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NONEMPTY INTERSECTION OF LONGEST PATHS IN A GRAPH WITH A SMALL MATCHING NUMBER

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Abstract. A maximum matching of a graph G is a matching of G with the largest number of edges. The matching number of a graph G, denoted by $\alpha'(G)$, is the number of edges in a maximum matching of G. In 1966, Gallai conjectured that all the longest paths of a connected graph have a common vertex. Although this conjecture has been disproved, finding some nice classes of graphs that support this conjecture is still very meaningful and interesting. In this short note, we prove that Gallai's conjecture is true for every connected graph G with $\alpha'(G) \leqslant 3$.

Keywords: longest path; matching number

MSC 2010: 05C38, 05C70, 05C75

1. Introduction

Graphs in this paper are simple (without loops or parallel edges), finite and undirected. Let G be a graph with the vertex set $V(G)$ and edge set $E(G)$. Let v be a vertex of $V(G)$. The *neighborhood* of v in G, denoted by $N_G(v)$, is the set of vertices in $V(G)$ which are adjacent to v. The *degree* of v in G, denoted by $d_G(v)$, equals $|N_G(v)|$. A *matching* in a graph is a set of pairwise nonadjacent edges. A *maximum matching* is a matching with the largest number of edges. The *matching number* of G, denoted by $\alpha'(G)$, is the number of edges in a maximum matching of G.

The research on the intersection of longest paths in a graph has a long history. In particular, Gallai [5] proposed the following conjecture in 1966.

Conjecture 1.1 (Gallai [5]). *If G is a connected graph, then all the longest paths of G have a common vertex.*

Three years later, Walther [10] disproved Gallai's conjecture by exhibiting a counterexample on 25 vertices. Up to now, the smallest counterexample to Gallai's conjecture is a graph on 12 vertices (see Figure 1), which was found by Walther [11] and Zamfirescu [13] independently. One may find that this graph is somewhat interesting: for each vertex v of it, there is a longest path not containing v in it. Therefore, all the longest paths share no common vertex.

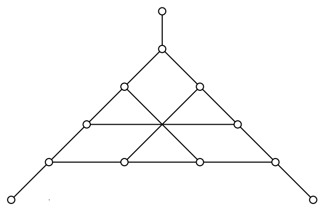

Figure 1. A counterexample to Gallai's conjecture on 12 vertices.

Since the answer to Gallai's conjecture is negative, it is natural to consider this problem by restricting the size of subsets of longest paths with a common vertex. First, it is an obvious fact that every two longest paths in a connected graph have a common vertex. (See also Exercise 2.2.13, page 52 in [2].) Zamfirescu [9], [13] asked the following challenging open problem: Do any three longest paths in a connected graph share a common vertex? This problem was presented as a conjecture in [6], and as an open problem in [2], [12]. For advancement on this problem, see [8] for details.

On the other hand, although Gallai's conjecture has been disproved, finding classes of graphs that support this conjecture is also very meaningful. An obvious such example is the class of trees. In 1990, Klavžar and Petkovšek [7] proved that Conjecture 1.1 holds on split graphs, and every connected graph such that each block is Hamiltonian-connected, almost Hamiltonian-connected or a cycle. As a corollary, Gallai's conjecture is true for the class of cacti. In 2004, Balister, Győri, Lehel, and Schelp [1] showed that circular arc graphs support Conjecture 1.1. In 2013, Rezende, Fernandes, Martin and Wakabayashi [3] proved that Conjecture 1.1 also holds on outer-planar graphs and 2-trees. In 2013, Chen, Ehrenmüller, Fernandes, Heise, Shan, Yang and Yates [4] furthermore proved that Gallai's conjecture is true for all series-parallel graphs (K_4-minor-free graphs). Since outer-planar graphs and 2-trees are K_4-minor-free, Chen et al. [4] extended Rezende et al.'s [3] results to a larger class of graphs.

Our main result is the following theorem.

Theorem 1.1. *If G is a connected graph with $\alpha'(G) \leqslant 3$, then all the longest paths of G have a common vertex.*

From Figure 1, we know that there is a counterexample to Gallai's conjecture such that its matching number is six. Thus, the following problem is proposed naturally.

Problem 1. Do all longest paths in a connected graph G with $\alpha'(G) \leqslant 5$ share a common vertex?

Note that if the answer to Problem 1 is yes, then the graph in Figure 1 is the smallest counterexample to Gallai's conjecture.

