014), 5, 774–785. DOI:10.14736/kyb-2014-5-0774
- [38] J. Sun, J. Wu, and D. Guo: Performance ranking of units considering ideal and anti-ideal DMU with common weights. *Appl. Math. Modelling* 37 (2013), 9, 6301–6310. DOI:10.1016/j.apm.2013.01.010
- [39] J. Sun, J. Wu, L. Liang, R. Y. Zhong, and G. Q. Huang: Allocation of emission permits using DEA: centralised and individual points of view. *Int. J. Product. Res.* 52 (2014), 2, 419–435. DOI:10.1080/00207543.2013.829592
- [40] S. Sun: Assessing computer numerical control machines using data envelopment analysis. *Int. J. Product. Res.* 40 (2002), 9, 2011–2039. DOI:10.1080/00207540210123634
- [41] K. Tone: A slacks-based measure of efficiency in data envelopment analysis. *Europ. J. Oper. Res.* 130 (2001), 3, 498–509. DOI:10.1016/s0377-2217(99)00407-5
- [42] Y. M. Wang, and K. S. Chin: Some alternative models for DEA cross-efficiency evaluation. *Int. J. Product. Econom.* 128 (2010), 1, 332–338. DOI:10.1016/j.ijpe.2010.07.032
- [43] Y. M. Wang and K. S. Chin: A neutral DEA model for cross-efficiency evaluation and its extension. *Expert Systems Appl.* 37 (2010), 5, 3666–3675. DOI:10.1016/j.eswa.2009.10.024
- [44] Y. M. Wang, K. S. Chin, and P. Jiang: Weight determination in the cross-efficiency evaluation. *Computers Industr. Engrg.* 61 (2011), 3, 497–502. DOI:10.1016/j.cie.2011.04.004
- [45] Y. M. Wang, K. S. Chin, and Y. Luo: Cross-efficiency evaluation based on ideal and anti-ideal decision making units. *Expert Systems Appl.* 38 (2011), 8, 10312–10319. DOI:10.1016/j.eswa.2011.02.116
- [46] Y. M. Wang, and Y. Luo: DEA efficiency assessment using ideal and anti-ideal decision making units. *Appl. Math. Comput.* 173 (2006), 2, 902–915. DOI:10.1016/j.amc.2005.04.023
- [47] J. Wu, L. Liang, and Y. Chen: DEA game cross-efficiency approach to Olympic rankings. *Omega* 37 (2009), 4, 909–918. DOI:10.1016/j.omega.2008.07.001
- [48] J. Wu, L. Liang, and F. Yang: Determination of the weights for the ultimate cross efficiency using Shapley value in cooperative game. *Expert Systems Appl.* 36 (2009), 1, 872–876. DOI:10.1016/j.eswa.2007.10.006
- [49] J. Wu, J. Sun, and L. Liang: Cross efficiency evaluation method based on weight-balanced data envelopment analysis model. *Computers Industr. Engrg.* 63 (2012), 2, 513–519. DOI:10.1016/j.cie.2012.04.017
- [50] J. Wu, J. Sun, Y. Zha, and L. Liang: Ranking approach of cross-efficiency based on improved TOPSIS technique. *J. Systems Engrg. Electron.* 22 (2011), 4, 604–608. DOI:10.3969/j.issn.1004-4132.2011.04.008
- [51] P. Zhou, B. Wang, and K. L. Poh: A survey of data envelopment analysis in energy and environmental studies. *Europ. J. Oper. Res.* 189 (2008), 1, 1–18. DOI:10.1016/j.ejor.2007.04.042

*Jiasen Sun, Research Center for Smarter Supply Chain, Soochow Think Tank & Business School, Soochow University, Suzhou 215021. P. R. China.*

*e-mail: jiasen@mail.ustc.edu.cn*

*Rui Yang, Business School, Soochow University, Suzhou 215021. P. R. China.*

*e-mail: ryang@suda.edu.cn*

*Xiang Ji, Corresponding Author. Business School, Central South University, Changsha 410012. P. R. China.*

*e-mail: signji@mail.ustc.edu.cn*

*Jie Wu, School of Management, University of Science and Technology of China, Hefei 230026. P. R. China.*

*e-mail: jacky012@mail.ustc.edu.cn*