Furthermore, since each graph in Figure 2 has the matching number at most 3, and it has K_4 as a minor, our result is not included in the previous results given by Chen et al. [4].

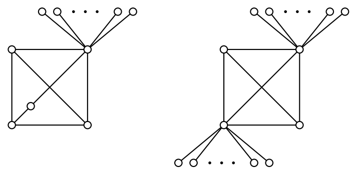

Figure 2.

Our proof of Theorem 1.1 will be given in the next section.

2. Proof of Theorem 1.1

We prove this theorem by contradiction. Let G be a counterexample. Since G is connected, if G has no cycle, then G is a tree, and therefore all the longest paths of G have a common vertex (a center vertex of G). So G has a cycle. Let $C = v_1v_2\dots v_rv_1$, $r \geqslant 3$, be a longest cycle of G, and $P = x_0x_1\dots x_s$ a longest path of G. We write $C[v_i, v_j]$ for the longer subpath of C between v_i and v_j, and $P[x_m, x_n]$ for the subpath of P between x_m and x_n, $1 \leqslant i, j \leqslant r$ and $0 \leqslant m, n \leqslant s$. Since $\alpha'(G) \leqslant 3$, we have that $r \leqslant 7$ and $s \leqslant 6$. If C is a Hamilton cycle, then every longest path of G is a Hamilton path, therefore all the longest paths have a common vertex. Thus C is not a Hamilton cycle. Let $R = G - V(C)$, and $u \in V(R)$.

Claim 2.1. $s \geqslant r$.

P r o o f. Since G is connected and C is not a Hamilton cycle, there is a vertex $y \in V(R)$ such that $yv_i \in E(G)$, where $v_i \in V(C)$. Then $yv_iv_{i+1}\dots v_{i-1}$ is a path of length r. Since P is a longest path of G, $s \geqslant r$. □

Claim 2.2. $|V(P) \cap V(C)| \geqslant 1$.

P r o o f. Suppose that $V(P) \cap V(C) = \emptyset$.

First we claim that $s \leqslant 4$. Assume that $s \geqslant 5$. Then x_0x_1, x_2x_3, x_4x_5, v_1v_2 are four independent edges, a contradiction. By Claim 2.1, we have $r \leqslant s \leqslant 4$. If $r = 4$, then v_1v_2, v_3v_4, x_1x_2, x_3x_4 are four independent edges, a contradiction. This implies that $r = 3$.

Since G is connected, there is a path P' connecting P and C. Suppose that the two end-vertices of P' are x_i and v_j. Now either $C[v_{j+1}, v_j]P'[v_j, x_i]P[x_i, x_s]$ or $C[v_{j+1}, v_j]P'[v_j, x_i]P[x_i, x_0]$ is a path of length at least 5, and longer than P, a contradiction. □

Claim 2.3. If there is a vertex $v \in V(G)$ such that $N_G(v) = \{v_1, v_2\}$, then every longest path of G containing v must also contain v_1 and v_2.

Proof. Let Q be a longest path such that $v \in V(Q)$. If v is an end-vertex of Q, without loss of generality supposing that v_2 is not on Q, then $Q \cup vv_2$ is a path longer than Q, a contradiction. Hence v is an internal vertex of Q. Since $d_G(v) = 2$, Q passes v_1 and v_2. □

By Claim 2.1 and $s \leqslant 6$, we have $r \leqslant 6$. Now we distinguish the following cases.

Case 1. $r = 3$.

Proof. By Claims 2.1 and 2.2, $s \geqslant 3$ and $|V(P) \cap V(C)| \geqslant 1$.

Claim 2.4. For any longest path Q of G, $|V(Q) \cap V(C)| = 1$ or $|V(Q) \cap V(C)| = 3$.

Proof. By Claim 2.2, $|V(Q) \cap V(C)| \geqslant 1$. If $|V(Q) \cap V(C)| = 2$, then without loss of generality, suppose that $v_1, v_2 \in V(Q)$. Now $v_1v_2 \in E(Q)$, since if $v_1v_2 \notin E(Q)$, then $v_1v_3v_2Q[v_2, v_1]v_1$ is a cycle longer than C, a contradiction. But now $(Q - v_1v_2) \cup v_1v_3v_2$ is a path longer than Q, a contradiction. □

Claim 2.5. If there is a longest path $Q = y_0y_1 \ldots y_s$ of G such that $|V(Q) \cap V(C)| = 1$, then for each vertex $v \in V(C) \setminus V(Q)$, $d_G(v) = 2$. Furthermore, all the longest paths of G share a common vertex.

Proof. First we claim that $s \leqslant 4$, since otherwise y_0y_1, y_2y_3, y_4y_5 and an edge in C are four independent edges, a contradiction. Furthermore, we have that $s = 4$. Otherwise, there will be a path longer than Q, a contradiction. Now $Q = y_0y_1y_2y_3y_4$. Since Q is the longest, $V(Q) \cap V(C) = \{y_2\}$. Without loss of generality, assume that $v_1 = y_2$.

Now we can check that for each $v \in \{v_2, v_3\}$, the following assertions hold:

(i) v is not adjacent to any vertex in $V(Q) \setminus y_2$, since $r = 3$;

(ii) v is not adjacent to any vertex in $V(G) \setminus (V(Q) \cup V(C))$, since if there is a vertex $z \in V(G) \setminus (V(Q) \cup V(C))$ such that $zv \in E(G)$, then either $\{y_0y_1, y_3y_4, y_2v_2, v_3z\}$ or $\{y_0y_1, y_3y_4, y_2v_3, v_2z\}$ is a set of independent edges of size four;

(iii) $d_G(v) = 2$, since (i) and (ii).

By Claim 2.3, every longest path of G containing v_2 (v_3) must also contain y_2 and v_3 (v_2). Therefore, all the longest paths of G contain y_2. □

Since G is a counterexample, by Claims 2.4 and 2.5, for every longest path Q of G, $V(C) \subset V(Q)$. But now all the longest paths of G contain $V(C)$, a contradiction. $\square$

Case 2. $r = 4$.

P r o o f. By Claims 2.1 and 2.2, $s \geqslant 4$ and $|V(P) \cap V(C)| \geqslant 1$.

Claim 2.6. For any longest path Q of G, $|V(Q) \cap V(C)| \geqslant 2$.

P r o o f. Let $Q = y_0 y_1 \dots y_s$ be a longest path of G such that $|V(Q) \cap V(C)| = 1$. Without loss of generality, assume that $V(Q) \cap V(C) = \{v_1\}$. If $s \geqslant 5$, then $\{y_0 y_1, y_2 y_3, y_4 y_5, v_2 v_3\}$ is a set of independent edges of size four, a contradiction. Thus $s \leqslant 4$. However, there is a path longer than Q, a contradiction. $\square$

Claim 2.7. For any two longest paths Q and Q' of G such that $|V(Q) \cap V(C)| = |V(Q') \cap V(C)| = 2$, we have $V(Q) \cap V(C) = V(Q') \cap V(C) = \{v_i, v_{i+2}\}$, $i \in \{1, 2\}$.

P r o o f. We claim that $\{v_1, v_2\}$ is not in $V(Q)$. Otherwise, if $v_1 v_2 \in E(Q)$, then $(Q - v_1 v_2) \cup v_1 v_4 v_3 v_2$ is a path longer than Q, a contradiction; if $v_1 v_2 \notin E(Q)$, then $Q[v_1, v_2] v_2 v_3 v_4 v_1$ is a cycle longer than C, a contradiction. Similarly, none of $\{v_2, v_3\}$, $\{v_3, v_4\}$, $\{v_4, v_1\}$ is contained in $V(Q)$. Since $|V(Q) \cap V(C)| = 2$, we have $V(Q) \cap V(C) = \{v_1, v_3\}$ or $V(Q) \cap V(C) = \{v_2, v_4\}$. Suppose that $V(Q) \cap V(C) = \{v_1, v_3\}$ and $V(Q') \cap V(C) = \{v_2, v_4\}$. Then $v_1 v_3$ cannot be an edge of Q, and $v_2 v_4$ cannot be an edge of Q', otherwise $(Q - v_1 v_3) \cup v_1 v_2 v_3$ or $(Q' - v_2 v_4) \cup v_2 v_3 v_4$ is a path longer than Q, a contradiction. Since the length of a longest cycle in G is 4, $v_1 w_1 v_3$ is a subpath of Q and $v_2 w_2 v_4$ is a subpath of Q', where $w_1, w_2 \in V(R)$. If $w_1 = w_2$, then $v_1 w_1 v_2 v_3 v_4 v_1$ is a cycle of length 5, a contradiction. If $w_1 \neq w_2$, then $v_1 w_1 v_3 v_2 w_2 v_4 v_1$ is a cycle of length 6, a contradiction. $\square$

Claim 2.8. If there is a longest path $Q = y_0 y_1 \dots y_s$ of G, such that $|V(Q) \cap V(C)| \leqslant 3$, then for each vertex $v \in V(C) \setminus V(Q)$, $d_G(v) = 2$. Furthermore, all the longest paths of G share a common vertex.

P r o o f. By Claim 2.6, $|V(Q) \cap V(C)| \geqslant 2$. If $|V(Q) \cap V(C)| = 2$, by Claim 2.7, set $V(Q) \cap V(C) = \{v_1, v_3\}$. If $|V(Q) \cap V(C)| = 3$, then exactly one of $\{v_1, v_2, v_3, v_4\}$ is not in $V(Q)$. Thus for each $v_i \in V(C) \setminus V(Q)$, v_{i-1}, $v_{i+1} \in V(Q)$, i is taken modulo 4. Since $r = 4$, $v_{i-1} w_i v_{i+1}$ is a subpath of Q in G, w_i may be a vertex of $V(C)$. Now v_{i-1} and v_{i+1} are not end-vertices of Q, since otherwise adding v_i to Q results in a longer path, a contradiction. Suppose that $v_{i-1} = y_k$, $v_{i+1} = y_j$, $0 < k < j < s$.

We can check that for each vertex $v_i \in V(C) \setminus V(Q)$, the following assertions hold:

(i) v_i is not adjacent to w_i, since otherwise $(Q - v_{i-1} w_i) \cup v_{i-1} v_i w_i$ is a path longer than Q;

(ii) v_i is not adjacent to any vertex in $Q[y_0, y_{k-1}] \cup Q[y_{j+1}, y_s]$, since otherwise if $w' \in Q[y_0, y_{k-1}] \cup Q[y_{j+1}, y_s]$ such that $v_i w' \in E(G)$, then either $v_i v_{i+1} Q[v_{i+1}, w'] w' v_i$ or $v_i v_{i-1} Q[v_{i-1}, w'] w' v_i$ is a cycle of length at least 5; and

(iii) v_i is not adjacent to any vertex in $V(G) \setminus (V(Q) \cup V(C))$, since if there exists a vertex $z \in V(G) \setminus (V(Q) \cup V(C))$ such that $zv_i \in E(G)$, then $s = 4$. Otherwise $\{y_0y_1, y_2y_3, y_4y_5, zv_i\}$ is a set of independent edges of size four, a contradiction. But now $zv_iv_{i-1}Q[v_{i-1}, v_{i+1}]v_{i+1}Q[v_{i+1}, y_s]$ is a path of length at least 5;

(iv) $d_G(v_i) = 2$, since (i), (ii) and (iii).

If $|V(Q) \cap V(C)| = 2$, by Claim 2.3, every longest path of G containing v_2 (v_4) must also contain v_1 and v_3. Therefore, all the longest paths of G contain v_1 and v_3.

If $|V(Q) \cap V(C)| = 3$, by Claim 2.3, every longest path of G containing v_i must also contain v_{i-1} and v_{i+1}. Therefore, all the longest paths of G contain v_{i-1} and v_{i+1}. □

Since G is a counterexample, by Claim 2.8, for every longest path Q of G we have $V(C) \subset V(Q)$. But now all the longest paths of G contain $V(C)$, a contradiction. □

Case 3. $r = 5$.

Proof. By Claims 2.1 and 2.2, we have that $s \geqslant 5$ and $|V(P) \cap V(C)| \geqslant 1$.

Claim 2.9. For any longest path Q of G, $|V(Q) \cap V(C)| \geqslant 3$.

Proof. Let $Q = y_0y_1 \ldots y_s$ be a longest path of G such that $|V(Q) \cap V(C)| \leqslant 2$. Then y_0y_1, y_2y_3, y_4y_5 and an edge in C are four independent edges, a contradiction. □

Claim 2.10. If there is a longest path $Q = y_0y_1 \ldots y_s$ of G such that $|V(Q) \cap V(C)| \leqslant 4$, then for each vertex $v \in V(C) \setminus V(Q)$, $d_G(v) = 2$. Furthermore, all the longest paths of G share a common vertex.

Proof. By Claim 2.9, $|V(Q) \cap V(C)| \geqslant 3$. Since $r = 5$, at least two vertices of $V(Q) \cap V(C)$ are adjacent in C. Without loss of generality, suppose that $v_1, v_2 \in V(Q)$. If $|V(Q) \cap V(C)| = 3$, then $v_3, v_5 \notin V(Q)$. Since if v_3 or $v_5 \in V(Q)$, hence $\{y_0y_1, y_2y_3, y_4y_5, v_4v_5\}$ or $\{y_0y_1, y_2y_3, y_4y_5, v_3v_4\}$ is a set of independent edges of size four, a contradiction. If $|V(Q) \cap V(C)| = 4$, then exactly one of $\{v_3, v_4, v_5\}$ is not in $V(Q)$. Thus for each $v_i \in V(C) \setminus V(Q)$, v_{i-1}, $v_{i+1} \in V(Q)$, $i \in \{3, 4, 5\}$. Since $r = 5$, $v_{i-1}w_iv_{i+1}$ or $v_{i-1}w_{1i}w_{2i}v_{i+1}$ is a subpath of Q, w_i, w_{1i}, w_{2i} may be vertices of $V(C)$. Now v_{i-1} and v_{i+1} are not end-vertices of Q, since otherwise adding v_i to Q results in a longer path, a contradiction. Suppose that $v_{i-1} = y_k$, $v_{i+1} = y_j$, $0 < k < j < s$.

We can check that for each vertex $v_i \in V(C) \setminus V(Q)$, the following assertions hold:

(i) v_i is not adjacent to w_i if $v_{i-1}w_iv_{i+1}$ is a subpath of Q or w_{i1}, w_{i2} if $v_{i-1}w_{i1}w_{i2}v_{i+1}$ is a subpath of Q since otherwise $(Q - v_{i-1}w_i) \cup v_{i-1}v_iw_i$ or $(Q - v_{i-1}w_{i1}) \cup v_{i-1}v_iw_{i1}$ or $(Q - v_{i+1}w_2) \cup v_{i+1}v_iw_{i2}$ is a path longer than Q;

(ii) v_i is not adjacent to any one of $\{y_{k-1}, y_{j+1}\}$ since otherwise $(Q - v_{i-1}y_{k-1}) \cup v_{i-1}v_iy_{k-1}$ or $(Q - v_{i+1}y_{j+1}) \cup v_{i+1}v_iy_{j+1}$ is a path longer than Q;

(iii) v_i is not adjacent to any vertex in $Q[y_0, y_{k-2}] \cup Q[y_{j+2}, y_s]$ since otherwise if $w' \in Q[y_0, y_{k-2}] \cup Q[y_{j+2}, y_s]$ such that $v_iw' \in E(G)$, then either $v_iv_{i+1}Q[v_{i+1}, w']w'v_i$ or $v_iv_{i-1}Q[v_{i-1}, w']w'v_i$ is a cycle of length at least 6; and

(iv) v_i is not adjacent to any vertex in $V(G) \setminus (V(Q) \cup V(C))$ since if there exists a vertex $z \in V(G) \setminus (V(Q) \cup V(C))$ such that $zv_i \in E(G)$, then $\{y_0y_1, y_2y_3, y_4y_5, zv_i\}$ is a set of independent edges of size four;

(v) $d_G(v_i) = 2$ since (i), (ii), (iii) and (iv).

If $|V(Q) \cap V(C)| = 3$, by Claim 2.3, every longest path of G containing v_3 (v_5) must also contain v_2 (v_1) and v_4. Therefore, all the longest paths of G contain v_4.

If $|V(Q) \cap V(C)| = 4$, by Claim 2.3, every longest path of G containing v_i must also contain v_{i-1} and v_{i+1}. Therefore, all the longest paths of G contain v_{i-1} and v_{i+1}. □

Since G is a counterexample, by Claim 2.10, for every longest path Q of G, $V(C) \subset V(Q)$. But now all the longest paths of G contain $V(C)$, a contradiction. □

Case 4. $r = 6$.

Proof. By Claim 2.1 and $s \leqslant 6$, we have $s = 6$. If there is an edge in $E(R)$, then together with three independent edges of C, there will be a set of independent edges of size four, a contradiction. Thus R is an independent set.

Let $x \in V(R)$. Suppose that $xv_1 \in E(G)$. For any vertex $y \in V(R) \setminus x$, if $yv_2 \in E(G)$, $yv_4 \in E(G)$ or $yv_6 \in E(G)$, there will be a set of independent edges of size four, a contradiction. Thus $N_C(R \setminus x) \subset \{v_1, v_3, v_5\}$. Furthermore, if $xv_2 \in E(G)$ or $xv_6 \in E(G)$, then $(C - v_1v_2) \cup v_1xv_2$ or $(C - v_1v_6) \cup v_1xv_6$ is a cycle longer than C, a contradiction. Thus $xv_2 \notin E(G)$ and $xv_6 \notin E(G)$. If $xv_4 \in E(G)$, then $V(R) = \{x\}$. Otherwise, we could find a set of independent edges of size four, a contradiction. Since $s = 6$ and $|V(G)| = |V(C)| + |V(R)| = 7$, every longest path of G is a hamilton path. Then all the longest paths of G have a common vertex, a contradiction. This implies that $xv_4 \notin E(G)$. Thus $N_C(R) \subset \{v_1, v_3, v_5\}$.

If all vertices in $V(R)$ are only adjacent to v_1, then noting that the length of a longest path in C is 5, every longest path of G contains a vertex of R. This implies that all the longest paths of G contain the vertex v_1. Since $s = 6$, every longest path of G must contain a vertex of R.

If one vertex of $V(R)$ is adjacent to v_3, then every longest path of G contains v_1 and v_3; otherwise, if there is a longest path Q of G not containing v_1, then Q contains at most 6 vertices, a contradiction. Furthermore, if one vertex of $V(R)$ is adjacent to v_5, then every longest path of G contains v_1, v_3 and v_5. Since in this situation, if $v_2v_4 \in E(G)$, $v_2v_6 \in E(G)$, or $v_4v_6 \in E(G)$, then $N_R(v_1) = N_R(v_3) = N_R(v_5) = \{x\}$; otherwise, there will be a path longer than P, a contradiction. But now, we could find a cycle longer than C, a contradiction. Thus, $v_2v_4, v_2v_6, v_4v_6 \notin E(G)$. Now, if there is a longest path Q of G not containing v_1, then Q contains at most 5 vertices, a contradiction. Thus, we have completed the proof. □

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References

[1] *P. N. Balister, E. Győri, J. Lehel, R. H. Schelp*: Longest paths in circular arc graphs. Comb. Probab. Comput. *13* (2004), 311–317.
[2] *J. A. Bondy, U. S. R. Murty*: Graph Theory. Graduate Texts in Mathematics 244, Springer, Berlin, 2008.
[3] *S. F. de Rezende, C. G. Fernandes, D. M. Martin, Y. Wakabayashi*: Intersecting longest paths. Discrete Math. *313* (2013), 1401–1408.
[4] *J. Ehrenmüller, C. G. Fernandes, C. G. Heise*: Nonempty intersection of longest paths in series-parallel graphs. Preprint 2013, arXiv:1310.1376v2.
[5] *T. Gallai*: Problem 4. Theory of Graphs (P. Erdős et al., eds.). Proceedings of the Colloquium on Graph Theory, held at Tihany, Hungary, 1966, Academic Press, New York; Akadémiai Kiadó, Budapest, 1968.
[6] *J. M. Harris, J. L. Hirst, M. J. Mossinghoff*: Combinatorics and Graph Theory. Undergraduate Texts in Mathematics, Springer, New York, 2008.
[7] *S. Klavžar, M. Petkovšek*: Graphs with nonempty intersection of longest paths. Ars Comb. *29* (1990), 43–52.
[8] *A. Shabbir, C. T. Zamfirescu, T. I. Zamfirescu*: Intersecting longest paths and longest cycles: A survey. Electron. J. Graph Theory Appl. (electronic only) *1* (2013), 56–76.
[9] *H.-J. Voss*: Cycles and Bridges in Graphs. Mathematics and Its Applications 49, East European Series, Kluwer Academic Publishers, Dordrecht; VEB Deutscher Verlag der Wissenschaften, Berlin, 1991.
[10] *H. Walther*: Über die Nichtexistenz eines Knotenpunktes, durch den alle längsten Wege eines Graphen gehen. J. Comb. Theory *6* (1969), 1–6. (In German.)
[11] *H. Walther, H.-J. Voss*: Über Kreise in Graphen. VEB Deutscher Verlag der Wissenschaften, Berlin, 1974. (In German.)
[12] *D. B. West*: Open Problems—Graph Theory and Combinatorics, Hitting All Longest Paths. http://www.math.uiuc.edu/˜west/openp/pathtran.html, accessed in January 2013.

[13] *T. Zamfirescu*: On longest paths and circuits in graphs. Math. Scand. *38* (1976), 211–239.

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